# Appendix for *Communication in Collective Bargaining*

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This Appendix supplements *Communication in Collective Bargaining*. We first characterize setter's belief updating and choice of the revised proposal. Specifically, we present Lemma 1 and a proof for Lemma 2. Second, we present the proofs for the results regarding the comparison between straw poll and binding referendum, i.e., Proposition 1, Proposition 2 and Corollary 1. We also formalize the expressions of the pay-off gain functions and restate equilibrium conditions accordingly. Third, we present proofs regarding necessary conditions and existence of equilibrium in straw poll, i.e., Proposition 4, Proposition 5, Proposition 7 and Proposition 8.

## Setter's Belief Updating and Choice of the Revised Proposal

Suppose that the voters use the cut-point $k \in (0, \overline{\theta}]$ in the first period. Upon $y$ positive votes, the setter knows that those $y$ voters' ideal points are weakly higher than $k$, and the other $(n-y)$ voters's ideal points are less than $k$. From the setter's perspective, the ideal point of a voter who casts a positive vote follows the conditional cumulative distribution $\widehat{F}(t;k) \triangleq \Pr(\theta_i \leqslant t \mid \theta_i \geq k)$; and the ideal point of a voter who casts a negative vote follows the conditional cumulative distribution $\widetilde{F}(t;k) \triangleq \Pr(\theta_i \leqslant t \mid \theta_i < k)$. Upon $y < q$ positive endorsements, the revised proposal $b_2$ gets approved if and only if the $(n-q+1)$ th smallest ideal point among the $(n-y)$ ideal points $\theta_i \mid_{\theta_i<k}$ prefers it to the *status quo*. Let $\widetilde{F}_{n-y,n-q+1}(t;k)$ denote the cumulative distribution function of this order-statistic random

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variable. Upon $y \geq q$ positive votes, the revised proposal gets accepted if and only if the $(y-q+1)$ th smallest ideal point among the $y$ ideal points $\theta_i \mid_{\theta_i \geq k}$ prefers it to the *status quo*. Let $\widehat{F}_{y,y-q+1}(t;k)$ denote the cumulative distribution function of this order-statistic random variable. For convenience, let's define

$$\Omega(t|y;k) \triangleq \begin{cases} \widetilde{F}_{n-y,n-q+1}(t;k) & \text{if } y < q \\ \widehat{F}_{y,y-q+1}(t;k) & \text{if } y \geq q \end{cases} . \tag{A1}$$

The setter uses her perception of the voters' cut-point to form her updated belief about the probability with which her revised proposal gets accepted. Specifically, upon receiving $y$ positive votes, the revised proposal $b_2$ is collectively accepted with probability $1-\Omega(\frac{1}{2}b_2|y;k)$. With this belief, we can formally derive the setter's best response to the voters' cut-point $k$ given $y$ positive votes

$$\beta(y;k) \triangleq \arg\max_{b\in[0,\bar{\theta}]}[1-\Omega(\frac{1}{2}b|y;k)]u_A(b) + \Omega(\frac{1}{2}b|y;k)u_A(0). \tag{A2}$$

Notice that $[1-\Omega(\frac{1}{2}b|y;k)]u_A(b)+\Omega(\frac{1}{2}b|y;k)u_A(0) = [1-\Omega(\frac{1}{2}b|y;k)][u_A(b)-u_A(0)]+u_A(0)$. We then have

$$\beta(y;k) = \arg\max_{b_2\in[0,\bar{\theta}]}[1-\Omega(\frac{1}{2}b_2|y;k)]\widetilde{u}_A(b_2), \tag{A3}$$

where

$$\widetilde{u}_A(b_2) = \psi_A(\theta_A - b_2) - \psi_A(\theta_A).^{1} \tag{A4}$$

We can verify that $\widetilde{u}_A(\cdot)$ is strictly increasing and weakly concave on $[0,\theta_A]$ and satisfies the condition $\widetilde{u}_A(0) = 0$. We characterize how $\beta(y;k)$ is determined in Lemma 1, leaving the proof in the Supplementary Appendix. We then characterize some properties of $\beta(y;k)$

[1] It is a strictly dominated choice for the setter to propose any policy strictly higher than her own ideal. Suppose $b_2 > \theta_A$. If $\Omega(\frac{1}{2}b_2|y;k) < 1$, then proposing $\theta_A$ not only increases the chance of approvement, but also increases the payoff of the setter when the proposal is accepted. If, however, $\Omega(\frac{1}{2}b_2|y;k) = 1$, the setter ends up with the status quo for sure and receives $u_A(0)$. By proposing any policy $x$ such that $\Omega(\frac{1}{2}x|y;k) < 1$, the setter can strictly increase her expected payoff. Thus, without loss of generality, we can focus on a smaller set $[0,\theta_A]$ to pin down the revised proposal. Therefore, the setter's utility function can be written without the notation of absolute value, i.e., $u_A(b_2) = \psi_A(\theta_A - b_2)$.

in Lemma 2.

**Lemma 1** *(Full Characterization of the Second-Period Proposal) Suppose $k$ is the cut-point that the setter believes that voters use, and $y$ is the number of positive votes. The second-period proposal $\beta(y;k)$ is characterized as follows:*

*(1) for $y \leq q-1$, $\beta(y;k) \in (0, \min\{2k, \theta_A\})$ and $\beta(y;k)$ is uniquely determined by*

$$\frac{1-\widetilde{F}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{n-y,n+1-q}(\frac{1}{2}\beta;k)} = \frac{1}{2}\frac{\widetilde{u}_A(\beta)}{\widetilde{u}'_A(\beta)}; \tag{A5}$$

*(2) for $y \geq q$,*

*(2.1) when $\frac{1}{2}\theta_A \leq k < \overline{\theta}$, we have $\beta(y;k) = \theta_A$; and*

*(2.2) when $0 < k < \frac{1}{2}\theta_A$, if $y = q$ and $k \geq \widehat{k}$, where $\widehat{k} < \frac{1}{2}\theta_A$ is uniquely determined by $\frac{1}{q}\frac{1-F(k)}{f(k)} = \frac{1}{2}\frac{\widetilde{u}_A(2k)}{\widetilde{u}'_A(2k)}$, we have $\beta(y;k) = 2k$; otherwise, we have $\beta(y;k) \in (2k, \theta_A)$, and $\beta(y;k)$ is uniquely determined by*

$$\frac{1-\widehat{F}_{y,y-q+1}(\frac{1}{2}\beta;k)}{\widehat{f}_{y,y-q+1}(\frac{1}{2}\beta;k)} = \frac{1}{2}\frac{\widetilde{u}_A(\beta)}{\widetilde{u}'_A(\beta)}; \tag{A6}$$

*furthermore, we have $\lim_{k \to (\frac{1}{2}\theta_A)^-} \beta(y;k) = \theta_A$.*

**Lemma 2** *Suppose $k$ is the cut-point that the setter believes that voters use, and $y$ is the number of positive votes. We have*

*(1) the revised proposal $\beta(y;k)$ is single valued;*[2]

*(2) $\beta(y;k)$ is increasing in the total positive votes $y$; when $y \leq q$ or $k < \frac{1}{2}\theta_A$, $\beta(y;k)$ is strictly increasing in $y$; and*

*(3) $\beta(y;k)$ is continuously differentiable (except for, at most, two points), continuous and increasing in $k$.*[3]

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[2] $\beta(y;k)$ is well defined except for when $k = \overline{\theta}$ (every voter casts a negative claim or vote) and $y \geq 1$ (the setter sees some positive claims or votes). The proposals in these cases depend on the off-the-equilibrium-path beliefs.

[3] When $y \leq q-1$, $\beta(y;k)$ is strictly increasing and continuously differentiable in $k$. When $y > q-1$, $\beta(y;k)$ has, at most, two kinks. When $k$ is between the two kinks, $\beta(y;k)$ is strictly increasing and continuously differentiable in $k$; when $k$ is outside of the interval bounded by the two kinks, $\beta(y;k)$ is constant within each of the two intervals.

**Proof of Lemma 2.**

(1) The result is directly implied by Lemma 1.

(2) We prove the results with four steps.

