Online Appendix: Value of Reverse Factoring under Make-to-Order Production Environments

Appendix A: Proofs

Proof of Proposition **3.1***(i)*

Let us first consider the cash flows to the SME under the conventional external financing case. Note that the free cash reserves continuously earn the risk-free rate. By assumption, if the demand arrives before $t = T - l_{\text{ext}}$, then the SME does not need to borrow, otherwise the SME borrows $L(r_f) = z + (c\mu - ye^{r_f})e^{r_f(T-\tau)}$ at time T and pays back a stochastic amount χ (due to credit risk) at time $t = \tau + l_{ext} > T$. Under the risk-neutral measure, the loan is priced by the bank so that:

$$
L(r_f) = \frac{E^{\mathcal{Q}}(\chi)}{e^{r_{\text{ext}}\Delta l_{\text{ext}}}},
$$

where $\Delta l_{ext} = (\tau + l_{ext} - T)$ is the loan duration. Hence for a given τ , from Figure 2, cash flows discounted to $t = 0$, is given by:

$$
V_{ext}^{sme}(.|\tau) = \frac{p_s \mu}{e^{r_f(\tau + l_{ext})}} - \frac{c\mu}{e^{r_f \tau}} + (\frac{L(r_f)}{e^{r_f T}} - \frac{E^{\mathcal{Q}}[\chi]}{e^{r_f(\tau + l_{ext})}}) I_{\{\tau > T - l_{ext}\}}
$$

\n
$$
= \frac{p_s \mu}{e^{r_f(\tau + l_{ext})}} - \frac{c\mu}{e^{r_f \tau}} + (\frac{L(r_f)}{e^{r_f T}} - \frac{e^{r_{ext}\Delta_{ext}}L(r_f)}{e^{r_f(\tau + l_{ext})}}) I_{\{\tau > T - l_{ext}\}}
$$

\n
$$
= \frac{p_s \mu}{e^{r_f(\tau + l_{ext})}} - \frac{c\mu}{e^{r_f \tau}} + (1 - \frac{e^{r_{ext}\Delta_{ext}}}{e^{r_f(\tau + l_{ext}-T)}}) \frac{L(r_f)}{e^{r_f T}} I_{\{\tau > T - l_{ext}\}}
$$

\n
$$
= \frac{p_s \mu}{e^{r_f(\tau + l_{ext})}} - \frac{c\mu}{e^{r_f \tau}} + (1 - e^{b_s \Delta_{ext}}) \frac{L(r_f)}{e^{r_f T}} I_{\{\tau > T - l_{ext}\}}
$$

\n
$$
(\Delta l_{ext} = \tau + l_{ext} - T, r_{ext} = b_s + r_f).
$$

Applying the first order Taylor series approximation with respect to (r_f, b_s) around $(0, 0)$ we obtain:

$$
\hat{V}_{ext}^{sme}(. | \tau) = p_s \mu (1 - r_f(\tau + l_{ext})) - c \mu (1 - r_f \tau) - b_s \Delta l_{ext}(z + c \mu - y) I_{\{\tau > T - l_{ext}\}}.
$$

Then, after taking the expectation with respect to the τ , *value* of the cash flows at $t = 0$ is

$$
\hat{V}_{ext}^{sme} = E_{\tau}[\hat{V}_{ext}^{sme}(.|\tau)] = E_{\tau}[p_s\mu(1 - r_f(\tau + l_{ext})) - c\mu(1 - r_f\tau) - b_{ext}\Delta l_{ext}(z + c\mu - y)I_{\{\tau > T - l_{ext}\}}],
$$

= $p_s\mu(1 - r_f(\overline{\tau} + l_{ext})) - c\mu(1 - r_f\overline{\tau}) - b_sLA(T - l_{ext}).$

where $\overline{\tau} = E(\tau)$, $L = L(r_f = 0) = z + c\mu - y$ and $A(T - l) = E_{\tau}[\Delta H_{\{\tau > T - l\}}]$ for $l = l_{ext}$, l_{rev} .

