Quantity Discounts: An Overview and Practical Guide for Buyers and Sellers

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Dedication

Chuck dedicates this manuscript to his wife Kim and sons Christopher and Mark. This daunting project required many late nights. He thanks them for their patience, and he promises to make it up to them someday.

Contents

Abstract

Sellers commonly provide price discounts for large orders; in fact, quantity discounts have existed as ubiquitous tools of commerce for hundreds of years. In some industries today, quantity discounts are more common than not in business-to-business transactions. For example, large discount retailers demand price breaks from their suppliers, and they typically then pass on a portion of the savings to final consumers. Practitioners face the issue from two sides. For sellers, what form of quantity discounts makes the most sense, and how should the discounts be priced? What role should quantity discounts play in the larger contract negotiation scheme? For buyers, how many units should be ordered when faced with a quantity discount schedule? And under what conditions should a buyer take the lead and attempt to negotiate a discount schedule from its supplier?

In this monograph, the authors critically review the existing quantity discount literature — particularly emphasizing the differences between historical academic focus and practical applications, as well as identifying areas that are still under-developed. They provide, in one location, the equations and major solution algorithms for the most popular and relevant quantity discount scenarios from both the buyer's and seller's perspectives.

Highlights

- p. 78: "A properly designed quantity discount schedule can represent a win-win for all parties — increasing total supply chain profits by enticing decisions that produce efficiencies and then sharing the benefits with all supply chain members."

- p. 10: "Quantity discount offerings that are aggregated over either items or time (or both) remain the least studied forms at present."

- p. 77: "It is beginning to become apparent that the most practical and most applied buyer's perspective models in industry are math programming models for supplier selection and order allocation."

- p. 28: "Supply chain coordination has been the major thrust of the vast academic research from the seller's and joint perspectives; however, the message is not getting through to practice."

- p. 104: "With approximately 30% of sellers and buyers claiming to base discounts and order quantities on *intuition* [Jackson, 2015], business people still need to be properly educated."

1

Introduction and Historical Background

1.1 Setting the Stage

For as long as anyone can remember, commerce has presented the opportunity for buyers to receive a lower price for purchasing a large quantity. Even in the foregone days of bartering, one can imagine Farmer Jones paying fewer chickens per cow to Farmer Brown if he bought 10 cows. Today, consumers cannot walk into most retail stores without seeing at least some sort of quantity discount opportunity. Even fast-food restaurants encourage us to eat and drink more by "upsizing" orders for just a few coins more.

As consumers, we are bombarded with a plethora of quantity discount opportunities on a daily basis. Quantity discounts in business-to-consumer (B2C) transactions appear in a variety of forms, including the classic all-units discount (Figure 1.1), consumer products at the grocery store priced differently per unit for different package sizes, and even receiving lower health insurance premiums for buying as a member of a group. Some "price club" stores such as Costco and Sam's Club are specifically designed to appeal to consumers willing to buy in bulk to receive substantial savings. Some common marketing tools have the effect of a quantity discount as well, including frequent flyer plans

Figure 1.1: Pricing at a golf range in Pullman, Washington. (Reprinted with permission of Palouse Ridge Golf Course: www.palouseridge.com/rates, 6/6/14).

Figure 1.2: Common B2C quantity discount offerings. (Reprinted with permission of Safeway, Inc: www.weeklyspecials.safeway.com/customer_Frame.jsp? drpStoreID=2639, 6/11/14; and Higginson's Home Center and Sleep Shop, *The Daily Evergreen*, 1/16/15).

from airlines, punch cards at the coffee shop, gasoline discounts for in-store merchandise purchases (e.g., Safeway grocery stores), cash-back rewards from credit cards, one-day sales, "buy one get one free" offers, and free shipping for large orders (Figure 1.2). Recently, firms such as Groupon allow consumers to form short-term "purchasing partnerships" to achieve a lower unit price on a variety of goods and services as long as the order quantity exceeds a minimum threshold.

Further up the supply chain, we see quantity discounts permeating business-to-business (B2B) transactions as well. Many, if not most,

companies receive a quantity discount for at least some of their purchases while extending a quantity discount to at least some of their customers. Mars, Inc. uses an auction format for supplier selection. Each of its suppliers submits a bid for annual contracts that often includes volume discounts for a single product or business volume discounts for a bundle of products [Hohner et al., 2003]. Mobile phone operators in Europe receive volume discounts from telecommunication carriers based on the number of international calls placed through a particular carrier [van de Klundert et al., 2005]. More generally, large retailers and manufacturers, such as Wal-Mart and Boeing, respectively, demand discounts from their suppliers based on their large volume of purchased products. In 2010, Wal-Mart tried to boost its negotiating power by partnering with PepsiCo to jointly purchase potatoes from the same supplier to achieve a larger quantity discount [Boyle and Wolf, 2010]. Meanwhile, a variety of large manufacturers and retailers have programs in place to offer volume discounts to external companies, schools, and non-profits. Examples of these programs include Apple's Volume Purchase Program[1] and Costco's Volume/Bulk Sales program (Figure 1.3), which appears to provide customized discounting for volume purchases.

