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The Economics of Tipping Points: Some Recent Modeling and Experimental Advances*

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ABSTRACT

This paper provides a review of the economics of tipping points in natural resources and climate change economics, examining recent advances in theoretical modeling and controlled experiments. We begin with the non-convexity models as a theoretical foundation, provide a typology of the resulting deterministic tipping points, and discuss their implications for management. Then, we focus on hazard rate modeling for optimal resource management with stochastic and unknown tipping points. We discuss Bayesian learning, strategic behavior among agents, and the advancement in integrated assessment modeling with multiple and interacting tipping points. Finally, we examine the new contributions of experimental economics to understanding

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decision-making processes in the presence of tipping points. The paper concludes by highlighting the main advances in the literature and outlining future research directions, ultimately aiming to encourage further investigation and the development of innovative tools to address global challenges.

Keywords: Climate change; experimental economics; hazard rate modeling; natural resources; regime shifts; tipping points

JEL Codes: C92, D81, Q01, Q28 Q54, Q56, Q57

1 Introduction

Managing pollution and natural resources in ecosystems is a challenging task as ecosystems may experience large, abrupt, and persistent changes, which can sometimes be unexpected. These changes, known as *regime shifts*, are set in motion when the system reaches a *tipping point*[1] (Scheffer and Carpenter, 2003; Scheffer *et al.*, 2001). For example, when the nutrient level in a shallow lake rises slowly due to external activities, it might eventually hit a tipping point that leads to an alteration in nutrient dynamics, transforming the lake from a clear regime to a turbid one. In a broader perspective, Lenton *et al.* (2008) identified *tipping elements* within the Earth's system, such as the Greenland ice sheets and the Amazon rainforests, each with unique tipping points.

Economics becomes relevant when weighing the trade-offs between potential benefits or costs of change-driving variables and the economic implications of regime shifts (de Zeeuw and Li, 2016). Using the shallow lake example, while clear lakes offer recreation and fishing, they might also serve agricultural wastewater disposal purposes. Yet, these benefits come with the caveat of possibly transitioning the lake to a murkier state. Management theories, grounded in conventional dynamic optimization, can sometimes miss the mark, presuming unique optimal solutions (Levin *et al.*, 2013; Starrett, 1972). This risks oversight of multiple potential outcomes. However, recent developments in the economics of tipping points address these challenges, embracing such *non-convexities*

[1] In this paper, we use the terms "tipping point" and "threshold" interchangeably.

in economic systems, which introduce intricate dynamics (Starrett, 1972).

Regime shifts do not always follow a linear path. In certain cases, even if the initial alterations are reversed, the original state might remain elusive unless one surpasses the original tipping point — a process termed as *hysteresis*. Some shifts are permanent, a situation dubbed *irreversibility*, while others might arise from inherent system discontinuities or factors such as depensation, where a species' survival hinges on maintaining a minimum population. Over time, human activities can compromise an ecosystem's resilience, making it more vulnerable to regime shifts. Recognizing and managing these shifts is thus paramount for optimal resource and environmental stewardship.

The idea of tipping points traces its roots back to chemistry and mathematics. Observations by Hoadley (1884) emphasized that tiny changes in reactants could trigger swift, irreversible reactions. Similarly, Poincaré (1885) showed how extremely small changes in initial conditions can induce unpredictable system behaviors, highlighting the importance of critical thresholds. In the economic context, tipping point concepts have advanced notably since the late twentieth century. Economic events, such as fads, technological adoptions, and financial bubbles, underscore the dynamics of tipping points. Schelling (1971) developed models capturing the rapid adoption of technologies or the proliferation of new consumer preferences. The world of financial economics, particularly during episodes like the dot-com or housing market bubbles, showcases tipping points where investor sentiments dramatically shift (Sornette, 2003).

In the recent literature, the domain of tipping point research in economics has been rapidly expanding, with scholars constructing diverse models to study deterministic and uncertain tipping points. Some research analyzes how tipping points influence optimal decision-making, while others employ experiments to delve into human behavior. This paper narrows its focus to resource, environmental, and climate economics. We provide an interpretive review of models spanning various tipping points, melding theoretical frameworks with game theory insights (Section 2). We analyze modeling strategies for stochastic or uncertain tipping points, utilizing hazard rate functions or endogenous discount rates (Section 3). Section 4 reviews advancements in integrated assessment modeling, emphasizing the application of theory through rigorous economic and climate modeling alongside numerical optimization methods. Section 5 explores the myriad behavioral responses to tipping points, gleaning insights from experimental research and discussing game theoretical applications. We conclude in Section 6, synthesizing our discussions and highlighting avenues for future research.

2 Non-Convexities and Deterministic Tipping Points

Here, we review the deterministic modeling approaches that incorporate the possibility of multiple equilibria. Scholars in economic growth theory (Skiba, 1978) and physics (Haken, 2012) showed that it is possible to apply dynamic optimization and optimal control theory in such contexts, giving hope for improved management solutions. We focus here on developments following the seminal works by Nævdal (2001), Mäler *et al.* (2003), Brock and Starrett (2003), and Wagener (2003). While this has partly been reviewed in de Zeeuw (2014), Crépin and Folke (2015), Li *et al.* (2018), Gromov and Upmann (2021), and Long (2021), we include here the most recent findings. We first describe the basic models of natural resources and pollution management with thresholds and develop a typology of the ways in which these types of problems have been modeled. Then we discuss the implications of tipping points for management and policy design.

2.1 The Basic Models of Resource Management with Thresholds

Most of the economic literature on these issues builds on dynamic optimal management problems that are structured as one objective consisting of a function that represents the aggregated sum of future values derived from managing the resource (1), and some dynamic conditions restricting available options (see e.g., Table 1). We focus on cases where a threshold appears due to non-convexity in the restriction. In the basic models, the social planner maximizes the aggregated discounted present value function over time (1) subject to some constraints:

$$V\left(x_0\right) = \int_0^\infty B\left(q\left(t\right), x\left(t\right)\right) e^{-\rho t} dt \tag{1}$$

where ρ is the discount rate, q the control, which could be a harvest or a release of pollution, and x is the stock variable, which could be a stock of resource or pollution. The function $B(q(t), x(t))$ represents the value of the benefits derived from the ecosystem in each period of time, t. While tipping points and threshold phenomena may result from particular forms of objective functions or specific interactions between multiple linear dynamics (Lade *et al.*, 2013; Richter *et al.*, 2013), we focus here only on cases where the threshold dynamics stem from non-convexity in one of the constraints.

Many management problems with tipping points can be modeled using some variation of one of the dynamic restrictions in Table 1. These represent most of the commonly used ways of modeling the dynamics of problems with non-convexities, depending on how the threshold term is incorporated.

Many phenomena can be modeled using one dynamic equation exhibiting non-convexity, but others require a bit more complexity to encompass the full scale of dynamics that matter for the development of the system. For example, many problems involve several resources interacting with each other. At least three categories can be identified in the literature and we give a sample of those here.

Interactions of multiple variables are modeled using linked dynamic equations. In situations when at least one of these variables exhibits non-convexities, the system as a whole could have tipping points. For example, Crépin (2007) investigated the interactions between coral, fish,

Table 1: General ways of modeling regime shifts in one dimension in different types of systems. x denotes a stock variable, t time, u external loading, δ rate of loss, v maximum rate of the internal loading, z threshold value, r intrinsic growth rate, K carrying capacity and α the sharpness of the shift between the low and high level of some critical variable. Resource growth could also be modeled with a sharp threshold like for pollution recipients for large enough values of α.

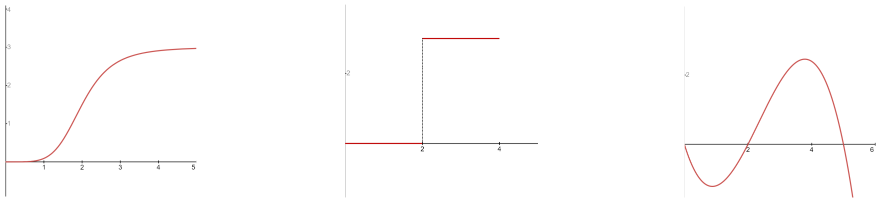
System of focus	Equation	References
Pollution in recipients (e.g., Lakes)	$\frac{dx}{dt} = u - \delta x + v\frac{x^\alpha}{z^\alpha + x^\alpha}$	Mäler *et al.* (2003); Wagener (2003)
Pollution in recipients with sharp thresholds (e.g., Climate, water bodies)	$\frac{dx}{dt} = u - \delta x$ if $t \leq \tau$ $\frac{dx}{dt} = u - \delta x + \beta$ if $t > \tau$	Crépin and Nævdal (2020); Nævdal (2001)
Resource growth (e.g., Fisheries)	$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - v\frac{x^\alpha}{z^\alpha + x^\alpha}$	Crépin (2007)
Resource growth with depensation	$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)\left(\frac{x}{z} - 1\right)$	Clark (1974); Crépin (2003)

and algae in a model of coral reefs. Investigating fish dynamics using only one equation is enough to replicate tipping dynamics in the model. However, the three-species model offers more nuances and allows us to also encompass that species interactions can influence the location of the threshold. Hence, ignoring them risks underestimating the proximity of the threshold and thus triggering an unwanted regime shift. In a forest where birch, spruce, and moose interact with each other and spruce exhibits depensation dynamics, Crépin (2003) illustrated the important role of moose in influencing whether the system will exist with an abundant spruce cover or whether the landscape will be much more open with abundant moose instead.

The literature has also put a focus on differences in the time scales of interactions. Even if the fast part of the system exhibits threshold dynamics, tipping may not be triggered until some slow variable passes a threshold that triggers regime shifts in the fast variable dynamics. For example, some coral reefs reach such a low stock level that species get less protection from predators (Crépin, 2007) or the climate becomes so warm that many ecosystems exhibit enough stress to tip to another regime (Crépin and Nævdal, 2020).

Just like changes in slow variables, spatial connectivity also has the potential to trigger a regime shift in a system that seemed relatively robust to change. Previous research illustrates the complex role of system connections in that respect, using case study analysis (Kinzig *et al.*, 2006), empirical data analysis and network modeling (Rocha *et al.*, 2018), and generalized modeling (Wunderling *et al.*, 2022). Crépin and Rocha (2021) illustrate how spatial connectivity can either trigger regime shifts or prevent them also in managed systems in pollution recipients and ecosystems producing natural resources.