Similarly, the value of the cash flows under reverse factoring is

$$
\hat{V}_{rev}^{sme} = p_s \mu (1 - r_f(\overline{\tau} + l_{rev})) - c\mu (1 - r_f \overline{\tau}) - (b_o + k)LA(T - l_{rev}).
$$
 Note that we assume there is no default risk for the corporation. However, our proof and analysis can be extended for the case with corporation bankruptcy. This requires replacing the payment of account receivables ($p_s \mu$) with an appropriate stochastic cash flow and defining a stochastic asset process for the corporation just like the SME. For simplicity we skipped this case.

Then, the expected benefits of the SME from reverse factoring are determined by:

$$
\pi^{sme} = \hat{V}_{rev}^{sme} - \hat{V}_{ext}^{sme} = b_s A (T - l_{ext}) - (b_o + k) A (T - l_{rev}) L - p_s \mu r_f (l_{rev} - l_{ext}).
$$

Note that for notational simplicity we present \hat{V}^{sme}_{rev} and analogous terms as V^{sme}_{rev} in the main text.

Proof of Proposition **3.1***(ii)*

Under conventional external financing, the value of the cash flows to the corporation, for a given τ , at $t = 0$ is given by (note that there is no default risk for the corporation and corporation does not need external funds):

$$
V_{ext}^{oem}(.\mid \tau)=-\frac{P_s\mu}{e^{r_f(\tau+l_{ext})}}.
$$

Applying the first order Taylor series approximation with respect to *r^f* around 0, we get:

$$
\hat{V}_{ext}^{oem} = -E_{\tau}[p_s \mu (1 - r_f(\tau + l_{ext}))] = -p_s \mu (1 - r_f(\overline{\tau} + l_{ext})).
$$

Similarly, the value of the cash flows under reverse factoring is:

$$
\hat{V}_{rev}^{oem} = -p_s \mu (1 - r_f (\overline{\tau} + l_{rev})).
$$

Accordingly, the benefits of the corporation from reverse factoring are determined by:

$$
\pi^{oem} = \hat{V}_{rev}^{oem} - \hat{V}_{ext}^{oem} = p_s \mu r_f (l_{rev} - l_{ext}).
$$

Proof of Proposition **3.1***(iii)*

Adding up the corporation's and SME's benefits in parts (*i*) and (*ii*) gives the desired result.

Proof of Proposition **3.2**

Directly follows from the proof of Proposition 3.1.

Proof of Proposition **3.3***(i)*

The optimal contract for the SME is obtained by maximizing the SME's benefits with respect to the contract terms subject to the participation constraint of the corporation and non-negative *k*:

$$
\hat{\pi}^{sme} = \max_{k, l_{rev}} \pi^{sme} = \max_{k, l_{rev}} L(b_s A(T - l_{ext}) - (b_o + k)A(T - l_{rev})) - p_s \mu r_f (l_{rev} - l_{ext})
$$

s.t. $l_{rev} \ge l_{ext}, k \ge 0$.

Observing that the first partials of the objective function are non-positive proves the desired result:

$$
\frac{\partial \pi^{sme}}{\partial k} = -b_o A (T - l_{rev}) < 0.
$$
\n
$$
\frac{\partial \pi^{sme}}{\partial l_{rev}} = -(b_o + k) L \frac{\partial A (T - l_{rev})}{\partial l_{rev}} - p_s \mu r_f < 0 \qquad (\frac{\partial A (T - l_{rev})}{\partial l_{rev}}) > 0.
$$

Proof of Proposition **3.3***(ii)*

The optimal contract for the corporation is obtained by maximizing the corporation's benefits with respect to the contract terms subject to the participation constraint of the SME and nonnegative bank fees:

$$
\hat{\pi}^{oem} = \max_{k, l_{rev}} \pi^{oem} = \max_{k, l_{rev}} p_s \mu r_f (l_{rev} - l_{ext})
$$

s.t. $Lb_s A(T - l_{ext}) \ge L(b_o + k) A(T - l_{rev}) + p_s \mu r_f (l_{rev} - l_{ext})$
 $k \ge 0.$

It is clear that the objective function is increasing in *lrev* and constant in *k*. Now observe that the right hand side of the SME's participation constraint is monotone increasing in *lrev* and *k*. This implies that optimal *k* is zero, and the optimal *lrev* makes the SME participation tight.

