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Issue Unbundling via Citizens' Initiatives: Technical Appendix

Abstract

This appendix provides proofs of the results presented in the paper "Issue Unbundling via Citizens' Initiatives."

In this appendix we provide proofs of the results presented in the paper "Issue Unbundling via Citizens' Initiatives." The numbering of the results is the same as in the paper.

PROOF OF PROPOSITION 1

We will show that the game in which the majority groups in each party simultaneously choose candidates has a unique Nash equilibrium which involves Party A selecting a candidate of type (l, t_A^*) and Party B a candidate of type (r, t_B^*). The first point to note is that for the majority group in each party, any strategy involving the selection of a candidate who does not share its preferred public spending preferences is strictly dominated. We prove this only for Party A, the argument for Party B being identical. It is easy to show that the strategy $(r, -t_A^*)$ is strictly dominated by the strategy (l, t_A^*), so we concentrate on showing that the strategy (r, t_A^*) is dominated by (l, t_A^*). When Party B selects candidates of types (l, t_A^*), $(l, -t_A^*)$ and (r, t_A^*), this is clear. The non-obvious case is that in which Party B selects a candidate of type $(r, -t_A^*)$ and $t_A^* = p$. A majority member of Party A obtains an expected payoff

$$\psi(\gamma^l - \gamma^r)[b(g^*(l), l) + \theta_p] + [1 - \psi(\gamma^l - \gamma^r)]b(g^*(r), l)$$

from choosing a candidate of type (l, p). The payoff from choosing a type (r, p) candidate is

$$b(g^*(r), l) + \psi(\gamma_p - \gamma_a)\theta_p.$$

Subtracting the latter from the former, the difference can be expressed as:

$$\psi(\gamma^l - \gamma^r)\Delta b(l) - [\psi(\gamma_p - \gamma_a) - \psi(\gamma^l - \gamma^r)]\theta_p.$$

This is positive by Assumption 1.

Now consider the game in which the majority members of Party A select from the strategies (l, t_A^*) and $(l, -t_A^*)$, while the majority members of Party B select from the strategies (r, t_B^*) and $(r, -t_B^*)$. Then we claim that for the majority group of each party, selecting a candidate who does not share its preferred regulatory preferences is strictly dominated. Consider Party A and the strategy $(l, -t_A^*)$. Selecting a candidate of type (l, t_A^*) has no impact on the probability that Party A wins (which is positive) and leads to a strictly higher payoff if Party A wins. Similarly for Party B.

It follows that the game in which the majority groups in each party simultaneously choose candidates is solvable by iterated (strict) dominance. The solution involves Party A selecting a candidate of type (l, t_A^*) and Party B a candidate of type (r, t_B^*). This is then the unique Nash equilibrium of the game. ■

PROOF OF PROPOSITION 2

We need to show that, under Assumption 2, the unique equilibrium involves the majority members of Party A selecting a type (l, a) candidate, while the majority members of Party B select a type (r, a) candidate. We first demonstrate that this is an equilibrium. We show only that it is a best response for the majority members of Party A to select a type (l, a) candidate when Party B selects a type (r, a) candidate. The argument for Party B is similar.

The expected payoff of a majority member of Party A when the two parties select candidates of type (l, a) and (r, a) respectively, is

$$\psi(\gamma^l - \gamma^r)b(g^*(l), l) + [1 - \psi(\gamma^l - \gamma^r)]b(g^*(r), l).$$

Since $\psi(\gamma^l - \gamma^r) > 0$, this payoff exceeds that from Party A selecting a type (r, a) candidate. If Party A were to select a type (l, p) candidate, it would lose the votes of the rational type (l, a) voters. The expected payoff of a majority member of Party A would be:

$$\psi(\gamma_p^l - (\gamma_a^l + \gamma^r))[b(g^*(l), l) + \theta_p] + [1 - \psi(\gamma_p^l - (\gamma_a^l + \gamma^r))]b(g^*(r), l).$$

Subtracting the latter from the former, the difference between the two payoffs is

$$[\psi(\gamma^l - \gamma^r) - \psi(\gamma_p^l - (\gamma_a^l + \gamma^r))]\Delta b(l) - \psi(\gamma_p^l - (\gamma_a^l + \gamma^r))\theta_p,$$

which is positive by Assumption $2(ii)$. If Party A were to select a type (r, p) candidate, the election would simply be a referendum on the regulatory issue. The expected payoff of a majority member of Party A would be:

$$b(g^*(r), l) + \psi(\gamma_p - \gamma_a)\theta_p.$$

Subtracting this from the proposed equilibrium payoff yields

$$\psi(\gamma^l - \gamma^r)\Delta b(l) - \psi(\gamma_p - \gamma_a)\theta_p,$$

which is positive by Assumption $2(i)$. Thus, (l, a) is a best response to (r, a) for the majority members of Party A.

We next show that Party A selecting a type (l, a) candidate and Party B selecting a type (r, a) is the only equilibrium. Let (k_A, t_A) and (k_B, t_B) be an equilibrium. Suppose first that $k_A = k_B = l$. Then, we claim that $t_A = t_B = p$.