Surprisingly, few ways to model dynamics exhibiting thresholds have been used in the literature (Crépin and Folke, 2015; Gromov and Upmann, 2021; Li *et al.*, 2018). These fall into two main categories depending on whether the thresholds occur as a result of internal system dynamics with non-convexities (see, e.g., Mäler *et al.*, 2003) or whether it happens because there is a jump in some variable in the restriction (see, e.g., Nævdal, 2001). Often the models rely on similar functional forms to introduce a threshold even if the systems studied are very different.



(a) Holling type III: $b\frac{x^\alpha}{z^\alpha+x^\alpha}$

(b) Sharp threshold: 0 if $x \leq z; b$ if $x > z$

(c) Depensation: $rx\left(1-\frac{x}{K}\right)\left(\frac{x}{z}-1\right)$

Figure 1: Different ways to model thresholds. Here $b = 3$ represents some maximum value of the state variable x, $z = 2$ some threshold value, $\alpha = 5$ the sharpness of the shifts, $r = 3$ the intrinsic growth rate, and $K = 5$ the carrying capacity. Figures were drawn using arbitrary parameter values in Desmos graphic calculator https://www.desmos.com/calculator and PowerPoint.

Figure 1 illustrates the main ways to model thresholds that we found in the literature. Non-convexities in system dynamics are often modeled using either S-shaped or sigmoid functions (Brock and Starrett, 2003) to mimic the impacts of variables that accumulate in a system. Below some inflection point, impacts are low. Above the inflection point, impacts are relatively high and asymptotically approaching some maximum value. The presence of such a functional relationship does not necessarily imply that a threshold will occur, but it can lead to threshold behavior in many systems for some ranges of parameter values. The inflection point represents then a critical threshold around which substantial change occurs.

Several functional forms are used in the literature to mimic this type of dynamics. A common function is the Holling type III functional form (Figure 1a). Originally designed to represent predation, it has been extensively used to model many types of resource or pollution dynamics with a threshold. Here b can represent the maximum level of some accumulation process and z the value of the stock at the threshold when this process switches from low to high level.

The Holling type III term can incorporate a broad range of steepness of a shift between high and low levels thanks to the parameter α. It can range from a very smooth transition around the threshold z for low values of α such as in Mäler *et al.* (2003) all the way to an abrupt jump in x variable when α goes to infinity as illustrated in Figure 1b. This corresponds to the type of thresholds modeled in Nævdal (2001). Other S-shape functions such as variations of the sigmoid function have also been used, for example, in climate models. However, it is not always as easy to vary the steepness of the transition between low and high levels of stock and the value of growth at zero stock is typically not zero.

If instead, some species need some minimum number of individuals to mate, their growth can exhibit depensation, as illustrated in Figure 1c. In such a situation, r denotes the intrinsic growth rate, K is the carrying capacity of the environment in which the population lives, and z the critical mass needed for the population to actually start growing. Note that this equation reaches zero growth for population values of x equal to 0, K or z. Examples of depensation models include Clark (1974) and Crépin (2003). Depensation implies a tipping point below which the stock may collapse to zero. In contrast, the other ways of representing thresholds illustrated in Figure 1 introduce the possibility of multiple equilibria with positive stock levels.

2.2 Management and Policies with Regime Shifts

The standard toolbox for managing pollution and resources from ecosystems often routinely assumed away the possibility of regime shifts. In contrast to these "well-behaved" resource management problems, the optimal solution of resource management models with a continuous dynamic restriction exhibiting non-convexities (see rows 1, 3 and 4 of Table 1) can be qualitatively dependent on parameters, and past events (Brock and Starrett, 2003; Crépin, 2003; Kiseleva and Wagener, 2010; Mäler *et al.*, 2003; Wagener, 2003). Kiseleva and Wagener (2010) and Wagener (2020) identified three types of optimal solution structures. Only one of them implies steering the system toward one equilibrium, no matter its earlier state. The two other solutions exhibit history dependence in the sense that it is optimal to steer the system to one of two possible equilibria depending on the initial conditions. In the special cases when the initial condition is at an indifference point, then the social planner is free to decide which equilibrium to target, while if the initial condition is the threshold pollution level and it happens to be a repelling equilibrium, the optimal policy will remain there as long as it is not perturbed.

There is a trade-off between social preferences illustrated by the social costs of pollution and economic factors represented by the discount factor. For example, if the relative cost of pollution decreases, the social planner would have to become less myopic in order for the same equilibrium outcome to remain optimal. Already in 1974, Clark showed that if and only if a renewable resource exhibited depensation dynamics, then this would create a discontinuity in the yield-effort situation. If effort exceeds a critical level, the population will collapse, and the change may even be irreversible.

In problems with discontinuities in the dynamic restriction, the optimal solution looks a bit different. For example, Nævdal (2001) studied a similar problem to the one studied by Mäler *et al.* (2003) but in his setup, the lake dynamics could jump in a discontinuous way between clear and murky regimes. He identified six different control situations. Most of these correspond to the optimal outcomes identified by Brock and Starrett (2003), Mäler *et al.* (2003), Wagener (2003), and Kiseleva and Wagener (2010). In two cases, the system could remain on the same side of the threshold (below or above) throughout the whole period. These two cases being straightforward were not analyzed further in the paper. Two other cases implied crossing the threshold once only, either from below if the system started with a low nutrient and eutrophying the lake was optimal or from above if the system started with murky water and reaching a clear lake was optimal. These situations also correspond to those identified in the continuous case, where it may be optimal to cross the ecological threshold.

The final case is one when it is optimal to approach the threshold and cross it, over and over, for an infinite number of times. Such a case does not exist with a continuous constraint. However, let us compare it to the case when the Skiba point and the threshold coincide in the continuous case. An equilibrium exists in the continuous case, while it does not exist in the discontinuous case so if dynamics are attracted toward it, it becomes a chattering control: on each side of the threshold, there are gains to be made from jumping to the other side so it is optimal to jump back and forth. In contrast, in the continuous case, the equilibrium/threshold could be an indifference point, in which case the system will not stay there or it could be an unstable threshold point, in which case the system stays there provided no perturbation occurs (Kiseleva and Wagener, 2010). The major difference is that those dynamics outside of this threshold are attractive in the discontinuous case, while they are repelling in the continuous case.

No matter how the threshold is modeled, resource management problems with tipping points tend to require other policy approaches than those without and other ways to evaluate them. Policies adjusting the price of a resource or pollution may not succeed in steering toward efficient resource allocation with regime shifts, quantity regulations could be more appropriate (Arrow *et al.*, 2003; Crépin *et al.*, 2012; Dasgupta, 2021; Dasgupta and Mäler, 2003; Weitzman, 1974). The mere possibility of tipping points implies the need for different approaches to evaluating policy reforms. Indices tailored to such purposes would need, for example, to represent useful measurements for evaluations even outside of equilibrium given that most systems would not be in equilibrium most of the time.

Many sub-optimal outcomes occur because people do not cooperate or coordinate their actions when they interact with each other strategically. For example, people collectively managing a lake would likely trigger a regime shift because each of them would have incentives to pollute the lake more than their optimal share of pollution if the social optimum prevailed. The more people, the higher the risk of non-cooperation (Mäler *et al.*, 2003). Building on Mäler *et al.* (2003) and Kiseleva and Wagener (2010), Wagener (2015) provided an overview of common pool resource games in the lake model. It includes both open-loop game solutions where the players decide their strategy once and for all in the first period (Mäler *et al.*, 2003) and closed-loop solutions when the players update their strategy in each period (Kossioris *et al.*, 2008) as well as comparisons of different tax schemes to correct the market imperfections in this context (Kossioris *et al.*, 2011). Mäler *et al.* (2003) compared constant and state-dependent taxes, while Kossioris *et al.* (2011) compared constant, linear, quadratic, and cubic taxes and showed that none of them was perfect and the marginal benefit of a tax scheme was decreasing with increasing tax complexity.

In a context, where grasslands are shared in common property for grazing animals, strategic interactions between cattle owners could lead either to over-exploitation, or to people being too careful to avoid the regime shifts (Crépin and Lindahl, 2009). In both cases, the market failure could lead to an equilibrium in the same regime as in the optimal solutions if the failure was not too severe. However, for severe failures, alternate regime outcomes could emerge, either to an overgrazed grassland if it was optimal to keep the land with high grass or an under grazed grassland if it was optimal to over-exploit the grassland.

Tipping points can also contribute to the easier formation of a coalition that lead to cooperation, because they increase the incentives to keep the system in the desired regime. This has been shown, for example, for climate negotiations (Wagener and de Zeeuw, 2021). Hence forming a coalition could be a valid strategy to shift a system from an unwanted regime to a desired one. However, once the system has transitioned, it is also necessary to form a new stable coalition to prevent shifting back.

Sometimes tipping points do not necessarily influence policy outcomes. For example, in eco-evolutionary games (Tilman *et al.*, 2020), the integrated strategic and environmental dynamics depend on incentives to lead or follow behavior and on the relative speed of environmental and strategic change, no matter whether there are tipping points, or how they are distributed.

So how do tipping points and regime shifts influence policy design? An important aspect of that question relates to the best strategy to recover to a desirable regime would the system have passed a tipping point and be on its way to reaching an unwanted regime. Heijdra and Heijnen (2013) showed, for a shallow lake with a tipping point linked to an industry with capital stock, that the best approach would be to administer a policy in two steps. In a socially optimal benchmark, a government could optimally allocate investment. In contrast, in the competitive equilibrium, the government could help mitigate pollution from the industry by forcing it to abate emissions. Such a policy would aim to push the system back to the desirable regime as fast as possible, even if that was linked to heavy costs. Once the shift has happened the second part of the policy would consist of a more standard policy aiming at correcting the market imperfections so that the system would remain in the desired state. They showed later that abatement would be more efficient than a tax in this context (Heijdra and Heijnen, 2014).