Proof of Proposition **3.3***(iii)*

The optimal contract for the supply chain is obtained by maximizing the total supply chain benefits with respect to the contract terms subject to the participation constraints of the SME and corporation, and non-negative bank fees:

$$
\hat{\pi}^{total} = \max_{k, l_{rev}} \pi^{total} = \max_{k, l_{rev}} L\left(b_s A(T - l_{ext}) - (b_o + k)A(T - l_{rev})\right)
$$

s.t. $L\left(b_s A(T - l_{ext}) - (b_o + k)A(T - l_{rev})\right) \ge r_f p_s \mu(l_{rev} - l_{ext}),$

$$
l_{rev} \ge l_{ext}, k \ge 0.
$$

First, observe that the objective function is decreasing in *k* and *lrev*, and the right hand side of the SME's participation constraint is monotone increasing in *k* and *lrev*. Together with the constraints $l_{rev} \ge l_{ext}$ and $k \ge 0$, this implies the desired result.

Proof of Corollary **3.1**

Recall that the corporation`s maximum benefit is given by

$$
p_{s}r_{f}\mu(l_{rev}^{*}-l_{ext}) = b_{s}A(T-l_{ext})L - b_{o}A(T-l_{rev}^{*})L.
$$
 Then,

$$
\frac{\hat{\pi}^{oem}}{\hat{\pi}^{i\text{total}}} = \frac{p_{s}r_{f}\mu(l_{rev}^{*}-l_{ext})}{(b_{s}-b_{o})A(T-l_{ext})L} = \frac{b_{s}A(T-l_{ext})L - b_{o}A(T-l_{rev}^{*})L}{b_{s}A(T-l_{ext})L - b_{o}A(T-l_{ext})L} = \frac{b_{s}A(T-l_{ext}) - b_{o}A(T-l_{rev}^{*})}{b_{s}A(T-l_{ext})L}.
$$

Since
$$
\frac{\partial A(T-l)}{\partial l} > 0
$$
, we have $b_o A(T-l_{rev}^*) > b_o A(T-l_{ext})$ which implies that $\frac{\hat{\pi}^{oem}}{\hat{\pi}^{total}} < 1$. The

sensitivity results can be easily established by checking the first derivative and are omitted here for brevity.

Proof of Proposition **4.1***(i)*

The tree diagram below describes the possible set of borrowing scenarios under stochastic demand for $l=l_{ext}$ and l_{rev} representing the conventional external financing and reverse factoring respectively.

 Figure A.1. The SME's Borrowing Amount under base-case with stochastic demand

Then, for a given realization of stochastic parameters, following the developments in the proof of Theorem 1, the discounted value of the cash flows under conventional external financing is:

$$
V_{ext}^{sme}(. | \tau, \xi) = p_s \xi e^{-r_f(\tau + l_{ext})} - c \xi e^{-r_f \tau} + (\frac{(z + c \xi e^{r_f(T-\tau)} - y e^{r_f \tau})}{e^{r_f \tau}} - \frac{(z + c \xi e^{r_f(T-\tau)} - y e^{r_f \tau}) e^{r_{ext}\Delta l_{ext}}}{e^{r_f \tau}}) I_{\Omega_i(l_{ext})}
$$

+
$$
\left(\frac{c \xi - y e^{r_f \tau}}{e^{r_f \tau}} - \frac{(c \xi - y e^{r_f \tau}) e^{r_{ext}l_{ext}}}{e^{r_f (\tau + l_{ext})}} + \frac{z}{e^{r_f \tau}} - \frac{z e^{r_{ext}\Delta l_{ext}}}{e^{r_f (\tau + l_{ext})}}\right) I_{\Omega_2(l_{ext})}
$$

+
$$
(\frac{c \xi - y e^{r_f \tau}}{e^{r_f \tau}} - \frac{(c \xi - y e^{r_f \tau}) e^{r_{ext}l_{ext}}}{e^{r_f (\tau + l_{ext})}}) I_{\Omega_3(l_{ext})},
$$

where
$$
\Omega_1(l_{ext}) = \{\xi, \tau : c \xi \leq y e^{r_f \tau}, z + c \xi e^{r_f (T-\tau)} - y e^{r_f \tau} \geq 0, \tau \geq T - l_{ext}\},
$$

$$
\Omega_2(l_{ext}) = \{\xi, \tau : c \xi \geq y e^{r_f \tau}, \tau \geq T - l_{ext}\},
$$

$$
\Omega_3(l_{ext}) = \{\xi, \tau : c \xi \geq y e^{r_f \tau}, \tau \leq T - l_{ext}\},
$$