If $t_A = t_B = a$, then either party could increase the payoff of its majority members by selecting a pro-regulation candidate. Similarly, if either $(t_A, t_B) = (p, a)$ or $(t_A, t_B) = (a, p)$, then assuming that $\psi(\gamma_p - \gamma_a) < 1$ the Party running the anti-regulation candidate could improve the payoff of its majority members by running a pro-regulation candidate. If $\psi(\gamma_p - \gamma_a) = 1$, then when $(t_A, t_B) = (p, a)$, Party B could improve its payoff by running a type (r, p) candidate. When $(t_A, t_B) = (a, p)$, Party B could improve its payoff by running a type (r, a) candidate, since Assumption 2(i) guarantees that the payoff from such a deviation

$$\psi(\gamma^l - \gamma^r)b(g^*(l), r) + (1 - \psi(\gamma^l - \gamma^r))b(g^*(r), r)$$

exceeds the "equilibrium" payoff

$$b(g^*(l), r) + \theta_p.$$

But if $t_A = t_B = p$, then the majority members of Party B can improve their payoff by running a type (r, p) candidate. A similar argument rules out the possibility that $k_A = k_B = r$.

Suppose then that $k_A \neq k_B$. Then, it must be the case that $k_A = l$ and $k_B = r$. Suppose then that $(t_A, t_B) \neq (a, a)$. We cannot have that $(t_A, t_B) = (p, p)$ since Assumption 2(iii) implies that the majority members of both parties would gain by running an anti-regulation candidate. But if either $t_A = p$ and $t_B = a$ or $t_A = a$ and $t_B = p$, then Assumption 2(ii) implies that the majority members of the party running the pro-regulation candidate could improve their payoffs by running an anti-regulation candidate. Thus, we must have that $(t_A, t_B) = (a, a)$. ■

PROOF OF PROPOSITION 3

The proof is similar to the proof of Proposition 2 and hence is omitted.

PROOF OF PROPOSITION 4

Let I_a be a variable that takes on the value 1 when the anti-regulation initiative is proposed and 0 when it is not. Similarly, let I_p be a variable which takes on the value 1 when the pro-regulation initiative is proposed and 0 when it is not. Any equilibrium is characterized by three things. First, functions $x_a(k_A, t_A, k_B, t_B, I_a, I_p)$, $x_p(k_A, t_A, k_B, t_B, I_a, I_p)$, $x_A(k_A, t_A, k_B, t_B, I_a, I_p)$ and $x_B(k_A, t_A, k_B, t_B, I_a, I_p)$ describing the interest group's contributions to the initiative campaigns and the two parties' candidates, for any given types of candidates selected and initiative proposals. Thus, x_a denotes the money spent buying votes in favor of the anti-regulation initiative and x_p denotes the money spent buying votes against the pro-regulation initiative. By definition, $x_a = 0$ if $I_a = 0$ and $x_p = 0$ if $I_p = 0$. Second, a function $\rho(k_A, t_A, k_B, t_B)$ giving the probability of each possible initiative proposal $(I_a, I_p) \in \{0,1\}^2$ for any given types of candidates. Thus, for example, $\rho(k_A, t_A, k_B, t_B)(1,1)$ is the probability that both anti and pro-regulation initiatives are proposed. Third, a pair of candidates $(\widehat{k}_A, \widehat{t}_A)$ and $(\widehat{k}_B, \widehat{t}_B)$. Formally, therefore, any equilibrium may be summarized by $\{(x_a(\cdot), x_p(\cdot), x_A(\cdot), x_B(\cdot)); \rho(\cdot); (\widehat{k}_A, \widehat{t}_A, \widehat{k}_B, \widehat{t}_B)\}$.

Consider then, a particular equilibrium $\{(x_a(\cdot), x_p(\cdot), x_A(\cdot), x_B(\cdot)); \rho(\cdot); (\widehat{k}_A, \widehat{t}_A, \widehat{k}_B, \widehat{t}_B)\}$. Let $\widehat{\pi}_t$ be the equilibrium probability that the regulatory policy outcome is $t \in \{0,1\}$. We must show that $\widehat{\pi}_1 = \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))$. The proof will proceed by contradiction, so suppose that $\widehat{\pi}_1 \neq \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))$. There are four possibilities: (i) both initiatives are proposed in equilibrium; (ii) only the anti-regulation initiative is proposed; (iii) only the pro-regulation initiative is proposed; and (iv) neither initiative is proposed. We will rule each of these out in turn, which will yield our contradiction.

We will make use of the following additional notation: $\pi_J(I_a, I_p)$ will denote the probability that Party J's candidate wins when the candidate pairs are $(\widehat{k}_A, \widehat{t}_A)$ and $(\widehat{k}_B, \widehat{t}_B)$ and the initiative proposals are (I_a, I_p); π_a will denote the probability that the anti-regulation initiative receives majority support when the candidate pairs are $(\widehat{k}_A, \widehat{t}_A)$ and $(\widehat{k}_B, \widehat{t}_B)$ and $(I_a, I_p) = (1,0)$; and π_p the probability that the pro-regulation initiative receives majority support when the candidate pairs are $(\widehat{k}_A, \widehat{t}_A)$ and $(\widehat{k}_B, \widehat{t}_B)$ and $(I_a, I_p) = (0,1)$. Naturally, all these probabilities take into account the interest group's contribution behavior as specified by $(x_a(\cdot), x_p(\cdot), x_A(\cdot), x_B(\cdot))$.