In the context of amenity-led growth, Chen *et al.* (2012) showed the need to take into account behavioral responses to change in ecosystem services derived from a lake exhibiting a eutrophication threshold. Policies that ignore the feedback between urbanization and water quality might unintentionally generate boom-bust cycles of regional growth and decline, with a long-term trend toward economic decline. A policy accounting for this feedback would instead foster a balanced regional growth maintaining essential ecosystem services.

In general, complex system interactions involving potential regime shifts in some parts of the system require careful investigations to identify potential outcomes. Figueiredo and Pereira (2011) and Ospina *et al.* (2019) illustrated some issues related to deforestation and rural development using simplified mathematical models of deforestation including tipping point dynamics and connections to urbanization and rural migration. Lopez *et al.* (2019) analyzed the joint implementation of climate mitigation and adaptation measures in the land use sector in the presence of ecological thresholds. They found that no matter whether a regime shift is observed or not, interventions involving coordination between agriculture and forestry generated the most synergies, suggesting that effective policy integration requires looking at the land-use sector as a landscape rather than isolated components (e.g., agriculture and forestry sectors).

Institutional restrictions tend to limit the available management choices. When the ecological system is convex–concave, institutional restrictions could generate non-convexities, simply because managers may not have enough control of the optimal path to guide the system across its entire state space. Hence rather than trying to control convex–concave systems with limited control options, it may be better to include institutional design as one possible variable to manage these types of systems (Fenichel and Horan, 2016). This result is a dynamic equivalent to the Tinbergen rule (1956) stating that each separate externality would require separate controls to be manageable.

3 Hazard Rate Models and Uncertain Tipping Points

Many tipping points in natural resource systems are inherently stochastic or their locations are not known with certainty, making traditional optimization models inadequate. To address this issue, alternative models have been developed that incorporate hazard rate functions. This area of research was first explored by Kamien and Schwartz (1971) and has since been expanded upon by numerous researchers, including

Clarke and Reed (1994), Cropper (1976), Tsur and Zemel (1996), and Nævdal (2006). In this section, we examine the canonical model for resource management, outline its applications and extensions, and discuss its implications for game theory in the context of strategic behavior.

3.1 A Canonical Model of Optimal Resource Management

As in Section 2.1, we use $x(t)$ to represent the resource stock at time $t \geq 0$ and z a critical threshold or tipping point below which the resource system would undergo a regime shift (cf. Li *et al.*, 2018). If the stock does not cross the threshold, the resource stock is assumed to follow a standard dynamics equation $dx/dt = f(x, q)$, where $q(t)$ denotes the harvest rate at time t. Initially, $x_0 \geq z$ is in the desirable initial regime. As time goes on, the harvests or other factors may drive the resource to cross its threshold z at some future time T. If this happens, the growth process may undergo an abrupt change with substantial economic losses. Although harvests may generate economic benefits $B(q, x)$, the resulting stock reduction may trigger a regime shift with an abrupt economic loss. In the basic model, the social planner maximizes the joint discounted present value function

$$W\left(x_0\right) = E_T\left[\int_0^T B\left(q\left(t\right), x(t)\right) e^{-\rho t} dt + \varphi\left(x\left(T\right)\right) e^{-\rho T}\right] \tag{2}$$

where ρ is the discount rate and the expectation operator E is taken with respect to the uncertain flip date T (cf. Tsur and Zemel, 1996). The first term in the block brackets represents the present value of the stream of harvest benefits over the period before any regime shift. The second term is the present value of the "scrap" value at the start of the post-event period, $\varphi\left(x\left(T\right)\right)$, which is the maximum achievable value at time T. With some mathematical manipulations, the objective function in (2) can also be expressed as:

$$W\left(x_0\right) = \int_0^\infty S\left(t\right)\left[B\left(q\left(t\right), x(t)\right) + h\left(x\left(t\right)\right)\varphi\left(x\left(t\right)\right)\right] e^{-\rho t} dt \tag{3}$$

where $h\left(x\left(t\right)\right)$ denotes a state-dependent hazard rate function, $h'\left(x\right) \leq 0$, and $S\left(t\right) = \exp\left(-\int_0^t h\left(x\left(s\right)\right) ds\right)$ represents the survival probability up to time t, with initial condition $S(0) = 1$. An alternative, a more insightful expression for the objective function is given by:

$$W\left(x_0\right) = \int_0^{\infty} \left[B\left(q\left(t\right), x(t)\right) + h\left(x\left(t\right)\right)\varphi\left(x\left(t\right)\right)\right] e^{-\int_0^t [\rho + h(x(s))]ds} dt \tag{4}$$

This expression features an augmented discount rate that incorporates the hazard rate $h\left(x\left(t\right)\right)$ in addition to the pure rate of time preference. The hazard rate is formally defined as:

$$h\left(x\left(t\right)\right) = \lim_{\Delta \to 0} \frac{\Pr\left(T \leq t + \Delta \left| T > t\right.\right)}{\Delta} = -\frac{d}{dt}\ln(S(t)) \tag{5}$$

which measures the conditional probability density of the occurrence of a flip event over an infinitesimally short interval Δ from time t, given survival up to time t.

Depending on the specification of the hazard function and additional constraints, the model can accommodate a diverse range of tipping point problems. Firstly, in contrast to the resource management problem mentioned earlier, if the stock negatively impacts utility, then the crossing of a potential tipping point will occur from below. A higher ambient pollution level, for example, implies a more detrimental health effect. For problems of this nature, the hazard rate can always be specified as an increasing function in the stock, with $h'(.) \geq 0$. This is generally the case for water pollution and carbon emission issues, among others.

Secondly, if the hazard rate is purely time-dependent with $h(t) \geq 0$, the survival probability becomes $S(t) = \exp(-\int_0^t (h(s))ds)$, with the occurrence date defined in the time domain. Examples include the failure of a light bulb and the occurrence of a hurricane or wildfire. As a special case, with a constant $h > 0$ over time, the survival probability is simply $S(t) = \exp(-ht)$ that would decline exponentially over time. It can be readily shown that the expected lifetime of the normal regime is $E(T) = 1/h$, which means that a larger hazard rate always implies a short lifetime of the normal regime. As the effective discount rate is larger, the future becomes less valued as compared to the case with no flip risk. Under certain assumptions, the presence of the exogenous hazard would imply a more aggressive resource extraction or carbon emission due to the shift of welfare weight from the future to the present.

Thirdly, in the scenario where the hazard rate is state-dependent, as in (3), we encounter a more interesting case comprising two distinct settings. The first setting involves a fixed but unknown critical threshold, with passive learning occurring over time (Tsur and Zemel, 1996). Given an initial resource stock x_0, if no flip has ever transpired, we can deduce that this is a safe stock level above the threshold; otherwise, if the stock has crossed the threshold from above, the system would have already flipped. In this case, if the resource stock increases from x_0 over time, the hazard rate $h(x) = 0$ and $h'(x) = 0$, as we are moving toward even safer positions. Ultimately, we would reach a safe steady-state x^* as if no threshold existed. Conversely, if the resource stock decreases over time from a higher initial level, the risk of flipping escalates, and the optimal policy becomes precautionary. While harvesting provides a benefit, the risk of a regime shift and potential future harvests also increases. The most effective strategy is to aim for an expected steady-state stock level, denoted as x^c.

For any initial stock within the interval $[x^*, x^c]$, the optimal strategy is to maintain the current position. Within this interval, the cost of a possible regime shift surpasses the benefit of harvesting, rendering it unprofitable to harvest more to reduce the stock. However, increasing the stock does not decrease risk but diminishes the potential benefit of harvests. Of course, as the system transitions from a higher x^0 to the expected steady-state x^c, there remains the risk of crossing the threshold, resulting in a regime shift.

The other setting pertains to a stochastic threshold (rather than the unknown one), whereby no stock level is absolutely safe. Even if a particular stock level has been attained in the past without instigating the event, we cannot dismiss the possibility of its occurrence at the same stock level in the future under less favorable exogenous circumstances. In comparison to the case with an unknown threshold, the hazard-rate function remains stationary without any learning opportunity, and the objective function (3) can be technically optimized as though it is a certainty-equivalent deterministic problem. This form of event uncertainty has been employed to model various resource-related situations, including nuclear risk control (Cropper, 1976), environmental pollution (Tsur and Zemel, 1998), and climate change economics (van der Ploeg and de Zeeuw, 2018).

3.2 Model Applications and Extensions

Using the benchmark model in (1)–(3) as a starting point, the recent literature has witnessed numerous applications and extensions. In this subsection, we explore a selection of noteworthy examples, emphasizing the significance of the hazard rate functions in optimal resource management, and resilience valuation, as well as the roles of learning by doing and flipping impact delays.

3.2.1 Natural Resource Management

Concerning resource management, the hazard rate function and post-event loss play an important role in policy choices. Within a linear fishery model framework, Polasky *et al.* (2011) investigated the influence of a stochastic tipping point on optimal harvests. The study encompasses four typical cases, combining two post-event damages, i.e. stock collapse and negative shocks on system dynamics, with exogenous and endogenous flipping probabilities. Among their findings, they discovered that incorporating an endogenous hazard with a potential system dynamics shift necessitates a precautionary catching policy to preserve higher resource stock levels. Conversely, no such precautionary policy is required when the hazard rate remains a positive constant (exogenous risk of tipping). Intuitively, if there is a risk of an asteroid hit that humans cannot do anything about (the exogenous risk case), then people would tend to consume more before the event. Whereas if the risk depends on human actions such as the emission of CO_2 (endogenous risk), then we should act more precautionary. This insight is in principle correct but may be subject to certain subtle assumptions. For example, de Zeeuw and He (2017) expanded upon this work, and demonstrated that when the hazard depends on pollution, the resulting precaution could result in even lower emissions than what would be considered optimal within a high-damage regime. De Zeeuw and He (2017) refined these findings further, illustrating how they also rely on the elasticity of intertemporal substitution.