(2.1) When $y \leq q-1$, we know that $\beta(y;k) \in (0, \min\{2k, \theta_A\})$ and is uniquely determined by $\frac{1-\widetilde{F}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{n-y,n+1-q}(\frac{1}{2}\beta;k)} = \frac{1}{2}\frac{\widetilde{u}_A(\beta)}{\widetilde{u}'_A(\beta)}$. The left-hand side of the first-order condition is the inverse of the hazard rate function. It is a strictly decreasing function of $\beta$, as shown in Lemma 4 of the Supplementary Appendix. The right-hand side of the first-order condition $\frac{1}{2}\frac{\widetilde{u}_A(\beta)}{\widetilde{u}'_A(\beta)}$, is a strictly increasing function of $\beta$ when $\beta \in [0, \theta_A]$. Thus $\beta(y;k)$ is determined as the unique zero point of the strictly decreasing function $\frac{1-\widetilde{F}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{n-y,n+1-q}(\frac{1}{2}\beta;k)} - \frac{1}{2}\frac{\widetilde{u}_A(\beta)}{\widetilde{u}'_A(\beta)}$. Because $\frac{1-\widetilde{F}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}$ is strictly increasing in $y$ by Lemma 4, as the parameter $y$ in this decreasing function increases, the value of the function $\frac{1-\widetilde{F}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{n-y,n+1-q}(\frac{1}{2}\beta;k)} - \frac{1}{2}\frac{\widetilde{u}_A(\beta)}{\widetilde{u}'_A(\beta)}$ becomes larger, so that the zero point increases. Therefore, $\beta(y;k)$ should be strictly increasing in $y$ whenever $y \leq q-1$. Because $\beta(q-1;k) < 2k \leq \beta(q;k)$, we know that $\beta(y;k)$ is strictly increasing in $y$ whenever $y \leq q$.

(2.2) When $y \geq q$ and $\frac{1}{2}\theta_A \leq k \leq \overline{\theta}$, it is obvious that $\beta(y;k) = \theta_A$.

(2.3) Now suppose $y \geq q$, $k < \frac{1}{2}\theta_A$, and $\frac{1}{q}\frac{1-F(k)}{f(k)} > \frac{1}{2}\frac{\widetilde{u}_A(2k)}{\widetilde{u}'_A(2k)}$. In this case, $\beta(y;k)$ is determined by the first order condition. Because $\frac{1-\widehat{F}_{y,y-q+1}(\frac{1}{2}\beta;k)}{\widehat{f}_{y,y-q+1}(\frac{1}{2}\beta;k)}$ is strictly increasing in $y$ by Lemma 4, we can use the similar logic in (2.1) to show that $\beta(y;k)$ is strictly increasing in $y$ whenever $y \geq q$.

(2.4) Now suppose $y \geq q$, $k < \frac{1}{2}\theta_A$, and $\frac{1}{q}\frac{1-F(k)}{f(k)} \leq \frac{1}{2}\frac{\widetilde{u}_A(2k)}{\widetilde{u}'_A(2k)}$ (i.e., $k \geq \widehat{k}$). In the same way of (2.1), we can show that $\beta(y;k)$ is strictly increasing in $y$ whenever $y \geq q+1$. Furthermore we also know that $\beta(q+1;k) > 2k = \beta(q;k)$. Thus $\beta(y;k)$ is strictly increasing in $y$ whenever $y \geq q$.

Combining the results in (2.1)-(2.4), we know that $\beta(y;k)$ is increasing in $y$. In addition, when $y \leq q$ or $k < \frac{1}{2}\theta_A$, $\beta(y;k)$ is strictly increasing in $y$;

(3) We prove the results with three steps.

(3.1) When $y \leq q-1$, we know that $\beta(y;k) \in (0, \min\{2k, \theta_A\})$ and is uniquely determined by $\frac{1-\widetilde{F}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{n-y,n+1-q}(\frac{1}{2}\beta;k)} = \frac{1}{2}\frac{\widetilde{u}_A(\beta)}{\widetilde{u}'_A(\beta)}$. Because $\frac{1-\widetilde{F}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}$ is strictly increasing in $k$ by Lemma

4, as the parameter $k$ in this decreasing function increases, the value of $\frac{1-\widetilde{F}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{n-y,n+1-q}(\frac{1}{2}\beta;k)} - \frac{1}{2}\frac{\widetilde{u}_A(\beta)}{\widetilde{u}'_A(\beta)}$ becomes larger. Therefore, the zero point $\beta(y;k)$ is strictly increasing in $k$ whenever $y \leq q-1$. In addition, the continuous differentiability directly comes from the fact that $\frac{1-\widetilde{F}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{n-y,n+1-q}(\frac{1}{2}\beta;k)}$ and $\frac{1}{2}\frac{\widetilde{u}_A(\beta)}{\widetilde{u}'_A(\beta)}$ are twice continuously differentiable in $\beta$ and $k$.

(3.2) When $y > q$, and $0 < k < \frac{1}{2}\theta_A$, we can show the same results in the same way. When $y > q$ and $k \geq \frac{1}{2}\theta_A$, we have $\beta(y;k) = \theta_A$. Because $\lim_{k \to (\frac{1}{2}\theta_A)^-} \beta(y;k) = \theta_A$ by Lemma 1, we know that $\beta(y;k)$ is continuously differentiable (except for at $k = \frac{1}{2}\theta_A$), continuous and increasing in $k$. Specifically, when $k < \frac{1}{2}\theta_A$, $\beta(y;k)$ is strictly increasing in $k$.

(3.3) Now suppose $y = q$. When $k \in [\frac{1}{2}\theta_A, \overline{\theta})$, we have $\beta(q;k) = \theta_A$. When $k \in [\widehat{k}, \frac{1}{2}\theta_A)$, we have $\beta(q;k) = 2k$. When $k \in [0, \widehat{k})$, $\frac{1-\widehat{F}_{y,y-q+1}(\frac{1}{2}\beta;k)}{\widehat{f}_{y,y-q+1}(\frac{1}{2}\beta;k)}$ is independent of $k$ so that $\beta(q;k)$ does not depend on $k$. Therefore $\beta(y;k)$ has, at most, two kinks. When $k$ is between the two kinks, $\beta(y;k)$ is strictly increasing and continuously differentiable in $k$. When $k$ is outside of the interval bounded by the two kinks, $\beta(y;k)$ is constant within each of the two intervals.

■

## The Comparison between Straw Poll and Binding Referendum

**Proof of Proposition 1.**

(1) We first show that, straw poll with the initial proposal being $\widehat{b}_1$ makes the setter better off than the binding institution with the same initial proposal. The setter's expected payoff under the binding institution (with the initial proposal $\widehat{b}_1$ and the cutpoint $\widehat{k}$) is characterized by the following expression: $\sum_{j\geq q}\binom{n}{j}F(\widehat{k})^{n-j}(1-F(\widehat{k}))^j u_A(\widehat{b}_1) + \sum_{j<q}\binom{n}{j}F(\widehat{k})^{n-j}(1-F(\widehat{k}))^j \max_b\{[1-\Omega(\frac{1}{2}b|j;\widehat{k})]u_A(b) + \Omega(\frac{1}{2}b|j;\widehat{k})u_A(0)\}$, where $\Omega(\cdot|j;\widehat{k})$ is defined by Equation A1. With sincere voting, the payoff becomes $\sum_{j\geq q}\binom{n}{j}F(\widehat{k})^{n-j}(1-F(\widehat{k}))^j u_A(2\widehat{k}) + \sum_{j<q}\binom{n}{j}F(\widehat{k})^{n-j}(1-F(\widehat{k}))^j \max_b\{[1-\Omega(\frac{1}{2}b|j;\widehat{k})]u_A(b) + \Omega(\frac{1}{2}b|j;\widehat{k})u_A(0)\}$. It equals $\sum_{j\geq q}\binom{n}{j}F(\widehat{k})^{n-j}(1-F(\widehat{k}))^j[1-\Omega(\widehat{k}|j;\widehat{k})]u_A(2\widehat{k}) + \sum_{j<q}\binom{n}{j}F(\widehat{k})^{n-j}(1-F(\widehat{k}))^j \max_b\{[1-\Omega(\frac{1}{2}b|j;\widehat{k})]u_A(b) + \Omega(\frac{1}{2}b|j;\widehat{k})u_A(0)\}$. This is the same expected payoff for the setter in straw poll if she initially proposes $\widehat{b}_1$, while proposing $2\widehat{k}$ upon $j \geq q$ positive votes and proposing

$\beta(j;\widehat{k})$ respectively upon $j < q$ positive votes. Her expected payoff can be improved in straw poll if instead she proposes $\beta(j;\widehat{k})$ upon $j \geq q$ positive votes. That means $u_A(2\widehat{k}) \leq [1-\Omega(\frac{1}{2}\beta(j;\widehat{k})|j;\widehat{k})]u_A(\beta(j;\widehat{k})) + \Omega(\frac{1}{2}\beta(j;\widehat{k})|j;\widehat{k})]u_A(0)$, for $j = q, ..., n$. Lemma 1 suggests that $\beta(q;\widehat{k}) > 2\widehat{k}$ when $q < n$. Thus, we have $u_A(2\widehat{k}) < [1-\Omega(\frac{1}{2}\beta(j;\widehat{k})|j;\widehat{k})]u_A(\beta(j;\widehat{k})) + \Omega(\frac{1}{2}\beta(j;\widehat{k})|j;\widehat{k})]u_A(0)$, for $j = (q+1), ..., n$, providing $q < n$. As a result, in straw poll, the setter's expected payoff can be increased to $\sum_j \binom{n}{j} F(\widehat{k})^{n-j}(1-F(\widehat{k}))^j \max_b\{[1-\Omega(\frac{1}{2}b|j;\widehat{k})]u_A(b)+\Omega(\frac{1}{2}b|j;\widehat{k})u_A(0)\}$. When $q < n$, the welfare improvement is strict.