Possibility (i): $\rho(\widehat{k}_A, \widehat{t}_A, \widehat{k}_B, \widehat{t}_B)(1,1) = 1$.

When both initiatives have been proposed, the issue will be decided by which ever initiative passes. (Under our assumption that noise voters vote for one and only one initiative, one initiative must receive majority support.) The interest group will devote $x^*(\gamma_a - \gamma_p)$ to supporting the anti-regulation initiative (i.e., $x_a(\cdot) + x_p(\cdot) = x^*(\gamma_a - \gamma_p)$) and hence the probability that the pro-regulation initiative will win is given by $\widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))$. But this means that $\widehat{\pi}_1 = \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))$ - a contradiction.

Possibility (ii): $\rho(\widehat{k}_A, \widehat{t}_A, \widehat{k}_B, \widehat{t}_B)(1,0) = 1$.

In this case, a citizen of type (k,t) (not in the interest group) enjoys an equilibrium expected payoff:

$$\pi_A(1,0)b(g^*(\widehat{k}_A), k) + \pi_B(1,0)b(g^*(\widehat{k}_B), k) + \theta_t(1 - \pi_a)[\pi_A(1,0)r^*(\widehat{t}_A) + \pi_B(1,0)r^*(\widehat{t}_B)].$$

This reflects the fact that the initiative will settle the issue only if it passes. If it fails, an event with probability $1-\pi_a$, the issue will be decided by the winning candidate.

If a pro-regulation initiative were introduced, then both initiatives would be on the table and the issue will be decided by which ever initiative passes. The interest group will devote $x^*(\gamma_a - \gamma_p)$ to supporting the anti-regulation initiative and hence the probability that the pro-regulation initiative will win is given by $\widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))$. Thus, the expected payoff of a type (k,t) citizen would be

$$\pi_A(1,1)b(g^*(\widehat{k}_A), k) + \pi_B(1,1)b(g^*(\widehat{k}_B), k) + \theta_t \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p)).$$

Differencing, the gain in a type (k,t)'s citizen's expected payoff from the pro-regulation initiative being introduced is:

$$\chi(k) + \theta_t \kappa$$

where

$$\chi(k) = [\pi_A(1,1) - \pi_A(1,0)]b(g^*(\widehat{k}_A), k) + [\pi_B(1,1) - \pi_B(1,0)]b(g^*(\widehat{k}_B), k)$$

and

$$\kappa = \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p)) - (1-\pi_a)[\pi_A(1,0)r^*(\widehat{t}_A) + \pi_B(1,0)r^*(\widehat{t}_B)].$$

Given that $\rho(\widehat{k}_A, \widehat{t}_A, \widehat{k}_B, \widehat{t}_B)(1,0) = 1$, it must be the case that $\chi(k) + \theta_t \kappa \leq 0$ for all (k,t). If not, then for sufficiently small δ, it would be in some citizen's interest to propose the pro-regulation initiative and hence $(I_a, I_p) = (1,0)$ could not be generated by a pure strategy equilibrium of the game in which each citizen chooses whether or not to place an initiative. Observe that $\chi(k) < 0$ if and only if $\chi(-k) > 0$. Thus, if $\kappa > 0$ then $\chi(k) + \theta_p \kappa > 0$ for some k, while if $\kappa < 0$ then $\chi(k) + \theta_a \kappa > 0$ for some k. It follows that $\kappa = 0$. But this implies that

$$\widehat{\pi}_1 = (1-\pi_a)[\pi_A(1,0)r^*(\widehat{t}_A) + \pi_B(1,0)r^*(\widehat{t}_B)] = \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p)),$$

which is a contradiction.

Possibility (iii): $\rho(\widehat{k}_A, \widehat{t}_A, \widehat{k}_B, \widehat{t}_B)(0,1) = 1$.

In this case, a citizen of type (k,t) (not in the interest group) has an equilibrium expected payoff:

$$\pi_A(0,1)b(g^*(\widehat{k}_A), k) + \pi_B(0,1)b(g^*(\widehat{k}_B), k) + \theta_t\{\pi_p + (1-\pi_p)[\pi_A(0,1)r^*(\widehat{t}_A) + \pi_B(0,1)r^*(\widehat{t}_B)]\}.$$

The idea is that the only chance that the regulation will not be implemented is if the initiative fails, an event with probability $1-\pi_p$. In this event, regulatory policy is determined by the winning candidate.