Ren and Polasky (2014) established that the result of Polasky *et al.* (2011) is contingent on the standard linear-in-harvest benefit function. They demonstrated that by utilizing a more general utility function, the outcome becomes ambiguous, with both precautionary and more aggressive policies potentially being optimal. In a more comprehensive analysis, de Zeeuw and He (2017) examined the factors underlying the ambiguous result for a concave utility function. They established that while stock dependence of the hazard rate still leads to precaution, a consumption smoothing argument may drive optimal management in the opposite direction. The ultimate result is contingent on the relative strength of these two forces. When the elasticity of intertemporal substitution is sufficiently small, the first force dominates, and the precautionary principle applies; however, when the elasticity of intertemporal substitution is high, the precautionary principle still applies, but only if the effect of the stock variable on the hazard rate is large enough (i.e., $h'(x)$ must be sufficiently negative, as per Margolis and Nævdal (2008)). Along the same research direction, Engström and Gars (2016) explored how the risk of climate tipping events impacts optimal fossil-fuel use, carbon taxes, and fossil-fuel prices over time. They considered not only the direct effect of carbon stocks on the risk of regime shifts but also the indirect negative effect of flipping risk on the value of remaining fossil-fuel reserves. While the former implies precaution, the latter suggests more aggressive behavior, which could hold significant quantitative implications for climate policy analysis concerning catastrophic climate events.

More recently, Stecher and Baumgärtner (2022) developed a generic ecosystem model featuring fundamental alternate regimes and two distinct stochastic influences. This model can be utilized to investigate multiple management strategies related to identifying optimal management solutions in a stochastic context, computing the probability of a regime shift, or pinpointing empirical factors that precipitated a specific regime shift. The authors emphasized the importance of active resilience management for stochastic and multi-stable ecosystems. Particularly, managers must adapt to changing conditions, as previously safe strategies may no longer be viable if environmental circumstances have altered. Furthermore, their model can assist in calculating the probabilities of regime shifts associated with different types of management actions.

3.2.2 Resilience Valuation

In addition to resource management, the benchmark model in (1)–(3) has also been used to investigate the significance of ecological resilience and sustainable development in the recent literature. Formally, resilience is defined as the capacity of an ecosystem to respond to a perturbation or disturbance by resisting damage and recovering quickly (Folke and Gunderson, 2010; Holling, 1973). However, measuring and controlling resilience may prove challenging except in relatively simple systems. To render the value of resilience comparable with other capital stocks, Mäler (2008) conceptualized resilience as some additional capital stock and proposed employing its accounting prices for sustainability analysis. Under conditions of perfect knowledge and control, Mäler argued that the accounting price should be zero, as a small change in resilience would have no impact on the risk of a regime shift (cf. Tsur and Zemel, 1996). Nevertheless, with uncertainty regarding system dynamics and imperfect control, any change in resilience today may generally influence the likelihood of a future system flip, rendering resilience pricing a sensible approach.

Mäler and Li (2010) developed a resilience pricing method for assessing sustainability under the uncertainty of tipping points. Defining resilience as a stock variable, they attempted to evaluate its marginal contribution to the present discounted value of future services in mitigating the risk of regime shifts (c.f. Gjerde *et al.*, 1999). Subsequently, they incorporated the value of resilience into the inclusive wealth framework for dynamic welfare analysis (cf. Arrow *et al.*, 2003; Walker *et al.*, 2010). Li *et al.* (2016) explored the effects of regime shifts and resilience on fisheries management, utilizing the Argentinean Hake fishery as a case study. A bio-economic model was employed to analyze the consequences of potential fishery collapse and the resilience value per unit of fish stock when its level is low. Their findings indicate that the resilience value is not monotonic in the stock level, as both low and high stock levels result in low resilience value. Moreover, they compared value functions with and without potential flipping risks considered in optimal fishery management.

Building on a similar concept, Franklin and Pindyck (2018) constructed a model to determine the social cost of tropical deforestation, factoring in potential tipping points. Scientific models suggest that surpassing a critical threshold in tropical deforestation might alter precipitation patterns, catalyzing a transition from the tropical forest to a savanna state, and incurring substantial economic loss. While the precise tipping points remain elusive, escalating deforestation heightens the risk of reaching these thresholds by reducing general forest resilience. The authors posit that traditional marginal valuation methods, which calculate economic loss based on each additional hectare of deforested land, might considerably undervalue the true per-hectare cost when considering larger-scale deforestation. To formulate effective land-use strategies and ecosystem service payments, they compute the social cost of Amazon rainforest deforestation using an average-cost approach, which accounts for potential losses from shifts in ecological regimes post-threshold. Given that the marginal cost rises convexly with deforestation, the average valuation produces a deforestation cost substantially higher than its marginal counterpart. In contrast, Mäler and Li (2010) calculate the marginal contribution per unit of resilience stock to the expected present value of the future payoffs, taking into account the risk of flips within the model. Therefore, the average valuation in Franklin and Pindyck (2018) and the marginal valuation as in Mäler and Li (2010) should be in principle consistent with each other.

Along the same line, Li and Bali-Swain (2016) developed a dynamic stochastic general equilibrium (DSGE) model to investigate the interrelationships among growth, water resilience, and sustainability in South Africa. The model incorporates water resources as a production factor and underscored the significance of water resilience for sustainable development. The results demonstrate that enhancing water resilience can contribute to both economic growth and environmental sustainability, emphasizing the necessity for integrated water resource management strategies in developing countries. According to Castelli *et al.* (2022), this is the only attempt to study water resources in a DSGE framework.

3.2.3 The Role of Learning

Note that most examples mentioned above assume the hazard rate function is known with certainty. In reality, however, numerous situations exist where the hazard rate function may be imperfectly known, and learning becomes crucial for optimal decision-making, such as determining the optimal carbon taxes in climate economics. Different assumptions on learning, however, may have rather different implications for optimal decision-making. As mentioned earlier, Tsur and Zemel (1996) demonstrated a simple learning process regarding the unknown threshold. If no regime shift occurs at a certain resource stock level, then it is absolutely certain that the threshold has not been crossed yet, and it is safe to maintain that level.

Lemoine and Traeger (2014) expanded on this concept by incorporating a Bayesian learning mechanism within an augmented DICE model framework (see Nordhaus, 2008; Nordhaus, 2018) to model the climatic hazard rate function. They assume the prior probability distribution of the unknown temperature threshold to be uniformly distributed over a domain $[T_0, \bar{T}]$, with $\bar{T}$ representing the upper bound beyond which a climate regime shift would certainly occur. At any time t, if no regime shift is observed at temperature T_t, it can be inferred that T_t is perfectly safe. The posterior distribution of the unknown threshold would then be distributed over $[T_t, \bar{T}]$. Conditional on no regime shift up to time t, the hazard rate for the forthcoming period $[t, t+1]$ is modeled as $h(t) = \frac{T_{t+1} - T_t}{\bar{T} - T_t}$ for $T_{t+1} \leq \bar{T}$, and $h(t) = 1$ for $T_{t+1} > \bar{T}$. For all $h(t) < 1$, the hazard rate would increase in T_t, implying that the closer the temperature is to its upper bound, the larger the risk of a flip.

Intuitively, when a regime shift is expected to occur at the upper bound temperature level, the closer the actual temperature is to the bound, the more probable it is that a small temperature increase would cross the threshold. By considering such a learning process, they discover that the resulting optimal carbon tax should grow at a faster rate than the GDP growth rate.

Using a different Bayesian updating rule, Gerlagh and Liski (2018) demonstrated that the optimal carbon tax can grow at a slower rate than the economy. For simplification purposes, they modeled the hazard rate uncertainty as a discrete distribution of two states of nature, $h(t) = 0$ and $h(t) = \lambda > 0$, with $\mu_0 < 1$ as the prior probability for $h(t) = \lambda$, and $1 - \mu_0$ for $h(t) = 0$. It is important to note that this configuration does not necessitate that a regime shift will ever transpire in the future. The posterior probability is derived as

$$\mu_t = \Pr\left(p = \lambda \,|\text{no flip at } t\right) = \frac{\mu_0 (1-\lambda)^t}{(1-\mu_0) + \mu_0 (1-\lambda)^t} \tag{6}$$

which declines over time. If there is a chance of never experiencing a regime shift and no flip has been observed over an extended period until time t, one would become more optimistic about the future.

As the optimal carbon tax is proportional to the product of this posterior probability and the concurrent GDP at any time t, the decreasing μ_t implies that the optimal tax should grow slower than GDP over time, contrasting sharply with Lemoine and Traeger (2014). If the underlying true hazard rate increases over time for λ_t, Gerlagh and Liski (2018) also demonstrated that the optimal carbon tax could grow at a faster rate than the economy if the true hazard rate grows sufficiently quickly. This finding aligns with Lemoine and Traeger (2014) but for different reasons.

The learning rule in Gerlagh and Liski (2018) is related to the delay mechanism in the degree of "experimentation", specifically, even if a threshold is crossed but the effect is delayed, the regime shift would not have been observed. In such a situation, a larger degree of precaution is likely to be optimal, as illustrated by Crépin and Nævdal (2020). This is the case, for instance, with climate change, where thresholds can initiate slow processes such as the melting of Arctic summer sea ice or permafrost, which would commence long after they have been triggered. In these circumstances, Crépin and Nævdal (2020) proposed that the risk should be modeled differently. They introduced the concept of inertia risk, where inertia was modeled as an additional dynamic equation for system stress that introduces a delay in the system. They exemplified this approach by modeling pollution in a recipient using a sharp threshold, as depicted in the equations in the second line of Table 1, and add an equation modeling the level of stress, s in the system, as a function of the state variable x and parameters α representing inertia and γ, the system's capacity to absorb stress.

$$\frac{ds}{dt} = \alpha x - \gamma s, \quad (\alpha, \gamma > 0) \tag{7}$$

If stress surpasses some stochastic threshold, a regime shift occurs. The advantage of this approach is that it encompasses the possibilities of delays or interactions between rapid and slow dynamics in the system. Furthermore, in contrast to many standard models, the probability of a catastrophe occurring at some point in time is not either 0 or 1, it can actually span the entire interval $[0, 1]$, which is more realistic in many real-life situations.

3.2.4 Ambiguity and Impact Delay Issues

The domain of climate policy is replete with ambiguities in decision-making. Lemoine and Traeger (2016a) probed into ambiguity aversion in climate policy with tipping points, inferring that uncertainty aversion results in a slightly raised optimal carbon tax. This theme of ambiguity and uncertainty carries over to the timing and stringency of interventions. Sims and Finnoff (2016) emphasized the pitfalls of delaying investments due to unknown tipping points. Conversely, Agliardi and Xepapadeas (2022) postulated the urgency of immediate action given profound uncertainties, but hinted that policy stringency might necessitate subsequent revisions.

