Online Appendix

Details of Simulations for Table 1. We assume $G(\theta) = \frac{1}{2} + \theta,\ \theta \in [-\frac{1}{2}, \frac{1}{2}],\ F = u,\ V = 1$. So, from (4) in the paper, the equilibrium conditions determining $\Delta(0)$ are

$$\begin{aligned}
\Delta(0) &= \frac{\pi(1-\pi)(1-h(0))}{\pi + h(0)(1-\pi)} - \frac{\pi(1-\pi)(1-\widetilde{h}(0))}{\pi + \widetilde{h}(0)(1-\pi)} \\
\widetilde{h}(0) &= \frac{1-a_D}{1-a_C}, h(0) = \frac{a_D}{a_C} \\
a_C &= p + (1-p)V\Delta(0)(1-q) \\
a_D &= 1 - p + pV\Delta(0)(1-q)
\end{aligned}$$

where the last two equations are from (A4) in the Appendix. Again from (4), equilibrium conditions determining $\Delta(q)$ are;

$$\begin{aligned}
\Delta(q) &= \frac{\pi(1-\pi)(1-h(q))}{\pi + h(q)(1-\pi)} - \frac{\pi(1-\pi)(1-\widetilde{h}(0))}{\pi + \widetilde{h}(0)(1-\pi)} \\
\widetilde{h}(0) &= \frac{1-a_D}{1-a_C}, h(q) = \frac{a_D + (1-a_D)q}{a_C + (1-a_C)q} \\
a_C &= p + (1-p)V\Delta(q)(1-q) \\
a_D &= 1 - p + pV\Delta(q)(1-q)
\end{aligned}$$

Finally, the equilibrium conditions determining $\Delta_0(0)$ as the same as for $\Delta(0)$, except that $a_C = p + (1-p)V\Delta(0), a_D = 1 - p + pV\Delta(0)$.

Observable Actions and Payoffs. We begin with a definition of confirmation bias. Throughout, we assume that the voter is optimistic i.e. $\pi > 0.5$. We will also use the fact that in both of the equilibria constructed below, the $C-$type is: (i) more likely to choose $x = A$ than the $D-$type; (ii) more likely to generate a payoff $v = 1$ than the $D-$type. So, we say that the voter has *confirmation bias* if (i) when only the action is observed, he mis-classifies B as A with probability $q > 0$; (ii) if the payoff as well as the action is observed, he mis-classifies $(x, 0)$ as $(x, 1)$ with probability $q > 0$, $x = A, B$.

Our first case is where $\phi > 0.5$. We can then show;

Proposition A1. *If $\phi > 0.5$, then there is an equilibrium with the following structure. First, if payoffs are not observed, the voter re-elects the incumbent if he thinks he observes $x = A$, and if payoffs are observed, the voter re-elects the incumbent if he thinks he observes either $(A, 1)$ or $(B, 1)$. Second, the consonant type chooses $x = s,\ s = A, B$ with probability 1, and the dissonant type imitates him with probabilities $\lambda_A = F((1-q)V),\ \lambda_B = F((2\phi - 1)(1-q)V)$ if the state is A or B respectively.*

Proof of Proposition A1. (i) *Voter Updating and Re-election Rule.* Let $\pi(x)$ be the voter's posterior probability that the incumbent is good, conditional on x, and $\pi(x,v)$ be the voter's posterior probability that the incumbent is good, conditional on (x,v), where x, (x,v) are either the action or the action/payoff pair that the CB voter thinks he observes. Assume that the $C-$type always matches the action to the state, and that the $D-$type imitates him with probability λ_s and chooses $x \neq s$ with probability $1-\lambda_s$, $1 > \lambda_s > 0$.

Then, by straightforward application of Bayes' rule;

$$\begin{aligned} \pi(A) &= \frac{p\pi}{p\pi + (p\lambda_A + (1-p)(1-\lambda_A))(1-\pi)} \\ \pi(B) &= \frac{(1-p)\pi}{(1-p)\pi + ((1-p)\lambda_B + p(1-\lambda_B))(1-\pi)} \end{aligned} \tag{1}$$

and also

$$\begin{aligned} \pi(A,1) &= \frac{p\pi}{p\pi + p\lambda_A(1-\pi)} \\ \pi(B,1) &= \frac{(1-p)\pi}{(1-p)\pi + (1-p)\lambda_B(1-\pi)} \\ \pi(A,0) &= \pi(B,0) = 0 \end{aligned} \tag{2}$$

Now, note from (1), (2) that as $p > 0.5$, $1 > \lambda_s > 0$, then $\pi(A) > \pi > \pi(B)$, and also that $\pi(x,1) > \pi > \pi(x,0)$, $x = A, B$. So, conditional on the incumbent's assumed behavior, the voter will re-elect the incumbent; (i) iff $x = A$, if only actions are observed; (ii) iff $v = 1$, if both payoffs and actions are observed.

