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# Modeling, Prediction, Assortment and Price Optimization Under Consumer Choice Behavior

#### Chenxu Ke

Shanghai University of Finance and Economics kechenxu@mail.shufe.edu.cn

Anran Li

CUHK Business School anranli@cuhk.edu.hk

#### **Ruxian Wang**

Johns Hopkins University ruxian.wang@jhu.edu



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## Modeling, Prediction, Assortment and Price Optimization Under Consumer Choice Behavior

Chenxu Ke<sup>1</sup>, Anran Li<sup>2</sup> and Ruxian Wang<sup>3</sup>

<sup>1</sup>College of Business, Shanghai University of Finance and Economics, China; kechenxu@mail.shufe.edu.cn
<sup>2</sup>Department of Decisions, Operations and Technology, CUHK Business School, Hong Kong; anranli@cuhk.edu.hk
<sup>3</sup>Carey Business School, Johns Hopkins University, USA; ruxian.wang@jhu.edu

#### ABSTRACT

Understanding how consumers make choices is of paramount importance, as it offers insights into consumer purchase behavior across multiple products, enables accurate predictions of future demand, and informs strategic planning and policy formulation. The examination of discrete consumer choice models plays a central role in decoding the decision-making process, offering a clear perspective on how individuals navigate among multiple options. These models are instrumental in evaluating a wide range of consumer decisions, such as product selection, brand preference, and the impact of various factors on choice. With the growth of e-commerce and the increasing emphasis on data-driven modeling and decision-making, consumer choice models have garnered significant attention. This rising interest underscores their relevance in the digital marketplace and their contribution to

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a deeper understanding of consumer behavior. The objective of this work is to present a comprehensive overview of choice modeling, covering both the theoretical underpinnings of widely adopted discrete choice models (e.g., the multinomial logit model), and those integrating contemporary elements like network externalities and ranking effects. It also discusses optimal solutions or efficient approximation heuristics for price and assortment optimization problems, where consumer choice behavior is governed by various discrete choice models. To facilitate practical business applications, this work offers estimation strategies and techniques to address data-related issues. Additionally, it includes cutting-edge advancements such as artificial intelligence and deep learning, and outlines future trends in the realm of operations management with discrete choice models. By delving into the intricate details and mechanisms of these models, this work aims to shed light on the methodological foundations and practical implementations of consumer choice modeling. providing researchers, practitioners, and policymakers with valuable insights into this rich and evolving field.

## 1

#### Introduction

Discrete consumer choice models are an important aspect of decisionmaking, as they help us understand how people make choices when faced with multiple alternatives. These models can be used to assess various consumer decisions, such as product choice, brand preference, and the influence of various factors on consumer decision-making. They have far-reaching implications for a variety of fields, including economics, marketing, behavioral science, transportation, and operations. Consumer choice models have received considerable interest in recent times due to the growing prevalence of e-commerce and the increasing emphasis on data-driven modeling and decision-making in academia. Researchers are exploring the effectiveness of various consumer choice models in predicting consumer behavior, such as the Random Utility Models, and are also examining the impact of situational or contextual factors, such as network externality and the presence of others, on consumer choice behavior and firms' operational strategy.

The objective of this review paper is to present a comprehensive overview of the theoretical framework, model estimation techniques, challenges, recent advances, and future directions in the area of discrete consumer choice modeling. By dissecting these models and the me-

#### Introduction

chanics behind them, this paper aims to illuminate the methodological underpinnings and practical applications of discrete consumer choice modeling, offering researchers, practitioners, and policymakers valuable insights into this rich and evolving field.

This work is organized as follows. In the rest of this section, we provide a historical overview of the evolution of discrete consumer choice modeling and highlight key studies and models in this field. Section 2delves into the theoretical framework of the most important discrete choice model-the Multinomial Logit (MNL) choice model. Although criticized by its Independence of Irrelevant Alternative (IIA) property, MNL remains the most popular choice for modeling consumer behavior. Section 3 covers interesting extensions built upon the classic MNL model. Section 4 presents other discrete choice models including the Nested Logit and Mixed Logit models. Section 5 focuses on the pricing problems under the discrete choice models. Section 6 identifies and discusses the challenges of the assortment optimization problems under various consumer choice models. Section 7 focuses on model estimation techniques, such as maximum likelihood estimation and expectationmaximization (EM) algorithm. It also explores the recent advances in discrete consumer choice modeling, including the integration of artificial intelligence and machine learning. Finally, Section 8 points out opportunities for further research and then concludes the work with a summary of the key points and concluding remarks.

#### Historical Note on the Discrete Choice Model

The origins of discrete choice modeling can be traced back to the mid-20th century. Early developments in consumer theory were central in proposing that consumers' preferences could be inferred from their purchasing habits. Samuelson (1948) developed the concept of "revealed preference". This was a major step forward because it provided a way to model and predict consumer behavior based on observable data.

In the 1950s and 1960s, the field of psychology offered substantial contributions to the nascent theory of choice modeling. Notably, Thurstone (1927) proposed the Law of Comparative Judgment, which provided a mathematical expression for how individuals make choices.

He suggested that individuals' decisions are the result of a random utility, an innovative concept at the time.

During the same period, Luce (1959b) developed a probabilistic choice model, stating that the probability of choosing an item is a function of its relative attractiveness compared to other available alternatives. This was a critical development as it introduced the concept of relative decision-making, which forms the basis for modern discrete choice models.

However, it was in the 1970s that the Random Utility Maximization (RUM) model, as we know it today, was developed. Under the RUM, the decision makers select the alternative that yields the maximum realized consumption utility. McFadden (1974) made significant contributions by developing the Multinomial Logit (MNL) Model, a specific case of RUM, where the random components of utility are independent and identically distributed (i.i.d.) following a Gumbel distribution. The MNL model transformed choice behavior analysis and earned McFadden the Nobel Prize in Economic Sciences in 2000.

Since then, RUM has continued to evolve with researchers developing numerous variations and extensions to deal with its limitations and better align the model with observed behaviors. These include nested logit, mixed logit, and probit models, among others. Today, RUM models remain a key tool in the fields of economics, marketing, transportation, health economics, operations and more. The focus of this review paper is on introducing the modeling, estimation and optimization techniques under the RUM framework. In the next section, we will discuss the most classic RUM model—the MNL in more details.

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