In the realm of regime shifts in resource management, Deopa and Rinaldo (2020) pioneered a framework for regime shift detection, showcasing its application to the Cantareira water reservoir in São Paulo, Brazil. Delving deeper, van der Ploeg and de Zeeuw (2018) underscored the economic implications stemming from the delay between initiating a tipping point and its full impact. Meanwhile, van der Ploeg and de Zeeuw (2019) championed strategies like carbon taxation and adaptation capital investments as deterrents against climate tipping and productivity shocks.

A nuanced exploration by Arvaniti *et al.* (2023) shed light on the optimal management challenges confronting forward-looking, impatient users amidst uncertain growth in renewable resources. Their dynamic model, underpinned by hyperbolic discounting agents, uniquely revealed that resource users might not necessarily curtail their harvesting during regime shifts. Moreover, this study unveiled the intricate relationship between optimal extraction and impatience or present bias.[2] Additionally, the modeling of uncertainty through the lens of environmental shock distributions is aptly discussed by Margolis and Nævdal (2008).

3.3 Tipping Games and Strategic Behavior Modeling

As challenges in global resources become more intricate and interconnected, game theory has emerged as an invaluable tool for assessing

[2] Anchoring many of these discussions are foundational concepts such as hyperbolic discounting, as evidenced by Laibson (1997), Li and Löfgren (2000), and Karp and Tsur (2011).

shared common pool resources (Crépin and Lindahl, 2009; Mäler *et al.*, 2003; Wagener, 2013). Recent developments, as discussed in Section 2.2, have deepened this exploration, with the integration of regime shift risks and dynamic player interactions capturing notable attention. For more experimental evidence, see Section 5.

In the realm of international climate change policies, the critical question remains: Can countries effectively coordinate their mitigation efforts to avert potential catastrophes? Barrett (2013) approached this dilemma through a game-theoretical lens, suggesting that countries can indeed form mutual mitigation strategies when the prospective damages from climate catastrophes overshadow the costs of prevention. This paradigm holds true even when future damages are uncertain, given the assumption of risk-neutrality among countries. Barrett's core revelation, however, delves into the realm of threshold uncertainties. A known threshold promises better coordination, but once veiled in uncertainty, coordination becomes challenging — changing the situation from a mutual coordination game to the classic prisoner's dilemma scenario.

Extending upon Barrett's foundational work, Nkuiya (2015) introduced dynamic interplays in a world with stochastic thresholds. This model illustrated a global tapestry of identical countries, all of which benefit from carbon emissions yet face the looming peril of abrupt climate changes if a certain atmospheric carbon threshold is breached. Crucially, the study unveiled that responses to emission threats deviated notably when multiple countries were in play, compared to models focusing solely on individual nations.

Within this broader conversation on common resource management, Miller and Nkuiya (2016) underscored the dynamics of coalitions. Their findings suggest that the mere presence of an impending resource-related threat could realign coalition strategies, potentially enlarging coalition sizes or altering harvest decisions. Similarly, Diekert (2017) delved into resource dynamics, illustrating that while the specter of an impending disaster could foster cooperative behavior, outcomes heavily depended on the initial state of the resource.

In a more macroeconomic outlook, van der Ploeg (2016) navigated the intricate pathways of carbon taxes, aiming to counter potential climate cataclysms. By modeling a world with two representative regions, one developed (North) and one less developed (South), they ascertained that while cooperative tax strategies could see convergence, non-cooperative behaviors would likely lead to divergence, with the less-developed South bearing a heavier tax burden in the end.

Recent investigations by Besley and Dixit (2019) further extended this dialogue. In a world marred by potential environmental catastrophes, they unveiled that optimal policy responses hinged on cross-country interdependence and the diverse costs of mitigation. Moreover, Emmerling *et al.* (2021) emphasized the multifaceted nature of international climate cooperation. Their work illuminated that successful coalitions are influenced by myriad factors, from economic disparities to societal norms. Echoing these sentiments, Wagener and de Zeeuw (2021) advocated for a nuanced understanding of tipping points, urging policymakers to consider the intricate interplay of various agents when crafting policies for systems as complex and dynamic as our global climate.

4 Integrated Assessment Models and Joint Tipping Points

Integrated assessment models (IAMs), like the DICE model introduced by William Nordhaus in 1992, analyze the intricate interplay between economic activities and the climate system. The DICE model, blending economic growth and climate modules, has been crucial in understanding optimal climate strategies. However, its deterministic nature fails to consider uncertainties or catastrophic events, making it a potentially unreliable guide for climate policy, as indicated by Pindyck (2013). The recent literature integrates tipping points and regime shift risks in IAMs. Fundamentally, this modeling aligns with the theory from Section 3. Instead of the simplified benefit functions as in the previous section, researchers use detailed macroeconomic models calibrated with real data. These models account for intricate interactions and multiple state variables, but due to their complexity, they often demand advanced numerical methods to find optimal policy recommendations.

4.1 Multiple and Interacting Tipping Points

Multiple threats can occur concurrently, making it insufficient to address just one (Martin and Pindyck, 2015). Tsur and Zemel (2017) built on

Martin and Pindyck (2015)'s work, considering inter-temporal factors and endogenous hazards. Recent research underscores the potential for multiple, interconnected tipping points leading to nonlinear dynamics in both economic and natural systems. Rockström *et al.* (2009) listed nine Earth system processes, including climate change and biodiversity loss. Within identified boundaries, we live in a safe-operating space; surpassing them risks unknown tipping points with severe implications for ecosystems and human societies.

Boundaries' interactions have been studied, illustrating how one boundary change can destabilize another (Armstrong McKay *et al.*, 2022; Folke and Gunderson, 2010; Steffen *et al.*, 2015). For example, climate change affects biodiversity, which in turn impacts climate change. Land-use alterations impact freshwater systems, and excessive nitrogen use changes biodiversity dynamics, affecting nutrient cycling.

Interacting tipping points extend beyond ecosystems. Lenton *et al.* (2008) identified multiple Earth system elements, like the Greenland ice sheet and the Amazon rainforest. These elements possess both reinforcing and counteracting feedbacks (Lenton and Ciscar, 2013). For instance, permafrost thawing releases methane, a greenhouse gas, which can destabilize oceanic methane hydrates, releasing more methane. Contrarily, the West Antarctic ice sheet melting can affect the Southern Ocean carbon sink, potentially elevating atmospheric CO_2 and exacerbating climate change.

Additionally, Lemoine and Traeger (2016b) and Lemoine and Rudik (2017) considered tipping points in parameters, like feedback uncertainty. The interplay between these points can amplify or reduce each other. Understanding these various tipping points is vital for economic and climate policy decisions, emphasizing the need for comprehensive assessment modeling (van der Ploeg, 2016).

4.2 Advancements in Integrated Assessment Modeling

Recent advancements in IAMs have enriched climate studies, especially in the realm of climate tipping points. Lemoine and Traeger (2014) refined the established Nordhaus DICE model by introducing a general temperature tipping point coupled with a learning mechanism. While their economic module echoes the DICE model, the authors introduced a dynamic hazard rate function for climatic tipping. As discussed in

Section 3.2.3, this rate measures the yearly probability of a tipping event based on the current year's temperature. A unique feature is the dynamic adaptation of the risk distribution, adjusting based on observed temperatures over time. Such adjustments can indicate an increasing risk as temperatures near the upper bound for flipping. Furthermore, the model contemplates the ramifications of crossing tipping points, such as feedback mechanisms and atmospheric carbon persistence.

For computational ease, they reduced the climate module variables and structured the model around three continuous state variables, with a discrete variable indicating tipping status. They employed a backward recursion algorithm and recursive dynamic programming to solve the optimization challenges, parameterizing the value function using the Chebyshev methods. Despite simplifications, the computational demand remained significant. Their primary insights underscored the profound implications of tipping risks for carbon taxation and proposed mitigation measures.

In a similar study, Heutel *et al.* (2016) investigated the feasibility of solar geoengineering as an additional mitigation strategy. Their research indicated that prior to a tipping point, solar geoengineering could reduce mitigation needs and decrease temperatures while increasing carbon concentrations. However, the success of this strategy depends on the type of tipping point encountered. They also flagged associated risks, particularly the ecological consequences of elevated CO_2 levels and diminished luminosity (Barrett *et al.*, 2014).

In a different approach, Lontzek *et al.* (2015) studied specific climate tipping elements using the DSICE model by Cai *et al.* (2013). This model accommodated individual hazard rate functions for each tipping event and subsequent post-event losses. They deviated from Lemoine and Traeger (2014) by excluding a learning mechanism and instead, relied on expert opinions for parameter calibration. Their modeling of post-event impacts offered a distinct perspective, envisioning gradual consequences over time. Their conclusions stressed that the presence of multiple tipping points could significantly elevate carbon prices.

Adding to this discourse, Lemoine and Traeger (2016b) highlighted the intricacies of tipping point interactions. By introducing a direct productivity loss due to temperature rise and acknowledging interdependence among their tipping points, they devised an integrated model to discern optimal emission policies. Their findings illuminated strong interdependencies among various tipping points, emphasizing that addressing one could inadvertently impact others.

In a complementary study, Cai *et al.* (2016) examined the synergies of multiple tipping points, revealing that their interactions can significantly modify the social cost of carbon. Interestingly, their results suggested a balance in the overall impact due to compensating positive and negative interactions. Cai and Lontzek (2019) also proposed an extended IAM that merged both the economic and climate risks. Their intricate model with considerably more state variables, necessitating elaborate computational techniques, underscored the value of incorporating multiple tipping points in estimating the social cost of carbon.

Lastly, Dietz *et al.* (2021) executed a holistic analysis, considering all tipping elements highlighted by Lenton and Ciscar (2013). Their model's granularity covered climate damages across 180 countries. Their results affirmed the heightened economic implications of climate change, though they noted that the net interaction effect of tipping points on the social cost of carbon remained relatively modest.