(b) *Incumbent Behavior.* (i) Consider first the $C-$type. Then, there are two possible deviations from equilibrium behavior. The first is deviating to $x = A$ when $s = B$. The payoffs to deviating and not deviating are

$$(1-\phi)V + \phi qV, \;\; u + (1-\phi)qV + \phi V \tag{3}$$

respectively. This is because with deviation, there is no short-run payoff, but the incumbent is certainly re-elected if only actions are observed, and elected with probability q if payoffs are observed, because the voter will mis-classify payoff $v = 0$ as $v = 1$ with probability q. With no deviation, the reverse is true; the incumbent is certainly re-elected if the payoff is observed, and elected with probability q if only the action is observed, because the voter will mis-classify action $x = B$ as $x = A$ with probability q. Moreover, in this case, there is a short-run payoff u. So, from (3), we see that deviation never pays, whatever u, if $\phi \geq 0.5$.

The second possible deviation is deviating to $x = B$ when $s = A$. By the same argument, the payoffs to deviating and not deviating are

$$(1-\phi)qV + \phi qV,\ u + (1-\phi)V + \phi V$$

In this case, deviation clearly never pays.

(ii) Consider now the $D-$type. Here, we ask when he will want to imitate the $C-$type. Suppose first that $s = A$. Then the payoff from imitation and short-run optimization are

$$V,\ u + (1-\phi)qV + \phi qV$$

respectively. So, in this case, the $D-$type imitates when $u \leq (1-q)V$ and so imitates with probability $\lambda_A = F((1-q)V)$.

Suppose next that $s = B$. Then by a similar argument to part (b)(i) of the proof, the payoff from imitation and short-run optimization are

$$(1-\phi)qV + \phi V,\ u + (1-\phi)V + \phi qV$$

respectively. So, in this case, the incumbent when $u \leq (2\phi - 1)(1-q)V$ and so imitates with probability $\lambda_B = F((2\phi - 1)(1-q)V)$.

(c) Finally, note that as $\phi > 0.5$, and $\overline{u} > V(1-q)$, $1 > \lambda_A > \lambda_B > 0$ as required. So, we have shown that given the assumed incumbent behavior, the equilibrium voting rule is optimal for the voter, and given the voting rule, incumbents are optimizing as described in the Proposition. So, the proof is complete. □

Our second case is where $\phi < \min\left\{0.5,\ \frac{2p-1}{2(1-p)}\left(\frac{\overline{u}}{V(1-q)} - 1\right)\right\} \equiv \overline{\phi}$. We can then show;

Proposition A2. *Assume that $F(u) = u/\overline{u}$. For $0 \leq \phi < \overline{\phi}$, there is a political equilibrium with the following structure. First, if payoffs are not observed, the voter re-elects the incumbent if he thinks he observes $x = A$, and if payoffs are observed, the voter re-elects the incumbent if he thinks he observes either $(A, 1)$ or $(B, 1)$. Second, the dissonant type panders with probability $\lambda_D = F((1-q)V)$, and the consonant type panders with probability $\lambda_C = F((1-2\phi)(1-q)V)$. So, pandering by either type is decreasing in voter confirmation bias.*

Proof of Proposition A2. (a) *Voter Updating and Re-election Rule.* Assume that C and $D-$types pander - i.e. always choose A - with probabilities $\lambda_C, \lambda_D \in (0,1)$ respectively. Also, let a_C, a_D be the unconditional probabilities that C, D types choose action A. We will also assume that $a_C > a_D$ which requires:

$$a_C = \lambda_C + (1-\lambda_C)p > \lambda_D + (1-\lambda_D)(1-p) = a_D \tag{4}$$

Finally, define $\pi(x),\ \pi(x,v)$ as in the proof of Proposition **??** above. Then, by application of Bayes' rule;

$$\begin{aligned}\pi(A) &= \frac{a_C\pi}{a_C\pi + a_D(1-\pi)} \\ \pi(B) &= \frac{(1-a_C)\pi}{(1-a_C)\pi + (1-a_D)(1-\pi)}\end{aligned} \tag{5}$$

and

$$\begin{aligned}\pi(A,1) &= \frac{(\lambda_C p + 1 - \lambda_C)\pi}{(\lambda_C p + 1 - \lambda_C)\pi + \lambda_D p(1-\pi)} \\ \pi(B,1) &= 1 \\ \pi(A,0) &= \frac{\lambda_C(1-p)\pi}{\lambda_C(1-p)\pi + (1-p)(1-\pi)} \\ \pi(B,0) &= 0\end{aligned} \tag{6}$$

Now, note from (6) that as $\lambda_C, \lambda_D < 1,\ \lambda_C p + 1 - \lambda_C > \lambda_D p$, so $\Pr(C\,|A,1) \geq \pi$. Also, from (6) (5), as $\lambda_C < 1$, $\Pr(C\,|A,0) < \pi$. Finally, we are assuming that $a_C > a_D$, so that from (5), $\pi(A) > \pi > \pi(B)$. So, conditional on the incumbent's assumed behavior, the voter will re-elect the incumbent iff $x = A$, if only actions are observed, or iff $v = 1$, if both payoffs and actions are observed.