(2) Notice that given the continuation equilibrium strategies, even if there may be several initial proposal under binding referendum that maximizes the setter's expected payoff, they all end up with the same level of expected payoff.

As the last stage to complete the proof, we also need to show that there exists an initial proposal $\widehat{b}_1 \in [0,\overline{\theta}]$ that maximizes the setter's expected utility given all the continuation equilibrium strategies. Notice that the setter's expected utility is $\sum_j \binom{n}{j} F(k)^{n-j}(1-F(k))^j\{[1-\Omega(\frac{1}{2}\beta(j;k)|j;k)]u_A(\beta(j;k)) + \Omega(\frac{1}{2}\beta(j;k)|j;k)u_A(0)\}$. Lemma 1 suggests that $\beta(y;k)$ is continuous in $k$ so that the expected payoff is a continuous function of $k$, and therefore also continuous in $b_1$ because $k = \frac{1}{2}b_1$. A continuous function defined on a compact set always has a maximizer. As a result, there exists an equilibrium in straw that makes the setter better off than in any equilibria under binding referendum, providing that voters are non-strategic. ■

**Proof of Proposition 2.**

The main logic of the proof is already mentioned in the main part of the paper. Now we only need to deal with the technical part that $b_1 = \theta_A$ induces $k^*$ as a best response. Recall that $V^S_{diff}(\theta_i;k^*)$ ($V^B_{diff}(\theta_i;k^*,b_1)$) denotes the voter $i$'s payoff gain from casting a positive vote in the first period of *straw poll* (*binding referendum* with initial proposal $b_1$), given that the other players' actions and his own action in the second period follow the equilibrium strategy profile. To complete the proof, we don't need to write down the detailed expressions of $V^S_{diff}(\theta_i;k^*)$ and $V^B_{diff}(\theta_i;k^*,b_1=\theta_A)$. Instead, we only need to pin down their difference.

Specifically, we need to verify the following condition.

$$V^B_{diff}(\theta_i; k^*, b_1 = \theta_A) - V^S_{diff}(\theta_i; k^*) = \begin{cases} 0 & \text{if } \theta_i \geq \frac{1}{2}\theta_A \\ \binom{n-1}{q-1} F(k^*)^{n-q}[1-F(k^*)]^{q-1}\widetilde{F}_{n-q,n-q}(\frac{1}{2}\theta_A; k^*)[u_i(\theta_A) - u_i(0)] & \text{if } \theta_i < \frac{1}{2}\theta_A \end{cases}, \quad \text{(A7)}$$

where $\widetilde{F}_{n-q,n-q}(\cdot; k^*)$ denotes the cumulative distribution function of the $(n-q)$ th smallest order statistics from the $(n-y)$ i.i.d. random variables $\theta_i \mid_{\theta_i<k}$. Notice that the payoff gain is calculated based on various contingent situations.

Firstly, consider the contingent cases when the other $(n-1)$ voters cast $q$ or more positive votes for the initial proposal. Voter $i$'s decision does not change the outcome in binding referendum so that $V^B_{diff}(\theta_i; k^*, b_1 = \theta_A) = 0$. In the straw-poll environment, the setter in the second period will propose her ideal point and those $y$ voters will again approve it. Therefore, the utility difference in straw poll $V^S_{diff}(\theta_i; k^*) = 0$.

Secondly, consider the contingent cases when the other $(n-1)$ voters cast $(q-2)$ or less positive votes for the initial proposal. The continuation game in both institutions generate the same distribution of outcome. Therefore the two utility-difference functions are the same, i.e., $V^S_{diff}(\theta_i; k^*) = V^B_{diff}(\theta_i; k^*, b_1 = \theta_A)$.

Thirdly, consider the contingent case when there are exact $(q-1)$ positive votes among the other $(n-1)$ voters. If voter $i$ casts a negative vote, he induces the same distribution of outcome in the continuation game in both institutions. If voter $i$'s ideal point $\theta_i \geq \frac{1}{2}\theta_A$ and he casts a positive vote, he gets a payoff of $u_i(\theta_A)$ for sure in both institutions. The only complicated part is the situation when his ideal point $\theta_i < \frac{1}{2}\theta_A$ and he casts a positive vote. In binding referendum, he gets a payoff of $u_i(\theta_A)$ for sure. However, in the same situation of straw poll, voter $i$ understands that he will reject the setter's ideal point if she proposes it in the second period; meanwhile, among the other $(n-1)$ voter, only $(q-1)$ of them have ideal points above $\frac{1}{2}\theta_A$ for sure, and the rest $(n-q)$ voters may prefer the *status quo* to $\theta_A$. Whether the $\theta_A$ gets approved depends on whether the maximum ideal point among the $(n-q)$ ideal points with distribution $\widetilde{F}(\cdot;k^*)$ accepts it or not. Hence, the voter's expected payoff is $(1-\widetilde{F}_{n-q,n-q}(\frac{1}{2}\theta_A;k^*))u_i(\theta_A)+\widetilde{F}_{n-q,n-q}(\frac{1}{2}\theta_A;k^*)u_i(0)$. So when voter $i$'s ideal point $\theta_i<\frac{1}{2}\theta_A$, in the pivotal contingent case, the difference of the two utility-difference functions is $u_i(\theta_A)-[(1-\widetilde{F}_{n-q,n-q}(\frac{1}{2}\theta_A;k^*))u_i(\theta_A)+\widetilde{F}_{n-q,n-q}(\frac{1}{2}\theta_A;k^*)u_i(0)] = \widetilde{F}_{n-q,n-q}(\frac{1}{2}\theta_A;k^*)[u_i(\theta_A)-u_i(0)]$.

Notice that the *ex ante* probability of the pivotal contingent case is $\binom{n-1}{q-1}F(k^*)^{n-q}[1-F(k^*)]^{q-1}$. Thus, Equation A7 has been proved.

Based on Equation A7, we can check that the following conditions hold.

(a) $V^B_{diff}(\theta_i;k^*,b_1=\theta_A)=V^S_{diff}(\theta_i;k^*)\geq 0$, for any $\theta_i\geq k^*$;

(b) $V^B_{diff}(\theta_i;k^*,b_1=\theta_A)\leq V^S_{diff}(\theta_i;k^*)\leq 0$, for any $\theta_i<k^*$.

As a result, no type of voters has an incentive to deviate from the voting strategy with cut-point $k^*$ if the setter proposes her ideal point in the first period of straw poll. ■

**Statement of Equilibrium Conditions**

To characterize voters' incentive to vote in the first period, we need to analyze each voter's payoff gain from casting a positive vote instead of a negative one. Given the proposal in the second period $b_2$, we use $V(\theta_i,b_2,y')$ to denote the second-period continuation payoff of the voter with ideal point $\theta_i$ when: (1) all voters voter sincerely in the second period, and (2) there are $y'$ positive votes among the other $(n-1)$ voters in first-period voting. The function $V(\theta_i,b_2,y')$ should also depend on the voter's perception about the cut-point $k$ that the other $(n-1)$ voters use in the first period. To save the notation, we do not explicitly write it as an independent variable of the function. Let $V^S_{diff}(\theta_i;k)$ ($V^B_{diff}(\theta_i;k,b_1)$) denote the difference of voter $i$'s expected payoffs between casting a positive and a negative vote in the first period of *straw poll* (*binding referendum* with initial proposal $b_1$), taking into account that: (i) the other voters use cut-point $k$, and (ii) the setter maximizes her expected payoff in the second period given her belief of voters' first period cut-point $k$. Specifically, $V^S_{diff}(\theta_i;k)$ and $V^B_{diff}(\theta_i;k,b_1)$ can be expressed by the following two equations.

$$V^S_{diff}(\theta_i;k) = \begin{cases} \sum_{j\geq q}\binom{n-1}{j}F(k)^{n-1-j}[1-F(k)]^j[V(\theta_i,\beta(j+1;k),j)-V(\theta_i,\beta(j;k),j)] \\ +\binom{n-1}{q-1}F(k)^{n-q}[1-F(k)]^{q-1}[V(\theta_i,\beta(q;k),q-1)-V(\theta_i,\beta(q-1;k),q-1)] \\ +\sum_{j\leq q-2}\binom{n-1}{j}F(k)^{n-1-j}[1-F(k)]^j[V(\theta_i,\beta(j+1;k),j)-V(\theta_i,\beta(j;k),j)] \end{cases}, \tag{A8}$$

$$V^B_{diff}(\theta_i;k,b_1) = \begin{cases} \binom{n-1}{q-1}F(k)^{n-q}[1-F(k)]^{q-1}[u_i(b_1;\theta_i)-V(\theta_i,\beta(q-1;k),q-1)] \\ +\sum_{j\leq q-2}\binom{n-1}{j}F(k)^{n-1-j}[1-F(k)]^j[V(\theta_i,\beta(j+1;k),j)-V(\theta_i,\beta(j;k),j)] \end{cases}. \tag{A9}$$

Based on the above payoff-gain functions, the equilibrium conditions in both institutions can be reduced to the requirement about voters' first-period voting incentives as follows.