4.3 Model Limitations

Despite advancements, current modeling approaches continue to face several limitations. One such limitation is the uncertainty arising from underlying processes, parameterization, and assumptions employed to represent intricate climate systems (cf. Cai and Lontzek, 2019; Kriegler *et al.*, 2009; Lemoine and Traeger, 2016a). The choice of hazard rate function complexity is often subjective, potentially influencing analytical outcomes. Estimating hazard rate parameters remains challenging due to limited data and an incomplete comprehension of tipping point dynamics (Lenton *et al.*, 2008). As the risks and impacts of elevated future global temperature scenarios remain largely unknown, model functional forms must be inferred through logical reasoning and parameter calibration based on expert opinions.[3]

[3]Note that, at the local level, there are empirical studies concerning the impact of temperature changes on economic cost (c.f. Fisher and Le, 2014; Hsiang *et al.*, 2017), but these are limited to the local level and not applicable to general integrated assessment modeling.

Although simplification is necessary for computational tractability, it may result in the omission of crucial interactions essential to understanding tipping point dynamics (Calvin and Bond-Lamberty, 2018). IAMs may also need to capture more interactions among multiple tipping points, which could either amplify or dampen their impacts (Cai and Lontzek, 2019; Lemoine and Traeger, 2016b). Furthermore, the selection of time horizons and discount rates can profoundly influence model outcomes, particularly when examining the long-term effects of tipping points (Nordhaus, 2013). Different values for these parameters may yield substantially divergent policy recommendations.

We have seen that IAMs incorporating tipping points offer valuable insights into potential impacts and risks linked to climate change but they are not without significant limitations. To address these constraints and enhance our understanding of the complex dynamics associated with tipping points in the context of climate change, continued research and model development are imperative.

5 Using Controlled Experiments to Analyze Behavior in Games with Tipping Points

Recent years have seen a growing body of literature using controlled economic experiments to test behavior and outcomes in common pool resource (CPR) games and public good (PG) games, situations we highlight in Sections 2.2 and 3.3. In this section, we will present insights that have emerged from experimental work that study the behavior in these two different, but somewhat interrelated situations.

5.1 Economic Experiments

Experimental economics is the branch of economics that studies human behavior in a controlled setting and is typically used to test theoretical predictions, one reason being that people do not necessarily act according to theoretical assumptions. Through a random assignment of participants to different groups (called treatments), the researcher controls these different circumstances, meaning that the only difference between these groups is the variable of interest. This allows for the researcher to establish a causal link between observed outcomes/behavior and the variable of interest. Besides a random assignment of participants to the different treatment groups, the method emphasizes the importance of providing participants with sufficient incentives, in order to outweigh potential biases associated with decision costs. Moreover, this compensation should be directly linked to decisions taken to avoid hypothetical biases (Smith, 1976).

Since the first PG game experiment (Dawes *et al.*, 1977) and the first CPR game experiments (Jorgenson and Papciak, 1981; Ostrom *et al.*, 1992) were undertaken, a huge number of variants and extensions of CPR and PG games have been performed,[4] including games with tipping points. The overall aim of these experiments is to test and analyze behavior, in particular, to increase our understanding of when we can expect cooperative outcomes (and when we cannot, implying for example a need for policy), which in the presence of tipping points can be extra critical.

There are different ways to implement the PG and CPR experiments. Most of the experiments involving tipping points have been conducted in a laboratory setting with students as subjects, using abstract or framed instructions. Whereas an abstract framing means that the information and the instructions provided to the participants are as neutral as possible, e.g., speaking about rewards, costs, and benefits, framed experiments use instructions containing context-specific elements, e.g., informing the subjects of situations involving climate tipping points or certain resource dynamics. Some of the experiments, however, have been performed in the field with resource users.[5]

5.2 Public Good Games

In a linear PG game experiment, each participant decides how much to contribute to a PG which is shared by the group. The game is set up so that the marginal capital return from the PG is lower than the marginal private return from private consumption foregone. Although the socially preferred outcome is that everyone contributes, from the individual's perspective, the rational egoistic choice is then to contribute zero. The game can be set up to be a one-shot game or a repeated game. The repeated game can in turn be finite or indefinite. In theory, it can

[4]See, e.g. Lindahl *et al.* (2021) for an overview.

[5]See, Harrison and List (2004) for classification of economic experiments.

of course also be infinite although that is difficult, if not impossible, to mimic in an experiment. Empirical results from linear PG games experiments show that people contribute more than what theory predicts. If the game is repeated over several rounds, contribution levels decrease over the rounds. Moreover, many participants are so-called "conditional cooperators" whose contributions to the public good are conditioned on beliefs about the average group contribution. Conditional cooperation at high contribution rates can often be sustained through costly punishment of free-riders.[6]

A threshold in the PG game is often introduced to reflect the case that there is a certain contribution target that needs to be met for the good to be provided at all. This type of situation could for example resemble the amount of funding needed for an infrastructure project, for example, the building of a bridge, a railroad, etc. Unlike the linear PG game which has a unique dominant strategy equilibrium (non-contribution), a threshold PG can have two sets of Nash equilibria: Nash equilibria in which the threshold is not met, and the public good not provided, and Nash equilibria where the threshold is met and the public good is provided. Experimental results typically show that the public good is provided relatively frequently but that perfect coordination by groups on the efficient Nash equilibrium outcome is rare. However, contributions typically increase as the private return of the public good increases (cf. Croson and Marks, 2000).

5.2.1 Introducing Tipping Points in Collective Risk Dilemmas

In the context of sustainability challenges and potential collective risks and tipping points arising if the public good is not provided, e.g., in the form of a climate catastrophe if mitigation actions are not sufficient, a new set of threshold public good experiments have emerged with additional features to the game. For example, investments may be lost if the public good is not provided (no refunds), the value of the public good may not be known with certainty, and there may be a risk of losing also the value of the money not invested, meaning that the private good is at stake with a certain probability if the contribution target (the threshold) is not met.

[6] See, for example, Ledyard (1995), Fehr and Gächter (2000), and Chaudhuri (2011) for more details.

Players can coordinate to avoid dangerous climate risks when the tipping point is known. Milinski *et al.* (2008) is one of the first experimental studies of public goods provision games in this new context. The question they pose is whether or not a group of people would reach a collective target through individual contributions to prevent a catastrophe such as a dangerous climate change, and doing so, when they know they will lose all their remaining money with a certain probability if they fail to reach the target sum. They find that the higher the risk of the catastrophe, the more likely it is that the group will reach the target sum. That people are willing to invest in costly climate mitigation to avoid disaster is also confirmed by Andrews *et al.* (2018). They find that people will invest in high-risk high-reward technologies when more certain options will not be sufficient. These experimental findings are in line with theoretical results by Barrett (2013) highlighted in Section 3.3. In a later study, Milinski *et al.* (2016) experimentally analyzed the collective-risk social dilemma that involves representatives deciding on behalf of their fellow group members. In their setup representatives can be re-elected or voted out after each consecutive collective-risk game. The study reveals that people tend to favor representatives who employ extortion, or threat of punishment to reinforce cooperation. This can lead to higher levels of cooperation and better outcomes for the group.

The presence of an uncertain tipping point is a potential threat to coordination. The location of the critical threshold for avoiding the catastrophic event is in general not known with certainty. In such a situation theory predicts that an increase in threshold uncertainty will decrease equilibrium contributions when the public good value is sufficiently low. For international agreement, this implies that even though countries agree to a collective goal, they will tend to free-ride and expect others to contribute more (cf. Barrett and Dannenberg, 2014, also in Section 3.3). This result holds even when the decision is delegated to a representative (İriş *et al.*, 2019)

Barrett and Dannenberg (2012) confirm the theoretical predictions and find that uncertainty about the location of the threshold indeed turns the game back into a prisoner's dilemma, causing cooperation to collapse. Moreover, they also find that the magnitude of the negative impact following failure to reach a mitigation threshold has relatively little influence in terms of behavioral outcomes compared to the influence of uncertainty about the location of the threshold. In a related study, Hasson *et al.* (2010) also find support for the result that the magnitude of the disaster has little influence on behavior. In their study, Hasson *et al.* (2010) find no significant difference in the level of mitigation across different variations of magnitude of the disaster. They do so in a framework where they introduce a stochastic term to account for probabilistic destruction in a climate-change setting, where the probability density function is mapped to within-group levels of mitigation.

In another study, Dannenberg *et al.* (2015) show that the problem linked to uncertainty is potentially more serious with risk ambiguity, i.e., when players are not only unaware of the value of the threshold but also of its probability distribution. The authors can conclude, however, that early and credible commitment can help groups cope with the presence of uncertainty.

While Barrett and Dannenberg (2012, 2014) confirm the theoretical prediction involving uncertainty about the location of the threshold, McBride (2010) observes only limited experimental verification of the theoretical prediction. Using elicited beliefs data to represent subjects' beliefs, he notes that behavior is not consistent with expected payoff maximization, however, contributions are increasing in subjects' subjective pivotalness. Thus, wider threshold uncertainty will sometimes — but not always — hinder collective action. Moreover, Guilfoos *et al.* (2019) observe that, in the presence of threshold uncertainty, cheap talks and written communication can enhance the chance for groups to coordinate and reach the socially preferred equilibrium contribution.

Some (but not all) experimental results indicate that uncertainty in the location of the tipping points can discourage cooperation. Schmidt (2017) claims that all this may be due to the underlying static game model. In a dynamic model, with convex abatement cost in each period, which allows reallocation of abatement effort, Schmidt shows theoretically that it is perfectly possible for cooperation even in the presence of uncertainty tipping points. However, to our knowledge, there have been no experiments testing this setup.

The unequal distribution of wealth, mitigation costs, and benefits from climate action may present a barrier to coordination. There have been some attempts to experiment in a dynamic setting though, but focusing on heterogeneity. Feige *et al.* (2018) set up an experiment in a repeated setting where participants have different contribution costs.

They conclude that a non-binding unanimous voting procedure on contributions leads to frequent agreement on an optimal total contribution and high rates of compliance, even in the case of heterogeneous marginal contribution costs. Groups, however, that do not reach an agreement perform worse than the baseline treatments without a voting procedure.

Milinski *et al.* (2011) expanded their previous analysis (Milinski *et al.*, 2008) by using the same contextual set-up and by exploring how different types of climate targets can affect cooperation between rich and poor nations. They did so by introducing heterogeneous wealth and two-time horizons into their collective risk dilemma game. Their experimental results reveal that rich players are willing to substitute for missing contributions by the poor, provided there exist intermediate climate targets that, if not reached, are potentially followed by intermediate costly risks. Based on their results they put forth the hypothesis that intermediate targets can facilitate cooperation, however, they may not be sufficient for successful climate negotiations.