If the anti-regulation initiative were introduced, the expected payoff of a type (k,t) citizen would be

$$\pi_A(1,1)b(g^*(\widehat{k}_A),k)+\pi_B(1,1)b(g^*(\widehat{k}_B),k)+\theta_t\widehat{\psi}(\gamma_p-\gamma_a,-x^*(\gamma_a-\gamma_p)).$$

Differencing, the gain in the expected payoff of a type (k,t) citizen from the anti-regulation initiative being introduced is

$$\chi(k)+\theta_t\kappa$$

where

$$\chi(k)=[\pi_A(1,1)-\pi_A(0,1)]b(g^*(\widehat{k}_A),k)+[\pi_B(1,1)-\pi_B(0,1)]b(g^*(\widehat{k}_B),k)$$

and

$$\kappa=\widehat{\psi}(\gamma_p-\gamma_a,-x^*(\gamma_a-\gamma_p))-\{\pi_p+(1-\pi_p)[\pi_A(0,1)r^*(\widehat{t}_A)+\pi_B(0,1)r^*(\widehat{t}_B)]\}$$

Again, it must be the case that $\chi(k)+\theta_t\kappa\leq 0$ for all (k,t) which implies that $\kappa=0$. If $\kappa=0$, then

$$\widehat{\pi}_1=\pi_p+(1-\pi_p)[\pi_A(0,1)r^*(\widehat{t}_A)+\pi_B(0,1)r^*(\widehat{t}_B)]=\widehat{\psi}(\gamma_p-\gamma_a,-x^*(\gamma_a-\gamma_p)),$$

which is a contradiction.

Possibility (iv): $\rho(\widehat{k}_A,\widehat{t}_A,\widehat{k}_B,\widehat{t}_B)(0,0)=1$.

In this case, a citizen of type (k,t) (not in the interest group) has an equilibrium expected payoff:

$$\pi_A(0,0)b(g^*(\widehat{k}_A),k)+\pi_B(0,0)b(g^*(\widehat{k}_B),k)+\theta_t[\pi_A(0,0)r^*(\widehat{t}_A)+\pi_B(0,0)r^*(\widehat{t}_B)].$$

If an anti-regulation initiative were introduced, the expected payoff of a type (k,t) citizen would be

$$\pi_A(1,0)b(g^*(\widehat{k}_A),k)+\pi_B(1,0)b(g^*(\widehat{k}_B),k)+\theta_t(1-\pi_a)[\pi_A(1,0)r^*(\widehat{t}_A)+\pi_B(1,0)r^*(\widehat{t}_B)].$$

Thus, the gain in expected payoff for a type (k,t) citizen from the anti-regulation initiative is

$$\chi(k)+\theta_t\kappa$$

where

$$\chi(k) = [\pi_A(1,0) - \pi_A(0,0)]b(g^*(\widehat{k}_A), k) + [\pi_B(1,0) - \pi_B(0,0)]b(g^*(\widehat{k}_B), k)$$

and

$$\kappa = (1-\pi_a)[\pi_A(1,0)r^*(\widehat{t}_A) + \pi_B(1,0)r^*(\widehat{t}_B)] - [\pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B)].$$

Similarly, if a pro-regulation initiative were introduced, the expected payoff of a type (k,t) citizen would be

$$\pi_A(0,1)b(g^*(\widehat{k}_A), k) + \pi_B(0,1)b(g^*(\widehat{k}_B), k) + \theta_t\{\pi_p + (1-\pi_p)[\pi_A(0,1)r^*(\widehat{t}_A) + \pi_B(0,1)r^*(\widehat{t}_B)]\}$$

and the gain in expected payoff from the pro-regulation initiative is

$$\widehat{\chi}(k) + \theta_t\widehat{\kappa}$$

where

$$\widehat{\chi}(k) = [\pi_A(0,1) - \pi_A(0,0)]b(g^*(\widehat{k}_A), k) + [\pi_B(0,1) - \pi_B(0,0)]b(g^*(\widehat{k}_B), k)$$

and

$$\widehat{\kappa} = \pi_p + (1-\pi_p)[\pi_A(0,1)r^*(\widehat{t}_A) + \pi_B(0,1)r^*(\widehat{t}_B)] - [\pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B)].$$

Since $\rho(\widehat{k}_A, \widehat{t}_A, \widehat{k}_B, \widehat{t}_B)(0,0) = 1$, it must be the case that $\chi(k) + \theta_t\kappa \leq 0$ and $\widehat{\chi}(k) + \theta_t\widehat{\kappa} \leq 0$ for all (k,t). This implies (i) that $\chi(l) = \chi(r) = 0$; (ii) that $\kappa = 0$; (iii) that $\widehat{\chi}(l) = \widehat{\chi}(r) = 0$; and (iv) that $\widehat{\kappa} = 0$.

If $\widehat{k}_A \neq \widehat{k}_B$, (i) implies that $\pi_J(1,0) = \pi_J(0,0)$ and (iii) implies that $\pi_J(0,1) = \pi_J(0,0)$. But then (ii) implies

$$(1-\pi_a)[\pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B)] = [\pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B)],$$

which means either that $\pi_a = 0$ or that $\pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B) = 0$. Similarly, (iv) implies that

$$\pi_p + (1-\pi_p)[\pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B)] = [\pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B)],$$

which means either that $\pi_p = 0$ or that $\pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B) = 1$. Since we know that $\pi_p > 0$, it must be the case that $\pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B) = 1$ and $\pi_a = 0$. The former equality implies that:

$$\widehat{\pi}_1 = \pi_A(0,0)r^*(\widehat{t}_A) + \pi_B(0,0)r^*(\widehat{t}_B) = 1,$$

while the latter equality implies that $\gamma_p - \gamma_a \geq \frac{1-\mu}{\mu}$. But in this case, $\widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p)) = 1$, which means that $\widehat{\pi}_1 = \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))$ - a contradiction.