Next, we separately approximate the indicator sets and the discounted cash flows with the first order Taylor series approximation with respect to (r_f, b_s) around $(0, 0)$. Then, after taking the expected values over the random quantities, we obtain:

$$
\hat{V}_{ext}^{sme} = E_{\xi,\tau} [p_s \xi (1 - r_f(\tau + l_{ext})) - c \xi (1 - r_f \tau) - b_s \Delta l_{ext} (z + c \xi - y) I_{\hat{\Omega}_{\xi}(l_{ext})} - (b_s l_{ext}(c \xi - y) + b_s \Delta l_{ext} z) I_{\hat{\Omega}_{\xi}(l_{ext})} \n- b_s l_{ext}(c \xi - y) I_{\hat{\Omega}_{\xi}(l_{ext})}].
$$

1 2 3 $where \quad \hat{\Omega}_1(l_{ext}) = {\xi, \tau : c\xi \leq y(1 + r_f\tau), z + c\xi(1 + r_f(T - \tau)) - y(1 + r_fT) \geq 0, \tau \geq T - l_{ext}}$ $\hat{\Omega}_2(l_{ext}) = {\xi, \tau : c \xi \ge y(1 + r_f \tau), \tau \ge T - l_{ext}}$ $\hat{\Omega}_{3}(l_{ext}) = {\xi, \tau : c \xi \geq y(1 + r_f \tau), \tau \leq T - l_{ext}}.$

Similarly, the derivation for the case with reverse factoring is

$$
\hat{V}_{rev}^{sme} = E_{\xi,\tau}[p_s \xi (1 - r_f(\tau + l_{rev})) - c\xi (1 - r_f \tau) - (b_o + k)\Delta l_{rev}(z + c\xi - y)I_{\hat{\Omega}_{\xi(l_{rev})}} - ((b_o + k)l_{rev}(c\xi - y) + (b_o + k)\Delta l_{rev}z)I_{\hat{\Omega}_{\xi(l_{rev})}} - (b_o + k)l_{rev}(c\xi - y)I_{\hat{\Omega}_{\xi(l_{rev})}}].
$$

where
$$
\hat{\Omega}_{1}(l_{rev}) = \{\xi, \tau : c\xi \leq y(1 + r_f \tau), z + c\xi(1 + r_f (T - \tau)) - y(1 + r_f \tau) \geq 0, \tau \geq T - l_{rev}\},
$$

$$
\hat{\Omega}_{2}(l_{rev}) = \{\xi, \tau : c\xi \geq y(1 + r_f \tau), \tau \geq T - l_{rev}\},
$$

$$
\hat{\Omega}_{3}(l_{rev}) = \{\xi, \tau : c\xi \geq y(1 + r_f \tau), \tau \leq T - l_{rev}\}.
$$

Then, the expected benefit for the SME is given by $\pi^{sme} = \hat{V}_{rev}^{sme} - \hat{V}_{ext}^{sme}$:

$$
=E_{\xi,\tau}[p_{s}\xi(1-r_{f}(\tau+l_{rev}))-c\xi(1-r_{f}\tau)-(b_{o}+k)\Delta l_{rev}(z+c\xi-y)I_{\hat{\Omega}_{\xi(l_{rev})}}-(\frac{b_{o}+k}{l_{rev}(c\xi-y)}+(\frac{b_{o}+k}{l_{rev}z})I_{\hat{\Omega}_{\xi(l_{rev})}}]
$$

\n
$$
-(b_{o}+k)l_{rev}(c\xi-y)I_{I_{\hat{\Omega}_{\xi(l_{rev})}}}] - E_{\xi,\tau}[p_{s}\xi(1-r_{f}(\tau+l_{ext})) - c\xi(1-r_{f}\tau)-b_{s}\Delta l_{ext}(z+c\xi-y)I_{\hat{\Omega}_{\xi(l_{ext})}}
$$

\n
$$
-(b_{s}l_{ext}(c\xi-y)+b_{s}\Delta l_{ext}z)I_{\hat{\Omega}_{\xi(l_{ext})}}-b_{s}l_{ext}(c\xi-y)I_{\hat{\Omega}_{\xi(l_{ext})}}].
$$