A related study (Tavoni *et al.*, 2011) also investigates the effects of heterogeneity by distributing endowments unequally among the participants in their experimental groups. Each group can reach a fixed target sum, a threshold through successive money contributions, knowing that if they fail, they will lose all remaining money with a 50% probability. The authors find that inequality reduces the prospects of reaching the target but that communication can increase success substantially, eliminating inequality over the course of the game, with rich players signaling a willingness to redistribute early on.

An unequal distribution of benefits (in case of catastrophe avoidance) may also have a significant effect on contributions. Bosetti *et al.* (2017) investigated the likelihood of a sizable coalition forming under different distribution settings and found that a distribution of benefits in favor of early investors could positively affect the likelihood of a large enough coalition forming.

The experimental evidence presented so far confer the theoretical results mentioned in Section 3.3, suggesting that inequalities may present a barrier to cooperation, but not necessarily (see, e.g., Miller and Nkuiya, 2016; van der Ploeg, 2016).

Kline *et al.* (2018) points out the characteristic feature of the global climate change dilemma — the interdependence between the underlying economic development that drives anthropogenic climate change and the subsequent dilemma arising from the need to mitigate emissions. In other words, in a carbon-based economy, responsibility for climate change is a byproduct of economic development and is therefore endogenous to it. To capture this endogeneity, the authors combine these two elements into a 'compound climate dilemma' and conduct a series of experiments in the United States and China to test its implications for cooperation. They show that, even if the advantaged participants increase their willingness to cooperate, the accompanying decrease in cooperative behavior by the disadvantaged participants more than offsets it. In light of this interdependence, the basis upon which mitigation obligations should be differentiated becomes an additional dimension of conflict, with implications for domestic politics and international negotiations discussed.

Jacquet *et al.* (2013) present a laboratory experiment that aims to simulate the decision-making process of individuals in intra- and inter-generational contexts and sheds light on the role of discount rates in influencing the willingness to invest in mitigating and avoiding collective climate change risk. In their experimental set-up, participants can choose to cooperate or risk losing an additional endowment with a high probability. The rewards for defection are immediate, whereas the rewards for cooperation are delayed by 1 day, delayed by 7 weeks (intragenerational discounting), or delayed by several decades and spread over a much larger number of potential beneficiaries (inter-generational discounting). The results of the experiment demonstrate that inter-generational discounting leads to a marked decrease in cooperation: all groups failed to reach the collective target. Also, individuals with higher intra-generational discount rates tend to invest less in climate mitigation, but here the effect is weaker.

5.3 Common Pool Resource Games

There are essentially two types of CPR game designs that have been tested in experiments. In a CPR 'investment game' experiment, each participant decides how much to invest in two types of goods (or markets), where one of the goods is the CPR and the other one a private good. Investment in the CPR (such as allocating time to harvesting from the CPR) means more exploitative behavior. The socially preferred outcome is associated with more moderate investments in the CPR compared to the Nash prediction and the individually preferred choice based on a rational egoistic decision maker[7] (in finite games). In a CPR 'extraction/request game' experiment, each participant decides in each round how much of the CPR to extract. The socially preferred outcome is associated with less extraction of the CPR compared to the Nash prediction and individually preferred choice (in finite games).[8] Both these types of CPR games can be set up as a one-shot game or as multiple-period games. The investment-type of design has been more dominant in the economic strand of the CPR experimental literature, probably because it is the design that was introduced and used by Ostrom and the request game design was introduced by psychologists. However, the request game design is particularly interesting in the context of tipping points and regime shifts, since the purely theoretical models on CPR management and regime shifts are more similar to this type of design.

There have been a vast number of studies investigating behavior in both these types of games and using different variations to the design. There are some commonalities in the results such as, (1) over-exploitation is common, although not necessarily according to the Nash equilibrium prediction, (2) communication (cheap talk) increases levels of cooperation, and (3) people playing these games are willing to costly punish free-riders, which can also lead to higher levels of cooperation as compared to cases when no communication is allowed (cf. Ostrom, 2006).

The literature on CPRs with thresholds started with work by Budescu and colleagues (see Budescu *et al.*, 1992, 1995a,b; Rapoport *et al.*, 1992). In these studies, built on a request game design, users make requests from the resource and if the sum of these requests is less than or equal to the resource size, the subject receives their respective requests. But, if the sum of withdrawals exceeds the size of the resource, then the users do not receive anything. The game theoretic prediction (and experimental results) depend on whether or not users make simultaneous or sequential requests where simultaneous requests are more likely to lead to resource depletion (Budescu *et al.*, 1992). Moreover, uncertainty

[7]See Ostrom *et al.* (1992) for the first experimental application in a lab setting.

[8]See Jorgenson and Papciak (1981) for the first experimental application of a lab setting.

about the location of the threshold leads to more resource depletion (Budescu *et al.*, 1995a), unless decisions are sequentially made and only the second mover faces uncertainty (Lindahl and Johannesson, 2009).

Recently, there have been some contributions to this field extending the design by Budescu and colleagues. Ahsanuzzaman *et al.* (2022), for example, investigated and compared in a lab experiment responses to a known threshold, an uncertain threshold with a known probability distribution of possible thresholds, and an uncertain threshold with an unknown probability distribution (ambiguity) and also tested the effect of communication: They found that while threshold uncertainty (both risk and ambiguity) tends to increase CPR use, communication reduces the use of shared resources and increases social efficiency. This contrasts theoretical results showing that whereas increased risk may lead to more selfish behavior (i.e., to more consumption), increased ambiguity may have the opposite effect (communication is only cheap talk) (Aflaki, 2013).

Kidwai and de Oliveira (2020) introduced the case when the subjects do not know their exact group size and found that reducing threshold and group size uncertainty increases expected earnings from the resource. However, reducing threshold uncertainty is beneficial while tackling group size uncertainty requires a more nuanced approach, highlighting the importance of joint analysis.

In both the studies by Ahsanuzzaman *et al.* (2022) and Kidwai and de Oliveira (2020) the game was played multiple rounds but in each round, the resource was reset. Botelho *et al.* (2014) also used a request/extraction game design but with a dynamic model, where the duration of the game is determined endogenously by the users' collective decisions. They found that increased levels of uncertainty about the threshold level may lead to quicker depletion of a resource stock, but that players may also adopt strategy paths that guarantee the threshold will not be exceeded. When this uncertainty is reduced, they maintain a positive resource level for longer durations. Using a similar setup but with a one-shot game design, Maas *et al.* (2017) introduced an uncertain tipping point. In their model, two Nash equilibria exist. Both lead to a tragedy of the commons, but one is an inferior solution because it leads to assured resource destruction. Their experimental results show that uncertainty reduces coordination and increases the likelihood of resource destruction. However, taxes and sanctioning policies can prevent resource destruction.

5.3.1 Introducing Tipping Points and Path Dependency

Most of the CPR experiments referred to above assumed a static or even a fixed resource. Even in a multiple-period game, the game was often repeated, meaning that the resource was 'reset' every period. However, one could instead think of a dynamic game in which the resource itself changes over time and depends on decisions taken in previous rounds, which would be a more realistic description if the resource in question is a renewable natural resource, like a grassland, a fish stock or a forestry. In the previous decade, a 'new generation' of CPR experiments emerged. In this new generation of experiments, there has been a specific emphasis on including a more realistic description of the natural resource in the game designs (cf. Cardenas *et al.*, 2013; Janssen *et al.*, 2015; Lindahl *et al.*, 2021). Some of these designs have also included tipping points.

Differing outcomes in investment games — the role of field context. Cardenas *et al.* (2013) developed an experimental design that builds on the 'investment' type of design where participants make decisions on effort. The design incorporates inter-temporal dynamics and has features of path dependency of previous use including nonlinearity of payoffs (a tipping point). In the experiment each participant decides (privately without communication) in each round where in two different locations to put effort and how much effort to put in (low or high) to exert in the chosen location where there is a slightly higher return from a high effort compared to a low effort. The return from effort depends on the resources available in each location. However, when too much effort is put into one location the resource will move to the low availability for the next round. This situation can only be reversed when in two consecutive rounds little effort is put into the location. Thus, dependent on the behavior in previous rounds, participants are facing different states of resource availability with varying needs to cooperate, coordinate, and be patient (the design features are carefully explained in Castillo *et al.*, 2011). Cardenas *et al.* (2013) applied the design to different cases (e.g., fisheries and forestry) and found that high levels of earnings cannot be sustained even when rules on access (e.g., using a rotation scheme or quotas) have been imposed. Prediger *et al.* (2011) built on this design and investigated the impact of culture and ecology on cooperation in a CPR experiment. They compared the results of Namibian and South African farmers. They found that Namibian farmers manage to sustain the resource to a higher degree compared to South African farmers. They argued that the large difference between the two regions is due to a combination of different historical developments and ecological preconditions: Namibian resource users have longer experience in cooperative resource management and intact traditional norms.

Emerging cooperation in request games with path dependency and tipping points. The experimental paradigm developed by Lindahl and colleagues represents, as far as we know, the first attempts to incorporate tipping points in the ecosystem dynamics in a request game design (see Lindahl *et al.*, 2012, 2016). Being inspired by previous theoretical game theoretic dynamic models representing CPR situations with regime shifts (see e.g., Crépin and Lindahl, 2009; Mäler *et al.*, 2003), Lindahl and colleagues proposed a dynamic extraction/request game where the natural resource growth follows a logistic-type of dynamics but with discrete steps and also that one of these discrete steps is a sharp threshold. If the stock size falls below this critical threshold there is a regime shift in the ecosystem dynamics, to a state where the resource growth rate is significantly lower. They show that the game theoretic prediction is that when there is such a threshold in the resource dynamics, we can expect less over-exploitation (compared to a case when there is no such threshold in the dynamics). This is also confirmed in their experiment. A similar theoretical result in a CPR setting is found in Miller and Nkuiya (2016), see Section 3.3. for more details) Lindahl *et al.* (2016) also found, however, that the presence of the threshold leads to more efficient resource management among cooperative groups, and moreover that cooperation is endogenous to the resource dynamics. This is something that the theoretical model cannot predict. They attributed the new findings to effective communication among players. It is the threat of reaching a critical tipping point, beyond which the growth rate will drop drastically, that triggers more effective communication within the group, enabling stronger commitment for cooperation and more knowledge sharing, which together explains the results. A simplified version of the design used by Lindahl *et al.* (2016) is also used in a field setting with artisanal fishers in Thailand. In their study, Lindahl and Jarungrattanapong (2022) confirmed the lab results by Lindahl *et al.* and found that groups confronted with a potential tipping point are more likely to form cooperative agreements compared to groups not confronted with such a drop. However, they also found that many groups under-exploit the resource and that over-exploitation is driven by socioeconomic contextual factors.