**Remark 1** *Given an initial proposal* $b_1$, $k^*(b_1) \in (0, \overline{\theta}]$ *is an equilibrium cut-point if and only if the following conditions are satisfied: (i)* $V^\omega_{diff}(k^*(b_1); k^*(b_1), b_1) = 0$, *(ii)* $V^\omega_{diff}(\theta_i; k^*(b_1), b_1) \geq 0$, *whenever* $\theta_i \in (k^*(b_1), \overline{\theta}]$, *and (iii)* $V^\omega_{diff}(\theta_i; k^*(b_1), b_1) \leq 0$, *whenever* $\theta_i \in [0, k^*(b_1))$, *where* $\omega$ *indicates whether the institutional environment is straw poll (S) or binding referendum (B), and we define* $V^S_{diff}(\theta_i; k, b_1) \triangleq V^S_{diff}(\theta_i; k)$.

Note that the equilibrium cut-point $k^*(b_1)$ should depend on the institutional environment. To save the notation, we do not use an additional subscript indicating the institutional environment.

**Proof of Corollary 1.**

The proof consists of two parts. First, we construct the equilibrium in binding referendum given an equilibrium in straw poll. In our constructed equilibrium, the setter always has an option to propose her ideal point in the first period so as to replicate the same outcome in straw poll. Then it remains to show that there is an equilibrium that gives the setter the highest payoff among all possible equilibria (if any) in straw poll. When there are only finite equilibria in straw poll, the claim is obvious. So we only need to show the claim holds if there are infinitely many equilibria in straw poll.

(1) We construct the equilibrium of binding referendum in two different situations. The first situation is when $q \geq 2$. In this case, we need to specify certain off-the-equilibrium-path beliefs (not required by the equilibrium conditions) as a part of the construction. The other situation is when $q = 1$. In this case, the equilibrium construction does not involve the off-the-equilibrium-path beliefs (not required by the equilibrium conditions). In both cases, we need to specify the voters' strategy $k^*(b_1)$ when the setter does not propose her ideal point, i.e., $b_1 \neq \theta_A$.

(1.1) First of all, let's consider the case with $q \geq 2$. We will show that for any $b_1 \neq \theta_A$, a pooling equilibrium strategy $k^*(b_1) = \overline{\theta}$ can always be induced. When all the other voters use the strategy $k^*_B(b_1) = \overline{\theta}$, the difference of expected payoffs between making a positive and a negative vote for voter with ideal point $\theta_i$ is

$$V^B_{diff}(\theta_i; k^*(b_1) = \overline{\theta}, b_1) = V(\theta_i, \beta(1; \overline{\theta}), 0) - V(\theta_i, \beta(0; \overline{\theta}), 0), \tag{A10}$$

where the function $V(\theta_i, b_2, y')$ is defined before Equation A8. Recall that $\beta(1, \overline{\theta})$ is not well defined and depends on the off-the-equilibrium-path belief because it is the revised proposal when the setter receives one positive vote but knows that all voters reject the initial proposal for sure. As long as we set the off-the-equilibrium-path belief such that $\beta(1; \overline{\theta}) = \beta(0; \overline{\theta})$, no voter has an incentive to deviate from the pooling equilibrium strategy. As a result, $k^*_B(b_1) = \overline{\theta}$ can always be supported as a continuation equilibrium.

If there is an initial proposal $b'_1$ that can induce an interior cut-point $k'_0$ as well as a strictly higher expected payoff than what $\theta_A$ induces for the setter, then define $k^*(b'_1) = k'_0$. For all the other initial proposals except $b_1 = \theta_A$, define $k^*(b_1) = \overline{\theta}$. With the constructed cut-point strategy, $\theta_A$ and $b'_1$ (if any) are the only possible proposals for the setter to induce information disclosure. Thus, the optimal solution for the setter exists and must be either $\theta_A$ or $b'_1$.

(1.2) Now we consider the case when $q = 1$.

(1.2.1) We will show that the following cut-point strategy can be sustained as an equilibrium strategy:

(i) when $b_1 \geq 2\overline{\theta} - \beta\left(0; \overline{\theta}\right)$, the voters all reject the proposal, i.e., $k^*(b_1) = \overline{\theta}$;

(ii) when $\beta\left(0; \overline{\theta}\right) \leq b_1 < 2\overline{\theta} - \beta\left(0; \overline{\theta}\right)$, the cut-point $k^*(b_1) \in (\frac{1}{2}b_1, \overline{\theta})$ is well determined by the equation $k^*(b_1) = \frac{b_1 + \beta(0;k^*(b_1))}{2}$;

(iii) when $b_1 \leq \beta\left(0; \overline{\theta}\right)$, the cut-point $k^*(b_1) \in (0, \overline{\theta})$ is well determined by $b_1 = \beta\left(0; k^*(b_1)\right)$;

(iv) when $b_1 = 0$, every type of voters accepts it.

In the following, we verify each of the above cases. We know that the payoff-gain function of a voter is $F(k^*(b_1))[\psi_i(|b_1 - \theta_i|) - V(\theta_i, \beta(0; k^*(b_1)), 0)]$, where

$$V(\theta_i, \beta(0; k^*(b_1)), 0) = \begin{cases} \psi_i(|\beta(0; k^*(b_1)) - \theta_i|) & \text{if } \theta_i \geq \frac{1}{2}\beta(0; k^*(b_1)) \\ [1 - \widetilde{F}_{n-1,1}(\frac{1}{2}\beta(0; k^*(b_1)); k^*(b_1))]\psi_i(|\beta(0; k^*(b_1)) - \theta_i|) & \\ \widetilde{F}_{n-1,1}(\frac{1}{2}\beta(0; k^*(b_1)); k^*(b_1))\psi_i(|0 - \theta_i|) & \text{if } \theta_i < \frac{1}{2}\beta(0; k^*(b_1)) \end{cases}$$

Consider case (i) with $b_1 \geq 2\overline{\theta} - \beta\left(0; \overline{\theta}\right)$. For $\theta_i \leq \overline{\theta}$, we can verify that $V(\theta_i, \beta(0; k^*(b_1)), 0) \geq \psi_i(|\beta(0; k^*(b_1)) - \theta_i|)$. Also notice that $b_1 \geq 2\theta_i - \beta\left(0; \overline{\theta}\right)$ so that we have $\theta_i \leq \frac{1}{2}(b_1 + \beta\left(0; \overline{\theta}\right))$. Because $b_1 \geq 2\overline{\theta} - \beta\left(0; \overline{\theta}\right) \geq \beta\left(0; \overline{\theta}\right)$, we have $\psi_i(|\beta(0; k^*(b_1)) - \theta_i|) \geq \psi_i(|b_1 - \theta_i|)$ so that no type has an incentive to deviate.

Consider case (ii). We first show that the incentive compatibility conditions of the other types are automatically satisfied given the cut-point condition specified. Notice that $b_1 \geq \beta\left(0; \overline{\theta}\right) \geq \beta(0; k)$ for any $k$. For $\theta_i \geq k^*(b_1)$, we have $\theta_i \geq \frac{b_1 + \beta(0;k^*(b_1))}{2}$ so that $\psi_i(|\beta(0; k^*(b_1)) - \theta_i|) \leq \psi_i(|b_1 - \theta_i|)$. For $\theta_i < k^*(b_1)$, we can verify that $V(\theta_i, \beta(0; k^*(b_1)), 0) \geq \psi_i(|\beta(0; k^*(b_1)) - \theta_i|)$. Because $\theta_i < \frac{b_1 + \beta(0;k^*(b_1))}{2}$, we have $\psi_i(|\beta(0; k^*(b_1)) - \theta_i|) \geq \psi_i(|b_1 - \theta_i|)$. Thus, no other types have incentive to deviate from the cut-point strategy, and the cut-point is determined by $\psi_i(|\beta(0; k^*(b_1)) - k^*(b_1)|) = \psi_i(|b_1 - k^*(b_1)|)$, i.e., $k^*(b_1) = \frac{b_1 + \beta(0;k^*(b_1))}{2}$. Define a new function $H_1(k; b_1) = k - \frac{b_1 + \beta(0;k)}{2}$. When $k = \overline{\theta}$, we have $H_1(k; b_1) > 0$. When $k \leq \frac{1}{2}b_1$, we have $H_1(k; b_1) < 0$. Because $H_1(k; b_1)$ is a continuous function with respect to

$k$, its zero point $k^*(b_1)$ always exists in the interval $(\frac{1}{2}b_1, \overline{\theta})$. Thus $k^*(b_1)$ is well defined.