The omnipresence of quantity discounts points to the importance of addressing questions such as the following. For sellers, what form of quantity discounts makes the most sense, and how should the discounts be priced? Which discounts appear to be most effective? Why are their competitors offering quantity discounts, and should they follow suit? What role should quantity discounts play in the larger contract negotiation scheme? For buyers, how many units should be ordered when faced with a quantity discount schedule? What will this "good deal" on paper do to their inventory position and operating costs? Should they join a group purchasing organization to try to lower purchase prices? Under what conditions should a buyer take the lead and attempt to negotiate a discount schedule from its supplier? Finally, should sellers and buyers share information and work together to generate the

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[1]http://www.apple.com/business/vpp/.

Figure 1.3: Volume sales at Costco.com. (Reprinted with permission of Costco Wholesale Coporation: www.costco.com/volume-bulk-sales.html, 6/11/14).

most efficient ways to move product through the supply chain? Can coordination really help?

Several hundred operations management academic papers on quantity discounts have addressed these questions and more from the buyer's perspective, the seller's perspective, or a joint perspective. For buyers, the standard research approach introduces a plausible quantity discount scenario and then determines the best order quantity. Seller's perspective papers typically attempt to prescribe the size of discount that should be offered to increase demand, minimize logistics costs, or both. Joint perspective papers often focus on creating lot sizes for the parties that minimize total supply chain costs. Quantity discounts are then established to encourage the parties to purchase the appropriate quantities and to share the supply chain savings amongst the members.

Since the 1950s, the subject has been an important research topic appearing in the economics and marketing literature as well. While the operations models tend to be more cost-based, the economics and

marketing models tend to be more utility-based by viewing quantity discounts more as a way to alter underlying buyer preferences about the product. Many models demonstrate how firms can use quantity discounts to price-discriminate among customers. The operations literature tends to be mostly B2B-focused, whereas much of the economics and marketing literature is B2C-focused.

In the operations field, much of the core work for both determining optimal order sizes for buyers and for using quantity discounts for coordinating lot sizes in supply chains had been developed by the end of the last century. Several literature review papers address much of this work in various ways. Several examples are Aissaoui et al. [2007], Benton and Park [1996], Dolan [1987], Goyal and Gupta [1989], Munson and Rosenblatt [1998], and Sarmah et al. [2006].

The current century has witnessed a steady flow of papers continuing to appear. The research area remains vibrant with at least 10 quality publications appearing annually in the operations management field alone. Some provide new iterations for old problems, while others address emerging ideas related to quantity discounts. This monograph covers many of the papers that have been written in the last 15 years. Including results from a recent survey of practicing managers, we attempt to illuminate differences and similarities between historical academic focus and practical applications, as well as to identify areas that are still under-developed.

1.2 Aim and structure of this monograph

Munson and Rosenblatt [1998] conducted personal interviews with 39 executives to ascertain their experiences with quantity discounts either as buyers or sellers. Meanwhile, the authors reviewed the quantity discount literature and then attempted to identify gaps in the literature based on findings from the qualitative interviews. Since that time, researchers have continued to explore quantity discount issues, including some of the gaps identified in the article. This monograph updates and builds upon Munson and Rosenblatt [1998]. We update the literature and reports result from a new survey of practicing managers.

Space constraints prohibit us from citing all of the related literature. We include some samples from economics and marketing but focus primarily on papers from operations management. We offer ways to bridge remaining gaps between research and practice. In addition, we include formulas and solution methods for some of the fundamental quantity discount scenarios from both buyers' and sellers' perspectives. Sample solution methods provide enough detail to be able to be implemented by managers, and they provide a flavor of the variety and complexity of some of the core techniques appearing in the literature.

The rest of the monograph is organized as follows. Section 2 describes quantity discount conditions and realities, including the many reasons why companies offer them, the various quantity discount characteristics, results from surveys of practicing mangers, and a sampling of real-world discounts appearing in the literature. Section 3 provides a literature review for buyer's perspective models, while Section 4 provides a review for seller's perspective models. Section 5 concludes with several recommendations for future research. Finally, especially designed for instructors and practitioners, the Appendix illustrates how to implement several standard quantity discount models into Microsoft Excel with relative ease.

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