The experimental design by Lindahl *et al.* (2012, 2016) has been modified by Schill *et al.* (2015) to allow for risk, meaning that the participants know there is a tipping point in the resource dynamics with a certain probability (0.1, 0.5, 0.9, 1). The authors found some interesting behavior. Only when the threshold is certain or the risk is extremely high, people would be more prone to agree initially on a common exploitation strategy to avert the latent shift. They also show the positive collective action effect is influenced by how risk and probabilities are communicated and perceived by the users. Modifying the design by not only simplifying it but also introducing ambiguity, and taking it to the field in small-scale fishers, Schill and Rocha (2023) found that groups that are uncertain about the thresholds are likely to sustain higher stock levels, thus potentially averting regime shifts (see also Rocha *et al.*, 2020) for an individual-level analysis). However, community-level factors influence exploitation, and appear to limit or even eliminate treatment effects; highlighting the significant influence of context on behavior.

Regulation can promote more efficient outcomes. Lindahl *et al.* (2017) used the design by Lindahl *et al.* (2016) and tested in a lab setting how a CPR system regulated by a quota performs in comparison with an unregulated system when there is a potential regime shift in the ecosystem dynamics. They predict that the unregulated system will perform equally or worse with respect to inefficiencies stemming from over-exploitation. Contradictory to their theoretical prediction, however, the results reveal that regulated systems on average are associated with lower efficiency, which stems both from under and over-exploitation. They suggested that the outcome could be due to the (lack of) information with respect to the resource dynamics given to regulated groups, and/or that the responsibility for proper management in this case is transferred to the regulator.

This contrasts the results from Ntuli *et al.* (2023) that investigated the behavioral responses of real natural CPR users to three policy interventions — sanctioned quotas, information provisioning, and a combination of both. They focused on situations in which users find utility in multiple resources (pastures and wild animal stocks) that all stem from the same ecosystem with complex dynamics entailing a tipping point. They found that user groups are likely to manage these natural resources more efficiently when facing a policy intervention (either a sanctioned quota, receiving information about a drastic drop in the stocks' regrowth below a threshold, or a combination of both), compared to groups facing no intervention. A sanctioned quota is likely to perform better than providing information about the existence of a threshold. However, having information about the threshold also leads to higher efficiency and fewer depletion cases, compared to a situation without any intervention.

Jules *et al.* (2020) also used an extension of Lindahl *et al.* (2016) for a fisheries problem with sanctions (such as trade restrictions). In a lab setting, they also compared the case with no tipping point in the dynamics with a threshold in the dynamics but added an uncertainty treatment in which the location of the threshold is uncertain. They showed that the threat of economic sanctions can induce more cooperative behavior, less over-exploitation, and more precautionary management of resources. The result is reinforced by the uncertainty about the location of the tipping point.

The studies by Lindahl *et al.* (2016, 2017), Schill *et al.* (2015), Lindahl and Jarungrattanapong (2022), Schill and Rocha (2023), and Ntuli *et al.* (2023) all used variants of the same experimental design. Hence, it seems one can conclude that a known tipping point in the resource dynamics can lead to more cooperative outcomes and less over-exploitation (compared to a case when there is no such tipping point). How people respond to different types of uncertainty around the existence of a tipping point is ambiguous and can depend on the level and type of uncertainty but also on how these features are communicated. Policies, like quotas and sanctions, can help reduce over-exploitation even further. Moreover, field applications show that while some lab results can be confirmed, socio-economic contextual factors may influence how people play these games. It is also important to note that the results hinge on that participants are allowed to communicate, which means that one cannot draw any conclusion on how the results change when the communication is fixed. For example, the experiments by Cardenas *et al.* (2013), Castillo *et al.* (2011), and Prediger *et al.* (2011) showed that without communication and in the presence of a tipping point it is difficult for users to sustain high stock levels over time. It is interesting that the emergence of more or less effective communication, and hence cooperation found in the studies by Lindahl *et al.* (2016, 2017), Schill *et al.* (2015), and Lindahl and Jarungrattanapong (2022), is endogenous to the resource dynamics, which theory did not predict. This result is quite novel as communication is typically tested for as a treatment where the control is typically that communication is not allowed. To let communication be a variable (not controlled) could open up new types of experimental designs in other CPR games and PG games.

5.4 Linking Experimental Evidence with Theory

What links exist between purely theoretical studies and experimental ones? Firstly, it is worth noting that there is often not a one-to-one correspondence between the theoretical models referred to in Sections 2.2 and 3.3 and the experimental studies. This is because it is necessary to make simplifications when setting up an experiment. For one, infinite time horizons need to be replaced with definite or indefinite time horizons. Further, the choice sets of actors need to be limited in an experiment and the resource dynamics presented need to be simplified. In this respect, the two strands of literature complement each other. Theoretical models can account for and explore different management options and implications for quite complex resource dynamics. Experiments, on the other hand, make simplifications on these aspects but allow for more complex behaviors of actors. Experimental papers often have testable hypotheses formulated relying on game theory and rational actors. A realization is then typically that the assumptions of rational actors maximizing payoff and that 'talk is cheap' seldom hold. People in a CPR setting communicate, cooperate, and exploit less than predicted, and in PG settings contribute more than predicted.

Despite these differences, some of the theoretical results we refer to in Section 3.3 are confirmed in experiments as we have tried to highlight. For example, people can coordinate to avoid a tipping point in a PG setting but uncertainty around the location of the tipping point makes coordination more challenging and is sometimes even detrimental to coordination. Unequal distribution of wealth is another potential barrier to coordination. In a CPR setting, both theoretical results highlighted in Section 3.3 and experimental results show that resource users can avoid a known disaster (tipping point).

Another difference between the two strands of literature is that often, the studies are designed to answer different research questions. For example, a theoretical paper introducing a regime shift in a CPR setting typically explores different system outcomes under different management settings, comparing an optimal (sole-owner) outcome with the outcome of a non-cooperative CPR management setting (see, e.g., Mäler *et al.*, 2003 and Crépin and Lindahl, 2009) and maybe explores further the type of policy that could be implemented in such a setting. An experimental investigation of the same situation can instead be set up to explore under which conditions we can expect a cooperative outcome (that is not necessary). Sections 2.2 and 3.3 have nevertheless served as inspiration to the experimental community working on PG and CPR games, opening up new areas of research where more realistic ecosystem properties and 'realities' have been implemented in PG and CPR games, leading to new sets of timely and interesting research questions that would otherwise not have been formulated. The next step is perhaps that insights from experimental studies can inspire advances in theoretical modeling, for example, to introduce more realistic behavioral assumptions. We do not know of any such attempts in resource management models involving regime shifts and tipping points.

Another interesting area to explore would be to investigate how individuals perceive various aspects related to tipping points. The experiments that have been done are in a strategic setting involving some kind of social dilemma. But how does an individual perceive complex resource dynamics involving tipping points and their inherent uncertainties? Are there cognitive limitations and what are they? As far as we know only a few experimental studies investigate the (mis)perceptions individual resource users have about dynamic resources with smooth resource dynamics (see, e.g., Moxnes, 2000). This could be an additional area of research to explore. Insights from experiments can then be used to build more realistic models.

6 Concluding Remarks and Future Avenues

We have presented a comprehensive overview of the economics of tipping points, emphasizing its relevance in resource and climate economics. This multifaceted study covers deterministic tipping points, hazard rate models with inherent uncertainties, behavioral insights from experimental economics, and the contributions of integrated assessment modeling.

Our exploration of deterministic tipping points emphasizes the importance of non-convexities in understanding critical thresholds and enhancing resource and environmental management. Moving forward, we should expand threshold representations, consider spatial implications, and probe the interactions between varying threshold types. The importance of uncertain tipping points, as demonstrated by the hazard rate models, underscores the need for resilience against potential shifts. Future efforts should address multiple resilience stocks and explore the facets of adaptation.

While the theoretical models have allowed for a better representation of irreversible damages, challenges like uncertainties in process comprehension, the subjectivity of hazard rate function selection, and parameter estimation remain. There's a pressing need to refine these models, emphasizing the inclusion of more tipping elements, fostering active over passive learning approaches, and integrating experimental findings.

Our dive into experimental economics emphasizes that human behavior in the face of tipping points is multifaceted. The future should focus on bridging the gap between theory and experimental evidence, understanding decision-making nuances, and leveraging interdisciplinary methods to understand human motivations and actions.

Integrated assessment models with tipping points, while insightful, have intrinsic limitations, such as uncertainties in processes and challenges in modeling tipping points' interactions and post-event impact evaluation. One potential solution is a more integrated approach, combining experimental findings with model development, to better reflect real world scenarios.

In intertwining experimental evidence with theory, we notice some discrepancies. However, theoretical models and experimental setups are complementary: while the theoretical models provide a framework for optimal management options, experiments offer a lens into human behaviors. This relationship should be nurtured. Insights from experimental studies could refine theoretical modeling, incorporating more realistic behavioral assumptions. There is also an avenue to explore individual perceptions toward tipping points, taking into account cognitive limitations, which can inform both experimental and model designs.

To conclude, the multifarious nature of tipping points in resource and climate economics demands a composite approach. This review acts as a foundation, underlining current knowledge while pinpointing areas for further exploration. By converging insights from different models and experimental economics, we can aspire to a more nuanced understanding, paving the way for robust and informed policy-making. Tackling the intricacies of tipping points is challenging, but through persistent research, collaboration, and innovative thinking, we may move towards a sustainable future.

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