Consider case (iii) with $b_1 \in (0, \beta\left(0; \overline{\theta}\right))$. Define $k^*(b_1)$ such that $b_1 = \beta\left(0; k^*(b_1)\right)$. Because $\beta\left(0; k\right)$ is continuous and strictly increasing in $k$ with $\lim\limits_{k \to 0^+} \beta\left(0; k\right) = 0$, the cut-point $k^*(b_1) \in (0, \overline{\theta})$ in this case is then well defined. For $\theta_i \geq k^*(b_1)$, we have $V(\theta_i, \beta(0; k^*(b_1)), 0) = \psi_i(|b_1 - \theta_i|)$ and for $\theta_i < k^*(b_1)$, we have $V(\theta_i, \beta(0; k^*(b_1)), 0) \geq \psi_i(|b_1 - \theta_i|)$, so that no types have an incentive to deviate.

Consider case (iv). If $b_1 = 0$, no voter has an incentive to deviate from accepting the proposal if the others follow the cut-point $k^*(b_1) = 0$.

(1.2.2) Now we consider if there is an optimal proposal in the first period that maximizes her expected payoff given the cut-point specified above. In case (iii) and case (iv), the setter's expected payoff is no more than that in the game without any communication. Thus choosing $b_1 \leq \beta\left(0; \overline{\theta}\right)$ is a strictly dominated strategy. In case (i), the setter's expected payoff is the same as that in the game without any communication. Thus, choosing $b_1 \geq 2\overline{\theta} - \beta\left(0; \overline{\theta}\right)$ is not optimal for the setter either. Without loss of generality, the setter's choice of the initial proposal can be focused within the interval $[\beta\left(0; \overline{\theta}\right), \min\{2\overline{\theta} - \beta\left(0; \overline{\theta}\right), \overline{\theta}\}]$. Specifically, she maximizes her expected payoff subject the constraints that $b_1 \in [\beta\left(0; \overline{\theta}\right), \min\{2\overline{\theta} - \beta\left(0; \overline{\theta}\right), \overline{\theta}\}]$ and that $k^*(b_1)$ is determined by $k^*(b_1) = \frac{b_1 + \beta(0; k^*(b_1))}{2}$. The setter's expected welfare when voters use the cut-point $k$ is $U_A(k, b_1) = \sum_{j \geq q} \binom{n}{j} F(k)^{n-j}[1 - F(k)]^j u_A(b_1) + \sum_{j \leq q-1} \binom{n}{j} F(k)^{n-j}[1 - F(k)]^j \{[1 - \widetilde{F}_{n-j,n-q+1}(\frac{1}{2}\beta(j; k)); k)]u_A(\beta(j; k)) + \widetilde{F}_{n-j,n-q+1}(\frac{1}{2}\beta(j; k)); k)u_A(s)\}$. Notice that the cut-point specified by case (ii) may not be unique. However, this does not prevent the existence of equilibrium as long as the optimization problem $\max\limits_{(b_1,k) \in \Gamma} U_A(k, b_1)$ has a solution, where $\Gamma \triangleq \{(b_1, k) : b_1 \in [\beta\left(0; \overline{\theta}\right), 2\overline{\theta} - \beta\left(0; \overline{\theta}\right)]$ and $k = \frac{b_1 + \beta(0; k)}{2}\}$. We can verify that $U_A(k, b_1)$ is continuous in $(k, b_1)$. Also notice that $b_1 \in [\beta\left(0; \overline{\theta}\right), 2\overline{\theta} - \beta\left(0; \overline{\theta}\right)]$ so that $b_1$ is bounded. $k = \frac{b_1 + \beta(0; k)}{2} > \frac{\beta\left(0; \overline{\theta}\right)}{2}$, and $k = \frac{b_1 + \beta(0; k)}{2} \leq \overline{\theta}$, so that $k$ is also bounded. Because $k - \frac{b_1 + \beta(0; k)}{2}$ is continuos in $(k, b_1)$, $\Gamma$ is a closed set. Hence, $\Gamma$ is compact. Then we can conclude that $\arg\max\limits_{(b_1,k) \in \Gamma} U_A(k, b_1)$ should not be empty. Suppose $(b_1^0, k^0) \in \arg\max\limits_{(b_1,k) \in \Gamma} U_A(k, b_1)$. We can verify that the initial proposal $b_1^0$ together with the cut-point strategy specified above with $k^*(b_1^0) = k^0$ satisfy the equilibrium conditions.

(2) Now we show that there is an equilibrium with $k^* \geq \frac{\theta_A}{2}$ that gives the setter the highest payoff among all possible equilibria with $k^* \geq \frac{\theta_A}{2}$ (if any) in the straw poll. We only need to show the non-trivial case when there are infinitely many equilibria in straw poll. Because the setter's expected payoff is bounded by $\psi_A(0)$, the supremum for the setter's expected payoff induced by all the equilibria exists. Denote it by $\widetilde{U}_A^*$. By the definition of supremum, there exist a series of equilibria with cut-point $k_1^t \geq \frac{\theta_A}{2}$ such that $\widetilde{U_A}(k_1^t)$ converges to $\widetilde{U}_A^*$ as $t$ goes to infinity, where $\widetilde{U_A}(k)$ is the setter's expected payoff if the voters use cut-point $k$ in the first period. Because $k_1^t$ is bounded, there is a subsequence of $\{k_1^t\}$, denoted by $\{k_1^{t_m}\}$, that converges to some point $k^{**}$. We can check that $\widetilde{U_A}(\cdot)$ is continuous, therefore we have $\lim_{m \to +\infty} \widetilde{U_A}(k_1^{t_m}) = \widetilde{U_A}(k^{**})$. Because $\lim_{t \to +\infty} \widetilde{U_A}(k_1^t) = \widetilde{U}_A^*$ so we get $\widetilde{U_A}(k^{**}) = \widetilde{U}_A^*$. Because $k_1^t \geq \frac{\theta_A}{2}$, we have $k_1^{t_m} \geq \frac{\theta_A}{2}$ and $k^{**} \geq \frac{\theta_A}{2}$. It remains to show that $k^{**}$ forms an equilibrium. Notice that $V_{diff}^S(k_1^{t_m}; k_1^{t_m}) = 0$ and $V_{diff}^S(\theta_i; k)$ is continuous in $\theta_i$ and $k$. Hence, we have $V_{diff}^S(k^{**}; k^{**}) = 0$. We only need to show the other types do not have an incentive to deviate. Without loss of generality, suppose there is a type $\theta' > k^{**}$ such that $V_{diff}^S(\theta'; k^{**}) < 0$. Then for sufficiently large $m$, we have $\theta' > k_1^{t_m}$ and $V_{diff}^S(\theta'; k_1^{t_m}) < 0$. The two inequalities together are contradict to the fact that $k_1^{t_m}$ forms an equilibrium. As a result, $k^{**} \geq \frac{\theta_A}{2}$ must form an equilibrium and it gives the setter the highest expected payoff among all the equilibria with $k^* \geq \frac{\theta_A}{2}$ in straw poll. ■

## Necessary Conditions of Equilibria in Straw Poll

### Proof of Proposition 4.

We prove the result by contradiction. Suppose there is an equilibrium cut-point $k^* \leq \frac{1}{2}\theta_A$. We first pin down the condition for the type $k^*$ to be indifferent between saying "yes" and "no", i.e., $V_{diff}^S(k^*; k^*) = 0$.

Consider the case when the other voter says "yes". If voter $i$ says "yes," he induces a revised proposal $b_2^*(2)$. Because his ideal point is $k^* \leq \frac{1}{2}b^*(2)$, he ends up with a payoff of

$\psi_i(k^*)$. If, however, he says "no," he induces a proposal $b_2^*(1)$, which will be accepted by both voters. Thus, he ends up with $\psi_i(|k^* - b_2^*(1)|)$. His payoff gain from saying "yes" instead of "no" is therefore $\psi_i(k^*) - \psi_i(|k^* - b_2^*(1)|)$.

Consider the case when the other voter says "no". If voter $i$ says "yes," he induces an expected payoff $[1 - \widetilde{F}(\frac{1}{2}b_2^*(1); k^*)]\psi_i(|k^* - b_2^*(1)|) + \widetilde{F}(\frac{1}{2}b_2^*(1); k^*)\psi_i(k^*)$. If voter $i$ says "no," he induces an expected payoff $[1 - \widetilde{F}(\frac{1}{2}b_2^*(0); k^*)]\psi_i(|k^* - b_2^*(0)|) + \widetilde{F}(\frac{1}{2}b_2^*(0); k^*)\psi_i(k^*)$.