\nwhere $\hat{\Omega}_{\xi}(l) = \{\xi,\tau:\xi \leq y(1+r_{f}\tau)/c,\xi \geq (y(1+r_{f}\tau)-z)/(c(1+r_{f}(\tau-\tau))),\tau \geq T-l\},$
\n
$$
\hat{\Omega}_{2}(l) = \{\xi,\tau:\xi \geq y(1+r_{f}\tau)/c,\tau \geq T-l\},
$$

\n
$$
\hat{\Omega}_{3}(l) = \{\xi,\tau:\xi \geq y(1+r_{f}\tau)/c,\tau \leq T-l\}.
$$

for $l = l_{rev}$, l_{ext} .

After rearranging the terms and omitting the "^" for notational simplicity, we get:

$$
\pi^{sme} = b_s \Big(E_{\xi,\tau} [(z + c\xi - y)\Delta l_{ext} I_{\Omega_{\xi(l_{ext})}} + ((c\xi - y)l_{ext} + z\Delta l_{ext}) I_{\Omega_{\xi(l_{ext})}} + (c\xi - y)l_{ext} I_{\Omega_{\xi(l_{ext})}}] \Big) - (b_o + k) \Big(E_{\xi,\tau} [(z + c\xi - y)\Delta l_{rev} I_{\Omega_{\xi(l_{rev})}} + ((c\xi - y)l_{rev} + z\Delta l_{rev}) I_{\Omega_{\xi(l_{rev})}} + (c\xi - y)l_{rev} I_{\Omega_{\xi(l_{rev})}}] \Big) - p_s \mu r_f (l_{rev} - l_{ext}).
$$

where
$$
\Omega_1(l) = \{\xi, \tau : \xi \leq y(1 + r_f \tau) / c, \xi \geq (y(1 + r_f T) - z) / (c(1 + r_f (T - \tau))), \tau \geq T - l\},
$$
\n $\Omega_2(l) = \{\xi, \tau : \xi \geq y(1 + r_f \tau) / c, \tau \geq T - l\},$ \n $\Omega_3(l) = \{\xi, \tau : \xi \geq y(1 + r_f \tau) / c, \tau \leq T - l\},$

for $l = l_{rev}$, l_{ext} .

Letting $H(l) = E_{\xi, \tau}[(z + c\xi - y)\Delta U_{\Omega(l)} + ((c\xi - y)l + z\Delta l)I_{\Omega_2(l)} + (c\xi - y)lI_{\Omega_3(l)}]$ for $l = l_{rev}$, lext, we obtain

$$
\pi^{sme} = b_s H(l_{ext}) - (b_o + k)H(l_{rev}) - p_s \mu r_f (l_{rev} - l_{ext}).
$$

Proof of Proposition **4.1***(ii)*

Follows directly from the proof of Proposition 3.1(*ii*).

Proof of Proposition **4.1***(iii)*

Follows directly from Proposition 4.1(*i*) and 4.1(*ii*).

Appendix B: Exact Solutions

In this appendix we provide the exact versions of Propositions 3.1, 3.2 and 3.3, and compare them with the results in the body of the paper that are obtained by using the Taylor series approximation of the cash flows.

PROPOSITION B.1 (exact version of Proposition 3.1). *In a make-to-order business environment, the reverse factoring contract* $v = (r_{rev}, l_{rev})$ generates the following benefits for each party.

(i)For the SME,

$$
\pi^{sme} = E_{\tau}[L(r_f) e^{-r_f T} G(b_s, l_{ext})] - E_{\tau}[L(r_f) e^{-r_f T} G(b_o + k, l_{rev})] - p_s \mu(E_{\tau}[e^{-r_f(\tau + l_{ext})}] - E_{\tau}[e^{-r_f(\tau + l_{rev})}]),
$$

(ii) for the corporation, $\pi^{oem} = p_s \mu(E_{\tau}[e^{-r_f(\tau + l_{ext})}] - E_{\tau}[e^{-r_f(\tau + l_{rev})}]),$
(iii) for both firms in total, $\pi^{total} = E_{\tau}[L(r_f) e^{-r_f T} G(b_s, l_{ext})] - E_{\tau}[L(r_f) e^{-r_f T} G(b_o + k, l_{rev})].$
where $G(b_o + k, l_{rev}) = (e^{(b_o + k)\Delta l_{rev}} - 1)I_{\{\tau > T - l_{rev}\}}, G(b_s, l_{ext}) = (e^{b_s \Delta l_{ext}} - 1)I_{\{\tau > T - l_{ext}\}}, \Delta l_{ext} = (\tau + l_{ext} - T),$
 $\Delta l_{rev} = (\tau + l_{rev} - T).$