If $\widehat{k}_A = \widehat{k}_B$, then (i) and (iii) are automatically satisfied. If $\widehat{t}_A = \widehat{t}_B$, then $\pi_J(1,0) = \pi_J(0,1) = \pi_J(0,0) = \frac{1}{2}$. A similar logic to that used above, implies that $\widehat{t}_A = \widehat{t}_B = p$ and $\pi_a = 0$. These equalities in turn imply that

$$\widehat{\pi}_1 = 1 = \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p)),$$

which is a contradiction. If $\widehat{t}_A \neq \widehat{t}_B$, then either $(\widehat{t}_A, \widehat{t}_B) = (p, a)$ or $(\widehat{t}_A, \widehat{t}_B) = (a, p)$. In either case, we have that

$$\widehat{\pi}_1 = \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p)),$$

which is a contradiction. ■

PROOF OF COROLLARY

Under the conditions of Proposition 1, the maximum probability that the regulatory outcome is congruent with the views of the majority (i.e., $r = 1$) without initiatives is, by Proposition 1, $\max\{\psi(\gamma^l - \gamma^r), 1 - \psi(\gamma^l - \gamma^r)\}$. With initiatives, the probability that the regulatory outcome is congruent is, by Proposition 4, $\psi(\gamma_p - \gamma_a)$. This exceeds $\max\{\psi(\gamma^l - \gamma^r), 1 - \psi(\gamma^l - \gamma^r)\} = \max\{\psi(\gamma^l - \gamma^r), \psi(\gamma^r - \gamma^l)\}$ since $\gamma_p - \gamma_a$ exceeds $\max\{\gamma^l - \gamma^r, \gamma^r - \gamma^l\}$ and $\psi(\cdot)$ is increasing on the interval $[\frac{-(1-\mu)}{\mu}, \frac{1-\mu}{\mu}]$.

Under the conditions of Proposition 2, the probability that the regulatory outcome is congruent without initiatives is, by Proposition 2, 0. With initiatives, the probability that the regulatory outcome is congruent is, by Proposition 4, $\psi(\gamma_p - \gamma_a) > 0$. Under the conditions of Proposition 3, the probability that the regulatory outcome is congruent without initiatives is, by Proposition 3, 0. With initiatives, the probability that the regulatory outcome is congruent is, by Proposition 4, $\widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_p - \gamma_a)) > 0$. ■

PROOF OF PROPOSITION 5

For sufficiently small δ, we must demonstrate the existence of an equilibrium $\{(x_a(\cdot), x_p(\cdot), x_A(\cdot), x_B(\cdot)); \rho(\cdot); (k_A, t_A, k_B, t_B)\}$, in which $(k_A, t_A, k_B, t_B) = (l, p, r, p)$ and $\rho(l, p, r, p)(1, 0) = 1$ if $\frac{1-\mu}{\mu} > \gamma_p - \gamma_a$ and $\rho(l, p, r, p)(0, 0) = 1$ if $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$.

The first task is to define the interest group's campaign contributions. Here, it is not necessary to be specific. For any $(k_A, t_A, k_B, t_B, I_a, I_p)$, simply let $x_a(k_A, t_A, k_B, t_B, I_a, I_p)$, $x_p(k_A, t_A, k_B, t_B, I_a, I_p)$,

$x_A(k_A, t_A, k_B, t_B, I_a, I_p)$ and $x_B(k_A, t_A, k_B, t_B, I_a, I_p)$ be any 4-tuple of campaign contributions that maximize the interest group's expected payoff. Thus, if $(k_A, t_A, k_B, t_B, I_a, I_p) = (l, p, r, p, 0, 0)$ then $x_a = x_p = x_A = x_B = 0$; if $(k_A, t_A, k_B, t_B, I_a, I_p) = (l, a, r, p, 0, 0)$ then $x_a = x_p = x_B = 0$ and $x_A = x^*(\omega)$, where ω is the fraction of the population preferring $(g^*(l), 0)$ to $(g^*(r), 1)$; etc.

The next task is to define the initiative proposal function $\rho(k_A, t_A, k_B, t_B)$. This is more involved because we must make sure that initiative proposals are consistent with the pure strategy equilibria of a game in which each citizen, having observed the candidates put forward, chooses whether or not to place an initiative at cost δ. We distinguish four different possibilities: (1) $t_A = t_B = a$; (2) $t_A = t_B = p$; (3) $(t_J, t_{-J}) = (p, a)$ and $k_A = k_B$; and (4) $(t_J, t_{-J}) = (p, a)$ and $k_A \neq k_B$. As in the previous proposition, we will make use of the following additional notation: $\pi_J(I_a, I_p)$ will denote the probability that Party J's candidate wins when the candidate pairs are (k_A, t_A) and (k_B, t_B) and the initiative proposals are (I_a, I_p); π_a will denote the probability that the anti-regulation initiative receives majority support when the candidate pairs are (k_A, t_A) and (k_B, t_B) and $(I_a, I_p) = (1, 0)$; and π_p the probability that the pro-regulation initiative receives majority support when the candidate pairs are (k_A, t_A) and (k_B, t_B) and $(I_a, I_p) = (0, 1)$. These probabilities will, of course, be partially determined by the interest group's campaign contributions. We also let $\pi^* = \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))$ which is the probability that the regulation would be implemented if both initiatives are proposed (it is easy to check that $x_a(k_A, t_A, k_B, t_B, 1, 1) + x_p(k_A, t_A, k_B, t_B, 1, 1) = x^*(\gamma_a - \gamma_p)$).