Thus the indifference condition is $-[1 - F(k^*)][\psi_i(|k^* - b_2^*(1)|) - \psi_i(k^*)] + [F(k^*) - F(\frac{1}{2}b_2^*(1); k^*)][\psi_i(|k^* - b_2^*(1)|) - \psi_i(k^*)] - [F(k^*) - F(\frac{1}{2}b_2^*(0); k^*)][\psi_i(|k^* - b_2^*(0)|) - \psi_i(k^*)] = 0$. The equation can be rewritten as

$$[2F(k^*) - F(\frac{1}{2}b_2^*(1))][\psi_i(|k^* - b_2^*(1)|) - \psi_i(k^*)] = [F(k^*) - F(\frac{1}{2}b_2^*(0))][\psi_i(|k^* - b_2^*(0)|) - \psi_i(k^*)]. \tag{A11}$$

We can check that $|k^* - b_2^*(1)| < k^*$ and $|k^* - b_2^*(0)| < k^*$, so that we have $\psi_i(|k^* - b_2^*(1)|) > \psi_i(k^*)$ and $\psi_i(|k^* - b_2^*(0)|) > \psi_i(k^*)$. We also have $F(k^*) > F(\frac{1}{2}b_2^*(0))$. Thus, the right-hand side of Equation A11 is positive. So the left-hand side should also be positive, and we must have

$$2F(k^*) > F(\frac{1}{2}b_2^*(1)) + 1. \tag{A12}$$

(a) Whenever $F(\frac{1}{2}\theta_A) \leq \frac{1}{2}$, we get $2F(k^*) \leq 2F(\frac{1}{2}\theta_A) \leq 1$. This inequality implies that $2F(k^*) - F(\frac{1}{2}b_2^*(1)) < 1$, which contradicts the Inequality A12. As a result, any equilibrium cut-point $k^*$ must be strictly greater than$\frac{1}{2}\theta_A$ whenever $\theta_A \leq 2F^{-1}(\frac{1}{2})$.

(b) When $F$ is convex, we have $\frac{1}{2}F(\frac{1}{2}b_2^*(1)) + \frac{1}{2} \cdot 1 \geq F(\frac{1}{2} \cdot \frac{1}{2}b_2^*(1) + \frac{1}{2}\overline{\theta})$. By combining this condition with Inequality A12, we get $F(k^*) > F(\frac{1}{4}b_S^*(1) + \frac{1}{2}\overline{\theta})$. We then get $k^* > \frac{1}{4}b_2^*(1) + \frac{1}{2}\overline{\theta}$, which implies that $\frac{1}{2}\theta_A > \frac{1}{4}b_2^*(1) + \frac{1}{2}\overline{\theta} > \frac{1}{2}\overline{\theta}$. This is contradictory to $\theta_A \leq \overline{\theta}$. As a result, in any equilibrium, the symmetric cut-point $k^* > \frac{1}{2}\theta_A$ when $F(\cdot)$ is convex. ■

**Proof of Proposition 5.**

The second part is directly implied from the first result that $k^* < \frac{1}{2}\theta_A$ and Lemma 1. Thus, we only need to prove the first part. Notice that $\beta(\cdot; k)$ generally depends on $\overline{\theta}$ and $\theta_A$.

For simplicity we do not explicitly write $\overline{\theta}$ and $\theta_A$ in the expression of the function $\beta(\cdot;k)$. We need to show that $\exists M_0 > 0$ such that for all $\theta_A = \overline{\theta} \geq M_0$, we have $k^* < \frac{1}{2}\theta_A$. We prove the result by contradiction.

Suppose it is not true. Then there must exist a series of setter's ideal points (indexed by $t$, i.e., $\theta_A = \overline{\theta} = \theta_t$) with $\lim\limits_{t\to+\infty} \theta_t = +\infty$ and the associated equilibrium cut-point $k_t \geq \frac{1}{2}\theta_t$. In such an equilibrium with cut-point $k$, we have $V_{diff}(k;k) = (1-F(k))[2k\theta_A - \theta_A^2 - (2k\beta(1;k) - \beta(1;k)^2)] + [F(k) - F(\frac{1}{2}\beta(1;k))][2k\beta(1;k) - \beta(1;k)^2] - [F(k) - F(\frac{1}{2}\beta(0;k))][2k\beta(0;k) - \beta(0;k)^2]$.

(a) We can verify that $\lim\limits_{t\to+\infty} \beta(1;k_t)$ and $\lim\limits_{t\to+\infty} \beta(0;k_t)$ finitely exist (according to the first order conditions of setter's optimization), and $\lim\limits_{t\to+\infty} \beta(1;k_t) > \lim\limits_{t\to+\infty} \beta(0;k_t)$. Specifically, $\lim\limits_{t\to+\infty} \beta(1;k_t)$ is the same as the proposal in equilibrium when the setter (with a linear utility) faces one voter in the static game; and $\lim\limits_{t\to+\infty} \beta(0;\theta_t)$ is the same as the proposal in equilibrium when the setter (with a linear utility) faces two voters and unanimity rule in the static game.[4]

(b) The limit of the payoff gain

$$\lim_{t\to+\infty} \frac{V_{diff}(k_t;k_t)}{2k_t} = \begin{cases} \lim\limits_{t\to+\infty} \{(1-F(k_t))(\theta_t - \beta(1;k_t))[1-\frac{\theta_t}{2k_t}] \\ \quad -(1-F(k_t))(\frac{\theta_t}{2k_t} - \frac{\beta(1;k_t)}{2k_t})\beta(1;k_t)\} \\ +\lim\limits_{t\to+\infty} \{[F(k_t) - F(\frac{1}{2}\beta(1;k_t))][\beta(1;k_t) - \frac{\beta(1;k_t)^2}{2k_t}] \\ \quad -[F(k_t) - F(\frac{1}{2}\beta(0;k_t))][\beta(0;k_t) - \frac{\beta(0;k_t)^2}{2k_t}]\} \end{cases}.$$

Because $\frac{\theta_t}{2k_t} \leq 1$, we have $\lim\limits_{t\to+\infty}(1-F(k_t))(\theta_t - \beta(1;k_t))[1-\frac{\theta_t}{2k_t}] \geq 0$, and $\lim\limits_{t\to+\infty}(1 - F(k_t))(\frac{\theta_t}{2k_t} - \frac{\beta(1;k_t)}{2k_t})\beta(1;k_t) = 0$. Hence, we get $\lim\limits_{t\to+\infty} \frac{V_{diff}(k_t;k_t)}{2k_t} \geq \lim\limits_{t\to+\infty} \{[F(k_t) - F(\frac{1}{2}\beta(1;k_t))]\beta(1;k_t) - [F(k_t) - F(\frac{1}{2}\beta(0;k_t))]\beta(0;k_t)\}$. Notice that $\lim\limits_{t\to+\infty}[F(k_t) - F(\frac{1}{2}\beta(1;k_t))]\beta(1;k_t)$ represents

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[4] By Lemma 1, $\beta(1;k_t)$ and $\beta(0;k_t)$ are determined by $\frac{1-\widetilde{F}(\frac{1}{2}\beta(1;k_t);k_t)}{\widetilde{f}(\frac{1}{2}\beta(1;k_t);k_t)} = \frac{1}{2}\frac{\beta(1;k_t) - \beta(1;k_t)^2/2\theta_t}{1-\beta(1;k_t)/2\theta_t}$ and $\frac{1-\widetilde{F}_{2,1}(\frac{1}{2}\beta(0;k_t);k_t)}{\widetilde{f}_{2,1}(\frac{1}{2}\beta(0;k_t);k_t)} = \frac{1}{2}\frac{\beta(0;k_t) - \beta(0;k_t)^2/2\theta_t}{1-\beta(0;k_t)/2\theta_t}$ respectively. Let $t \to +\infty$ in both equations, we get $\frac{1-G(\frac{1}{2}\lim_{t\to+\infty}\beta(1;k_t))}{g(\frac{1}{2}\lim_{t\to+\infty}\beta(1;k_t))} = \frac{1}{2}\lim\limits_{t\to+\infty}\beta(1;k_t)$ and $\frac{1-G_{2,1}(\frac{1}{2}\lim_{t\to+\infty}\beta(0;k_t))}{g_{2,1}(\frac{1}{2}\lim_{t\to+\infty}\beta(0;k_t))} = \frac{1}{2}\lim\limits_{t\to+\infty}\beta(0;k_t)$ respectively. Thus, $\lim\limits_{t\to+\infty} \beta(1;k_t)$ is the same as the proposal in equilibrium when the setter (with a linear utility) faces one voter in the static game without communication; and $\lim\limits_{t\to+\infty} \beta(0;\theta_t)$ is the same as the equilibrium proposal when the setter (with a linear utility) faces two voters and unanimity rule in the static game without communication.