PROPOSITION B.2 (exact version of Proposition 3.2). *In a make-to-order business environment, the following participation constraints apply to the reverse factoring contract* $v = (r_{rev}, l_{rev})$ *:*

(i) for the SME,

$$
E_{\tau}[L(r_f) e^{-r_f T} G(b_s, l_{ext})] - E_{\tau}[L(r_f) e^{-r_f T} G(b_o + k, l_{rev})] \ge p_s \mu\Big(E_{\tau}[e^{-r_f (\tau + l_{ext})}] - E_{\tau}[e^{-r_f (\tau + l_{rev})}]\Big),
$$

(ii) for the corporation, $E_{\tau}[e^{-r_f (\tau + l_{ext})}] \ge E_{\tau}[e^{-r_f (\tau + l_{rev})}],$

(iii) for both firms in total, $E_r[L(r_f) e^{-r_fT} G(b_s, l_{ext})] \ge E_r[L(r_f) e^{-r_fT} G(b_o + k, l_{rev})]$.

PROPOSITION B.3 (exact version of Proposition 3.3). *In a make-to-order business environment, the reverse factoring contract* $v = (r_{rev}, l_{rev})$ *maximizes*

(i) the SME's expected benefit when $(r_{rev} = r_f + b_o, l_{rev} = l_{ext})$ and k=0. The maximum benefit for the SME is $E_{\tau}[L(r_f) e^{-r_f T} G(b_s, l_{ext})] - E_{\tau}[L(r_f) e^{-r_f T} G(b_o, l_{ext})],$

(ii) the corporation's expected benefit when $(r_{rev} = r_f + b_o, l_{rev} = l_{rev}^*)$ and k=0. The maximum benefit for the corporation, $p_s\mu\left(E_\tau[e^{-r_f(\tau+l_{ev})}]-E_\tau[e^{-r_f(\tau+l_{rev}^*)}]\right)$, where l^*_{rev} is the payment period that *makes the SME's participation constraint tight, i.e.,*

$$
E_{\tau}[L(r_f) e^{-r_f T} G(b_s, l_{ext})] - E_{\tau}[L(r_f) e^{-r_f T} G(b_o, l_{rev}^*)] = p_s \mu\Big(E_{\tau}[e^{-r_f(\tau + l_{ext})}] - E_{\tau}[e^{-r_f(\tau + l_{rev}^*)}]\Big),
$$

(iii)the total benefit when $(r_{rev} = r_f + b_o, l_{rev} = l_{ext})$ and k=0. The maximum supply chain benefit is $E_{\tau}[L(r_{f})\mathrm{e}^{-r_{f}T}G(b_{s},l_{ext})]-E_{\tau}[L(r_{f})\mathrm{e}^{-r_{f}T}G(b_{o},l_{ext})].$

The exact version of Proposition 3.1 in Appendix B is materially identical to the version obtained under the Taylor series approximation in the body of the manuscript. In Proposition B.1, financing costs are captured by the *G*(.) function which is, however, much harder to interpret than the expressions in Proposition 3.1.

Taylor series approximation only eliminates interest paid on interest, and hence has a minor effect on the benefits and the participation constraint. We also conduct a comprehensive numerical analysis to explore the effect of approximation. Below we present a representative set of these analysis. For the base-case parameter values, the approximate and exact participation constraints

are described below as a function of the risk-free interest rate (the key parameter affecting the approximation quality). We have conducted additional analyses by varying capital market frictions and working capital policy. These results are also presented below. Our numerical analysis confirms our analytical intuition that the gap between the exact and approximate participation constraints is minor, and in many cases (as shown below) not even discernible (see Figures B.1 and B.2).

Figure B.1. Approximate and Exact Participation Constraints for the SME: impact of r_f ($y=1000$)

Figure B.2. Approximate and Exact Participation Constraints for the SME: impact of WCP (r_f =0.02)