(b) *Incumbent Behavior.* (i) Consider first the $C-$type. If $s = A$, the best choice for the incumbent is unambiguously $x = A$ as it is both short-run optimal and ensures re-election. If $s = B$, then he gets

$$[1-\phi+\phi q]V,\ u + [(1-\phi)q + \phi]V \tag{7}$$

from $x = A$ and $x = B$ respectively. The first payoff is calculated as follows. With probability $1-\phi$, the voter observes only $x = A$ and will re-elect the incumbent. With probability $1-\phi$, the voter will observe $(x,v) = (A,0)$ but will mis-classify this as $(A,1)$ with probability q and re-elect the incumbent. Similarly, the second payoff is calculated as follows. With probability $1-\phi$, the voter observes only B but mis-classifies it as A with probability q and re-elects the incumbent. With probability ϕ , the voter observes $(x,v) = (B,\ 1)$ and the incumbent is re-elected. So, from (7), the consonant type will pander - i.e. choose $x = A$ when $s = B$ - if $u \leq (1-2\phi)(1-q)V$, and hence the probability of pandering is $\lambda_C = F((1-2\phi)(1-q)V)$

(ii) *The Dissonant Type.* If $s = A$, the payoffs from choosing $x = A, B$ are respectively

$$V,\ u + qV \tag{8}$$

The explanation is as follows. If A is chosen, with probability $1-\phi$, the voter observes only $x = A$ and will re-elect the incumbent, and with probability ϕ , the voter will observe $(A,1)$ and will also re-elect him. If B is chosen, with probability $1-\phi$, the voter observes B but mis-classifies it as A with probability

q, and re-elects the incumbent. With probability ϕ, the voter observes $(B, 0)$ but this will be mis-classified as $(B, 1)$ with probability q. In either case, the incumbent is re-elected with probability q. So, require simply $u \leq V(1-q)$, and so the dissonant type will pander if $u \leq V(1-q) \equiv \hat{u}_D$.

If $s = B$, then the short-run optimal action is A. However, in this case, we must check that it is also optimal overall. Payoffs from $x = A, B$ respectively are;

$$u + (1 - \phi + \phi q)V,\ ((1-\phi)q + \phi)V \tag{9}$$

The explanation is as in (b)(i) above of the proof. We see that (9) holds iff $\phi \leq 0.5$, which is assumed. So, we conclude that if $s = B$, the incumbent always chooses A.

(iii) Note that $0 < \lambda_C < \lambda_D < 1$ as in equilibrium, as $(1-q)V < \overline{u},\ \phi < 0.5$. This confirms the maintained assumption that $\Pr(C\,|A, 0\,) < \pi$, thus confirming the voter re-election rule. It remains to check that $0 < a_D < a_C < 1$ in equilibrium. But if $F = u/\overline{u}$, then:

$$\lambda_C = \frac{(1-2\phi)(1-q)V}{\overline{u}},\ \lambda_D = \frac{(1-q)V}{\overline{u}} \tag{10}$$

Plugging (10) into (4) and rearranging, we eventually see that $a_D < a_C$ requires

$$\frac{2p-1}{2(1-p)}\left(\frac{\overline{u}}{V(1-q)} - 1\right) > \phi$$

which holds by assumption. □

Simulations for the Welfare Effect of Confirmation Bias with Partially Observable Payoffs. We compute this for objective welfare. Define the re-election probabilities for the two types, depending on whether they pander "P", or do not pander, "N", as

$$\begin{aligned} r_{CP} &= r_{DP} = r_P = 1 - \phi + \phi p \\ r_{CN} &= (1-\phi)(p + (1-p)q) + \phi, r_{DN} = (1-\phi)(1-p+pq) \end{aligned} \tag{11}$$

Moreover, we assume $\delta = 1,\ E = 1$, implying $V = 2$, and that u is uniformly distributed on $[0, \overline{u}],\ \overline{u} = 3$ so from Proposition **??**, the pandering probabilities are;

$$\lambda_D = \frac{2(1-q)}{3},\ \lambda_C = \frac{2(1-2\phi)(1-q)}{3} \tag{12}$$

Then, voter welfare is

$$\begin{aligned} W_O(q) &= \pi\lambda_C(p + r_P + (1-r_P)\pi) + (1-\pi)\lambda_D(p + (1-r_P)\pi) \\ &\quad + \pi(1-\lambda_C)(1 + r_{CN} + (1-r_{CN})\pi) + (1-\pi)(1-\lambda_D)(1-r_{DN})\pi \end{aligned} \tag{13}$$

We then compute $W_O(q) - W_O(0)$ from (13) for the values $\phi \in [0, 0.5]$, $p \in (0.6, 1]$, holding other values fixed as specified above. Finally, it can be checked that for these parameter values, $\frac{2p-1}{2(1-p)} \left(\frac{\overline{u}}{V(1-q)} - 1 \right) \geq \frac{0.2}{2.0.4} \left(\frac{6}{2} - 1 \right) = 0.5$, ensuring that $\phi \leq \overline{\phi}$, as required.