the optimal expected payoff of the setter with a linear utility when she faces only one voter and no communication. This payoff should be strictly higher than the payoff if she chooses a different proposal $\lim_{t\to+\infty}\beta(0;k_t)$. Therefore, we get $\lim_{t\to+\infty}\frac{V_{diff}(k_t;k_t)}{2k_t} \geq \lim_{t\to+\infty}\{[F(k_t) - F(\frac{1}{2}\beta(1;k_t))]\beta(1;k_t) - [F(k_t) - F(\frac{1}{2}\beta(0;k_t))]\beta(0;k_t)\} > 0$. It implies that for a large enough $t$, the indifference condition is violated. This is in contradiction to the equilibrium requirement. As a result, $\exists M_0 > 0$ such that for all $\theta_A = \overline{\theta} \geq M_0$, we must have $k^* < \frac{1}{2}\theta_A$. ■

## Equilibrium Existence in Straw Poll

**Proof of Proposition 7.**

(1.1) First of all, we show that an indifferent type $k^*$ always exists. From Equation 2 and Equation 3, we get

$$\frac{V^S_{diff}(\theta_i;k^*)}{F(k^*)} = \begin{cases} [2\theta_i\theta_A - \theta_A^2] - [2\theta_i b_2^*(0) - b_2^*(0)^2] & \text{if } \theta_i \in [\frac{1}{2}\theta_A, \overline{\theta}] \\ [1-\widetilde{F}(\frac{1}{2}\theta_A;k^*)][2\theta_i\theta_A - \theta_A^2] & \\ \quad -[2\theta_i b_2^*(0) - b_2^*(0)^2] & \text{if } \theta_i \in [\frac{1}{2}b_2^*(0), \frac{1}{2}\theta_A) \\ [1-\widetilde{F}(\frac{1}{2}\theta_A;k^*)][2\theta_i\theta_A - \theta_A^2] & \\ \quad -[1-\widetilde{F}(\frac{1}{2}b_2^*(0);k^*)][2\theta_i b_2^*(0) - b_2^*(0)^2] & \text{if } \theta_i \in [0, \frac{1}{2}b_2^*(0)) \end{cases}. \tag{A13}$$

Because $k^* \geq \frac{1}{2}\theta_A$, the indifference condition of the equilibrium becomes $2k^*\theta_A - \theta_A^2 = 2k^*b_2^*(0) - b_2^*(0)^2$. Therefore, the equilibrium cut-point $k^*$ must satisfy $k^* = \frac{\theta_A+\beta(0;k^*)}{2}$. Define a new function $H(k) = k - \frac{\theta_A+\beta(0;k)}{2}$. We have $k > \frac{\theta_A+\beta(0;k)}{2}$ for $\forall k \geq \theta_A$, and $k < \frac{\theta_A+\beta(0;k)}{2}$ for $\forall k \leq \frac{1}{2}\theta_A$. Thus we know that $H(k)|_{k\geq\theta_A} > 0$ and $H(k)|_{k\leq\frac{1}{2}\theta_A} < 0$. Given that $H(k)$ is a continuous function with respect to $k$, an indifference type $k^*$ always exists. Furthermore we must also have: any equilibrium $k^* \in (\frac{1}{2}\theta_A, \theta_A)$.

(1.2) Although an indifferent type always exists, it does not necessarily indicate the existence of the informative cut-point equilibrium. We need to check the communication incentive of the other types. Given Equation A13, we know that the payoff-gain function $\frac{V^S_{diff}(\theta_i;k^*)}{F(k^*)}$ is piecewise linear so that we only need to check the incentives of three different types: $\theta_i = \frac{1}{2}\theta_A$, $\frac{1}{2}\beta(0;k^*)$ and 0. Specifically, we have: $\frac{V^S_{diff}(\theta_i=\frac{1}{2}\theta_A;k^*)}{F(k^*)} = -[\theta_A\beta(0;k^*) - \beta(0;k^*)^2] < 0$, and $\frac{V^S_{diff}(\theta_i=\frac{1}{2}\beta(0;k^*);k^*)}{F(k^*)} = [1-\widetilde{F}(\frac{1}{2}\theta_A;k^*)][\beta(0;k^*)\theta_A - \theta_A^2] < 0$.

It remains to check the incentive of the type $\theta_i = 0$. Notice that $\frac{V^S_{diff}(\theta_i=0;k^*)}{F(k^*)} = [1-\widetilde{F}(\frac{1}{2}\beta(0;k^*);k^*)]\beta(0;k^*)^2 - [1-\widetilde{F}(\frac{1}{2}\theta_A;k^*)]\theta_A^2$. Hence, no deviation condition for the type $\theta_i = 0$ is equivalent to the inequality

$$[F(k^*) - F(\frac{1}{2}\beta(0;k^*))]\beta(0;k^*)^2 \leq [F(k^*) - F(\frac{1}{2}\theta_A)]\theta_A^2. \tag{A14}$$

For convenience, we use two temporary notations: $z_1 \triangleq \beta(0;k^*), z_2 \triangleq \theta_A$. Then we have $k^* = \frac{z_1+z_2}{2}$. And the no-deviation inequality can be rewritten as: $F(k^*)(z_2^2 - z_1^2) + z_1^2F(\frac{1}{2}z_1) \geq F(\frac{1}{2}z_2)z_2^2$. Given that $F(\cdot)$ is convex on $[0,\theta_A]$, we have $F(k^*)\frac{z_2^2-z_1^2}{z_2^2} + \frac{z_1^2}{z_2^2}F(\frac{1}{2}z_1) \geq F(k\frac{z_2^2-z_1^2}{z_2^2} + \frac{z_1^2}{z_2^2}\frac{1}{2}z_1) = F(\frac{(z_2^2-z_1^2)+z_1z_2}{2z_2})$. Because $\frac{(z_2^2-z_1^2)+z_1z_2}{2z_2} \geq \frac{1}{2}z_2$, we have $F(k^*)\frac{z_2^2-z_1^2}{z_2^2} + \frac{z_1^2}{z_2^2}F(\frac{1}{2}z_1) \geq F(\frac{1}{2}z_2)$. After rearrangement, we get $F(k^*)(z_2^2-z_1^2)+z_1^2F(\frac{1}{2}z_1) \geq F(\frac{1}{2}z_2)z_2^2$, so that Inequality A14 is satisfied and the type $\theta_i = 0$ has no incentive to deviate.

(3) Now we show that as $\theta_A = \overline{\theta} \to +\infty$, the type $\theta_i = 0$ has an incentive to deviate, that is $[F(k^*) - F(\frac{1}{2}\beta(0;k^*))]\beta(0;k^*)^2 > [F(k^*) - F(\frac{1}{2}\theta_A)]\theta_A^2$.

(3.1) First we show that the limit of the left-hand side of the inequality is strictly positive. Since the indifferent type $k^* \geq \frac{1}{2}\theta_A$, we have $\lim_{\overline{\theta}=\theta_A\to+\infty} k^* = +\infty$. Because $\lim_{\overline{\theta}\to+\infty} F(\theta) = G(\theta)$ is well defined and $\lim_{\overline{\theta}=\theta_A\to+\infty} \frac{\psi_A(\theta_A-x)-\psi_A(\theta_A)}{-\psi_A'(\theta_A-x)} = \lim_{\overline{\theta}=\theta_A\to+\infty} \frac{2\theta_Ax-x^2}{2\theta_A-2x} = x$, by Lemma 1 we know that $\lim_{\overline{\theta}=\theta_A\to+\infty} \beta(0;k^*)$ is well defined and finite. Therefore, we have $\lim_{\overline{\theta}=\theta_A\to+\infty} [F(k^*) - F(\frac{1}{2}\beta(0;k^*))]\beta(0;k^*)^2 = [1 - G(\frac{1}{2}\lim_{\overline{\theta}=\theta_A\to+\infty} \beta(0;k^*))](\lim_{\overline{\theta}=\theta_A\to+\infty} \beta(0;k^*))^2 > 0$.