Possibility 1: $t_A = t_B = a$. In this case, we let $\rho(k_A, t_A, k_B, t_B)(0, 1) = 1$. To justify this, we need to show (i) that at least one citizen would gain from placing the pro-regulation initiative on the ballot, when $t_A = t_B = a$ and the anti-regulation initiative is not on the ballot and (ii) that no citizen would gain from placing the anti-regulation initiative on the ballot, when $t_A = t_B = a$ and the pro-regulation initiative is on the ballot. For (i), note that $x_p(k_A, a, k_B, a, 0, 1) = x^*(\gamma_a - \gamma_p)$ and hence placing the pro-regulation initiative on the ballot raises the probability of the regulation being enacted to π^*. It has no effect on which candidate wins, since both candidates hold identical positions on the regulation. For (ii), note that if the anti-regulation initiative were proposed, the regulation would be decided by the winning initiative. But the probability of the regulation being enacted with both initiatives on the ballot is π^*, which is exactly the same as without the anti-regulation initiative. Since the anti-regulation initiative has no effect on the election outcome, there is no gain from proposing it.

Possibility 2: $t_A = t_B = p$. If $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$, we let $\rho(k_A, t_A, k_B, t_B)(0,0) = 1$. This is justified by the fact that, for both types of initiative, placing one on the ballot has no effect on the probability that the regulation will be enacted (which is 1) and no effect on which candidate wins. If $\frac{1-\mu}{\mu} > \gamma_p - \gamma_a$, we let $\rho(k_A, t_A, k_B, t_B)(1,0) = 1$. To justify this, note first that placing the anti-regulation initiative on the ballot raises the probability of the regulation not being enacted to $1-\pi^*$ (since $x_a(k_A, p, k_B, p, 1, 0) = x^*(\gamma_a - \gamma_p)$) and has no effect on the election outcome. Second, the probability of the regulation being enacted with both initiatives on the ballot is π^* which is exactly the same as without the pro-regulation initiative. Since the pro-regulation initiative has no effect on the election outcome, there is no gain from proposing it.

Possibility 3: $(t_J, t_{-J}) = (p, a)$ and $k_A = k_B$. If $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$, we let $\rho(k_A, t_A, k_B, t_B)(0,0) = 1$. In this case, the only thing differentiating the candidates in the election is their position on regulation. Accordingly, the pro-regulation candidate will win with probability one and the regulation will be introduced. This is unchanged by either type of initiative being on the ballot. If $\frac{1-\mu}{\mu} > \gamma_p - \gamma_a$, matters are more complicated. If (a) $\pi_p + (1-\pi_p)\pi_J(0,1) \neq \pi^*$ and (b) $(1-\pi_a)\pi_J(1,0) \neq \pi^*$, then we let $\rho(k_A, t_A, k_B, t_B)(1,1) = 1$. Condition (a) ensures that at least one citizen gains from the anti-regulation initiative being proposed when the pro-regulation initiative is on the ballot and condition (b) ensures that at least one citizen gains from the pro-regulation initiative being proposed when the anti-regulation initiative is on the ballot.

If condition (a) does not hold but condition (b) holds, we let $\rho(k_A, t_A, k_B, t_B)(1,0) = 1$. Since the only thing differentiating the candidates is their position on regulation, $x_{-J}(k_A, t_A, k_B, t_B, 0, 0) = x^*(\gamma_a - \gamma_p)$ and hence $\pi_J(0,0) = \pi^* = \pi_p + (1-\pi_p)\pi_J(0,1)$. This implies that no citizen can gain from the pro-regulation initiative being on the ballot whether or not the anti-regulation initiative is on the ballot. To show that some citizen gains from the anti-regulation initiative being on the ballot when the pro-regulation initiative is not on the ballot, it is enough to show that $(1-\pi_a)\pi_J(1,0) \neq \pi_J(0,0)$. But this follows immediately from condition (b) and the fact that $\pi_J(0,0) = \pi^*$.

If condition (b) does not hold but condition (a) holds, we let $\rho(k_A, t_A, k_B, t_B)(0,1) = 1$. Again, since the only thing differentiating the candidates is their position on regulation, $x_{-J}(k', t_A, k', t_B, 0, 0) = x^*(\gamma_a - \gamma_p)$ and $\pi_J(0,0) = \pi^* = (1-\pi_a)\pi_J(1,0)$. This implies that no citizen can gain from the anti-regulation initiative being on the ballot whether or not the pro-regulation initiative is on the ballot. Thus, we just need to show that some citizen gains from the pro-regulation initiative being on the ballot when the anti-regulation initiative is not on the ballot. This follows from condition (a) and the fact that $\pi_J(0,0) = \pi^*$.

If neither condition (a) nor condition (b) holds, we let $\rho(k_A, t_A, k_B, t_B)(0,0) = 1$. In this case, we have that $\pi_J(0,0) = \pi^* = \pi_p + (1-\pi_p)\pi_J(0,1) = (1-\pi_a)\pi_J(1,0)$ and these inequalities imply that no citizen can gain from either type of initiative being on the ballot whether or not the other initiative is on the ballot.

Possibility 4: $(t_J, t_{-J}) = (p, a)$ and $k_A \neq k_B$. If $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$, we let $\rho(k_A, t_A, k_B, t_B)(0,1) = 1$ if $\pi_J(0,0) \neq \pi_J(0,1)$ or if $\pi_J(0,0) < 1$. Since the two candidates have different public spending preferences and the pro-regulation initiative would pass with probability one if proposed, at least one citizen can gain from placing the pro-regulation initiative on the ballot when the anti-regulation initiative is not on the ballot if $\pi_J(0,0) \neq \pi_J(0,1)$ or if $\pi_J(0,0) < 1$. No citizen can gain from proposing the anti-regulation initiative, since it will fail with probability one and have no effect on the candidate election. If $\pi_J(0,0) = \pi_J(0,1) = 1$, we let $\rho(k_A, t_A, k_B, t_B)(0,0) = 1$. This is justified by the fact that neither type of initiative will impact the likelihood of the regulation being implemented nor the public spending policy.

If $\frac{1-\mu}{\mu} > \gamma_p - \gamma_a$, we let $\rho(k_A, t_A, k_B, t_B)(1,1) = 1$ if (a) $\pi_J(0,1) \neq \pi_J(1,1)$ or $\pi_p + (1-\pi_p)\pi_J(0,1) \neq \pi^*$ and (b) $\pi_J(1,0) \neq \pi_J(1,1)$ or $(1-\pi_a)\pi_J(1,0) \neq \pi^*$. Since the two candidates have different public spending preferences and π^* is the probability that the regulation is enacted if both initiatives are proposed, condition (a) ensures that at least one citizen gains from the anti-regulation initiative being proposed when the pro-regulation initiative is on the ballot and condition (b) ensures that at least one citizen gains from the pro-regulation initiative being proposed when the anti-regulation initiative is on the ballot.

It is easy to show that either condition (a) or (b) must hold. If condition (a) does not hold, we let $\rho(k_A, t_A, k_B, t_B)(0,1) = 1$. The fact that $\pi_J(0,1) = \pi_J(1,1)$ and $\pi_p + (1-\pi_p)\pi_J(0,1) = \pi^*$ implies that no citizen can gain from the anti-regulation initiative being on the ballot when the pro-regulation initiative is on the ballot. Thus, we just need to show that some citizen who gains from the pro-regulation initiative being on the ballot when the anti-regulation initiative is not on the ballot. It is enough to show that either $\pi_J(0,1) \neq \pi_J(0,0)$ or $\pi_p + (1-\pi_p)\pi_J(0,1) \neq \pi_J(0,0)$. One of these inequalities must hold for if $\pi_J(0,1) = \pi_J(0,0)$ and $\pi_p + (1-\pi_p)\pi_J(0,1) = \pi_J(0,0)$ then, since $\pi_p > 0$, $\pi_J(0,1) = 1$. But since condition (a) does not hold, this implies that $\pi^* = 1$ which contradicts the fact that $\frac{1-\mu}{\mu} > \gamma_p - \gamma_a$.

If condition (b) does not hold, we let $\rho(k_A, t_A, k_B, t_B)(1,0) = 1$. The fact that $\pi_J(1,0) = \pi_J(1,1)$ and $(1-\pi_a)\pi_J(1,0) = \pi^*$ implies that no citizen can gain from the pro-regulation initiative being on the ballot when the anti-regulation initiative is on the ballot. Thus, we just need to show that some citizen who gains from the anti-regulation initiative being on the ballot when the pro-regulation initiative is not on the ballot. It is enough to show that either $\pi_J(1,0) \neq \pi_J(0,0)$ or $(1-\pi_a)\pi_J(1,0) \neq \pi_J(0,0)$. One of these inequalities must hold for if $\pi_J(1,0) = \pi_J(0,0)$ and $(1-\pi_a)\pi_J(1,0) = \pi_J(0,0)$ then, since $\pi_a > 0$, $\pi_J(1,0) = 0$. But since condition (b) does not hold, this implies that $\pi^* = 0$ which is a contradiction.

We now claim that given the contribution and initiative proposal functions just constructed, it is an equilibrium for the majority members of each party to select candidates of type (l,p) and (r,p) respectively. This will imply the proposition, because our specification of the initiative proposal function implies that $\rho(l,p,r,p)(0,0) = 1$ if $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$ and $\rho(l,p,r,p)(1,0) = 1$ if $\frac{1-\mu}{\mu} > \gamma_p - \gamma_a$ (see Possibility 2 above). We shall simply show that a candidate of type (l,p) is a best response for the majority members of Party A, the argument for Party B being similar.

Assuming that the majority members of Party A have regulatory attitude t, their payoff at the proposed equilibrium is

$$\psi(\gamma^l - \gamma^r)b(g^*(l),l) + [1 - \psi(\gamma^L - \gamma^R)]b(g^*(R),L) + \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))\theta_t.$$

This reflects the fact that $\widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p)) = 1$ if $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$ and $x_a(l,p,r,p,1,0) = x^*(\gamma_a - \gamma_p)$. There are three possible deviations that Party A's members might make and we go through each in turn.

The first deviation is to a candidate of type (l,a). If $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$, our specification of the initiative proposal function implies that $\rho(l,a,r,p)(0,1) = 1$ if $\pi_B(0,0) \neq \pi_B(0,1)$ or if $\pi_B(0,0) < 1$ and $\rho(l,a,r,p)(0,0) = 1$ if $\pi_B(0,0) = \pi_B(0,1) = 1$ (see Possibility 4). But since the pro-regulation initiative will pass with probability one, only the candidates public spending preferences are relevant for the election outcome if it is proposed. This means that $\pi_B(0,1) = 1 - \psi(\gamma^l - \gamma^r) < 1$, which implies that $\rho(l,a,r,p)(0,1) = 1$. Thus, the payoff to the majority members of Party A from deviating is

$$\psi(\gamma^l - \gamma^r)b(g^*(l),l) + [1 - \psi(\gamma^l - \gamma^r)]b(g^*(r),l) + \theta_t,$$

which is exactly their equilibrium payoff. If $\frac{1-\mu}{\mu} > \gamma_p - \gamma_a$, our specification of the initiative proposal function implies that $\rho(l,a,r,p)(1,1) = 1$ if (a) $\pi_B(0,1) \neq \pi_B(1,1)$ or $\pi_p + (1-\pi_p)\pi_B(0,1) \neq$

π^* and (b) $\pi_B(1,0) \neq \pi_B(1,1)$ or $(1-\pi_a)\pi_B(1,0) \neq \pi^*$. Thus, if conditions (a) and (b) hold, both initiatives will be proposed and the regulation will be decided by the winning initiative. The payoff to the majority members of Party A from deviating is

$$\pi_A(1,1)b(g^*(l),l) + \pi_B(1,1)b(g^*(r),l) + \pi^*\theta_t.$$

But, since $\pi_A(1,1) = \psi(\gamma^l - \gamma^r)$, this is exactly their equilibrium payoff.

If condition (a) does not hold, then $\rho(l,a,r,p)(0,1) = 1$. Thus, the payoff to the majority members of Party A from deviating is

$$\pi_A(0,1)b(g^*(l),l) + \pi_B(0,1)b(g^*(r),l) + [\pi_p + (1-\pi_p)\pi_B(0,1)]\theta_t.$$

But, if condition (a) does not hold, then $\pi_B(0,1) = \pi_B(1,1) = 1 - \psi(\gamma^l - \gamma^r)$ and $\pi_p + (1 - \pi_p)\pi_B(0,1) = \pi^*$. Thus, this payoff is exactly the equilibrium payoff. If condition (b) does not hold, then $\rho(l,a,r,p)(1,0) = 1$ and the payoff to the majority members of Party A from deviating is

$$\pi_A(1,0)b(g^*(l),l) + \pi_B(1,0)b(g^*(r),l) + (1-\pi_a)\pi_B(0,1)\theta_t.$$

But, if condition (a) does not hold, then $\pi_B(1,0) = \pi_B(1,1) = 1-\psi(\gamma^l-\gamma^r)$ and $(1-\pi_a)\pi_B(0,1) = \pi^*$ and, again, this payoff is exactly the equilibrium payoff.

The second type of deviation is to a candidate of type (r,a). If $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$, our specification of the initiative proposal function implies that $\rho(r,a,r,p)(0,0) = 1$. Since $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$, and the only thing differentiating the candidates is their positions on the regulation, Party B's candidate will win with probability one. The payoff to the majority members of Party A from deviating is therefore

$$b(g^*(r),l) + \theta_t,$$

which is less than their equilibrium payoff.

If $\frac{1-\mu}{\mu} > \gamma_p - \gamma_a$, our specification implies of the initiative proposal function implies that $\rho(r,a,r,p)(1,1) = 1$ if (a) $\pi_p + (1-\pi_p)\pi_B(0,1) \neq \pi^*$ and (b) $(1-\pi_a)\pi_B(1,0) \neq \pi^*$. If condition (a) does not hold but condition (b) holds, $\rho(r,a,r,p)(1,0) = 1$. If condition (b) does not hold but condition (a) holds, $\rho(r,a,r,p)(0,1) = 1$, while if neither condition holds, $\rho(r,a,r,p)(0,0) = 1$. In all cases, the payoff to the majority members of Party A from deviating is

$$b(g^*(r),l) + \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))\theta_t,$$

which is less than their equilibrium payoff.

The final type of deviation is to a candidate of type (r, p). If $\frac{1-\mu}{\mu} \leq \gamma_p - \gamma_a$, our specification of the initiative proposal function implies that $\rho(r, p, r, p)(0, 0) = 1$. Since both parties' candidates are type (r, p), the payoff to the majority members of Party A from deviating is therefore

$$b(g^*(r), l) + \theta_t,$$

which is less than their equilibrium payoff. If $\frac{1-\mu}{\mu} > \gamma_p - \gamma_a$, $\rho(r, p, r, p)(1, 0) = 1$. The interest group will devote $x^*(\gamma_a - \gamma_p)$ to campaigning for the initiative's passage and the payoff to the majority members of Party A from deviating is therefore

$$b(g^*(r), l) + \widehat{\psi}(\gamma_p - \gamma_a, -x^*(\gamma_a - \gamma_p))\theta_t,$$

which is less than their equilibrium payoff. ■