(3.2) Now we show that $\lim_{\overline{\theta}=\theta_A\to+\infty} [F(k^*) - F(\frac{1}{2}\theta_A)]\theta_A^2 = 0$. Notice that $0 \leq [F(k^*) - F(\frac{1}{2}\theta_A)]\theta_A^2 \leq [1 - F(\frac{1}{2}\theta_A)]\theta_A^2 \leq [1 - G(\frac{1}{2}\theta_A)]\theta_A^2$. So it is sufficient to show that $\lim_{\theta_A\to+\infty}[1 - G(\frac{1}{2}\theta_A)]\theta_A^2 = 0$. For any $y \geq 0$, because $\int_0^{+\infty} x^2dG(x) - \int_0^y x^2dG(x) = \int_y^{+\infty} x^2dG(x) \geq y^2\int_y^{+\infty} dG(x) = y^2[1 - G(y)]$, we have $\int_0^{+\infty} x^2dG(x) \geq y^2[1 - G(y)] + \int_0^y x^2dG(x)$. Let

$y \to +\infty$ in the inequality, we get $\int_0^{+\infty} x^2 dG(x) \geq \lim_{y \to +\infty} y^2[1 - G(y)] + \int_0^{+\infty} x^2 dG(x)$. It implies $\lim_{y \to +\infty} y^2[1-G(y)] = 0$. Thus, we have $\lim_{\theta_A \to +\infty} [1-G(\frac{1}{2}\theta_A)]\theta_A^2 = 0$ and $\lim_{\overline{\theta}=\theta_A \to +\infty} [F(k^*) - F(\frac{1}{2}\theta_A)]\theta_A^2 = 0$. Therefore, the type $\theta_i = 0$ has an incentive to deviate from the informative cut-point equilibrium, as $\theta_A = \overline{\theta} \to +\infty$. As a result, when $\theta_A = \overline{\theta}$ and they are sufficiently large, the only possible symmetric cut-point equilibrium is that $k^* = \overline{\theta}$. ■

**Proof of Proposition 8.**

Each voter's expected payoff gain from casting a positive vote rather than a negative one is

$$V^S_{diff}(\theta_i; k^*) = (1 - F(k^*))\Delta_1(\theta_i) + F(k^*)\Delta_0(\theta_i), \tag{A15}$$

where $\Delta_1(\theta_i) \triangleq V(\theta_i, b_2^*(2), 1) - V(\theta_i, b_2^*(1), 1)$ and $\Delta_0(\theta_i) \triangleq V(\theta_i, b_2^*(1), 0) - V(\theta_i, b_2^*(0), 0)$. For convenience, let's also introduce the following six notations: $A_1 \triangleq [1-\widehat{F}(\frac{1}{2}b_2^*(2); k^*)]b_2^*(2) - b_2^*(1)$, $B_1 \triangleq [1-\widehat{F}(\frac{1}{2}b_2^*(2); k^*)]b_2^*(2)^2 - b_2^*(1)^2$, $A_0 \triangleq [1-\widetilde{F}(\frac{1}{2}b_2^*(1); k^*)]b_2^*(1) - [1-\widetilde{F}(\frac{1}{2}b_2^*(0); k^*)]b_2^*(0)$, $B_0 \triangleq [1 - \widetilde{F}(\frac{1}{2}b_2^*(1); k^*)]b_2^*(1)^2 - [1 - \widetilde{F}(\frac{1}{2}b_2^*(0); k^*)]b_2^*(0)^2$, $C_0 = [1 - \widetilde{F}(\frac{1}{2}b_2^*(0); k^*)]b_2^*(0)$, $D_0 = [1 - \widetilde{F}(\frac{1}{2}b_2^*(0); k^*)]b_2^*(0)^2$. Then we have

$$\Delta_1(\theta_i) = \begin{cases} 2A_1\theta_i - B_1 & \text{if} \quad \theta_i \geq \frac{1}{2}b_2^*(2) \\ -2b_2^*(1)\theta_i + b_2^*(1)^2 & \text{if} \quad \frac{1}{2}b_2^*(1) \leq \theta_i < \frac{1}{2}b_2^*(2) \\ 0 & \text{if} \quad \theta_i < \frac{1}{2}b_S^*(1) \end{cases}, \tag{A16}$$

$$\Delta_0(\theta_i) = \begin{cases} 2A_0\theta_i - B_0 & \text{if} \quad \theta_i \geq \frac{1}{2}b_2^*(1) \\ -2C_0\theta_i + D_0 & \text{if} \quad \frac{1}{2}b_2^*(0) \leq \theta_i < \frac{1}{2}b_2^*(1) \\ 0 & \text{if} \quad \theta_i < \frac{1}{2}b_2^*(0) \end{cases}. \tag{A17}$$

We have $b_2^*(1) > 0$, $C_0 > 0$ and $b_2^*(2) > b_2^*(1) > b_2^*(0)$. We can also verify that both $\Delta_1(\theta_i)$ and $\Delta_0(\theta_i)$ are continuous in $\theta_i$.

(1) Let's first establish the following claim: $k^*$ *forms a cut-point equilibrium if and only if* $V_{diff}(\theta_i = k^*; k^*) = 0$, that is, no other type of voters has an incentive to deviate from the

cut-point strategy once we find the existence of an indifferent type.[5] We prove this claim by three steps.

(1.1) In the first step, we show that $A_1 > 0$, i.e., $[1 - \widehat{F}(\frac{1}{2}\beta(2;k^*);k^*)]\beta(2;k^*) > \beta(1;k^*)$. Notice that $2k^* > \beta(1;k^*)$. Hence, a sufficient condition for $A_1 > 0$ to hold is: $[1 - \widehat{F}(\frac{1}{2}\beta(2;k^*);k^*)]\beta(2;k^*) \geq 2k^*$. This inequality is equivalent to $H_1(\beta(2;k^*);k^*) \geq H_1(2k^*;k^*)$, where $H_1(x;k) \triangleq [1 - \widehat{F}(\frac{1}{2}x;k^*)]x$. Similarly as in the proof of Lemma 1 in the Supplementary Appendix, we can show that the function $H_1(x;k)$ is single-peaked with respect to $x$. If $\arg\max_x H_1(x;k) \geq \beta(2;k)$, then we know that $\beta(2;k)$ and $2k$ are in the interval where $[1 - \widehat{F}(\frac{1}{2}x;k)]x$ is strictly increasing (in $x$) so that we will get the result. Thus, we need to show that $\arg\max_x H_1(x;k) \geq \arg\max_x [1 - \widehat{F}(\frac{1}{2}x;k)]^2[2\theta_A x - x^2]$.

Similarly as in the proof of Lemma 1 in the Supplementary Appendix, we can verify that $\arg\max_x H_1(x;k) = \min\{x \in [2k,+\infty) : \frac{1-\widehat{F}(\frac{1}{2}x;k)}{2\widehat{f}(\frac{1}{2}x;k)} - x \leq 0\}$, and $\arg\max_x [1-\widehat{F}(\frac{1}{2}x;k)]^2[2\theta_A x - x^2] = \min\{x \in [2k,\theta_A) : \frac{1-\widehat{F}(\frac{1}{2}x;k)}{2\widehat{f}(\frac{1}{2}x;k)} - \frac{2\theta_A x - x^2}{2\theta_A - 2x} \leq 0\}$. Notice that both $\frac{1-\widehat{F}(\frac{1}{2}x;k)}{2\widehat{f}(\frac{1}{2}x;k)} - x$ and $\frac{1-\widehat{F}(\frac{1}{2}x;k)}{2\widehat{f}(\frac{1}{2}x;k)} - \frac{2\theta_A x - x^2}{2\theta_A - 2x}$ are strictly decreasing functions of $x$. Because $\frac{2\theta_A x - x^2}{2\theta_A - 2x} \geq x$, we must have $\arg\max_x [1 - \widehat{F}(\frac{1}{2}x;k)]x \geq \arg\max_x [1 - \widehat{F}(\frac{1}{2}x;k)]^2[2\theta_A x - x^2]$. Thus, we have $A_1 > 0$.

(1.2) In the second step, we show that $A_0 > 0$, i.e., $[1 - \widetilde{F}(\frac{1}{2}\beta(1;k);k)]\beta(1;k) \geq [1 - \widetilde{F}(\frac{1}{2}\beta(0;k);k)]\beta(0;k)$. Similarly as in the last step, we can prove that $\arg\max_x [1 - \widetilde{F}(\frac{1}{2}x;k)]x \geq \arg\max_x [1-\widetilde{F}(\frac{1}{2}x;k)][2\theta_A x - x^2]$. Because $\beta(1;k) \geq \beta(0;k)$ and the function $[1 - \widetilde{F}(\frac{1}{2}x;k)]x$ is single peaked, we have $[1 - \widetilde{F}(\frac{1}{2}\beta(1;k);k)]\beta(1;k) \geq [1 - \widetilde{F}(\frac{1}{2}\beta(0;k);k)]\beta(0;k)$.

(1.3) In the third step, we complete the proof in (1) and show that the incentive compatibility constraints are automatically satisfied whenever an indifferent type exists. By the results above, we have: $A_1 > 0$, $A_0 > 0$, $C_0 > 0$ and $b_2^*(2) > b_2^*(1) > b_2^*(0)$. The shape of the conditional payoff-gain functions $\Delta_1(\theta_i)$ and $\Delta_0(\theta_i)$ are presented in Figure 1. $V_{diff}^S(\theta_i;k^*)$ is the weighted average of $\Delta_1(\theta_i)$ and $\Delta_0(\theta_i)$. Formally, we have

(i) when $\theta_i < \frac{1}{2}b_2^*(0)$, $V_{diff}^S(\theta_i;k^*) = 0$;

[5] The standard signaling games typically impose an exogenous single-crossing condition on the utility function. With that assumption, $V_{diff}^S(\theta_i;k^*)$ can be a monotonic function of $\theta_i$. However in our game, the shape of the function $V_{diff}^S(\theta_i;k^*)$ is endogenously determined and is not monotonic in $\theta_i$.

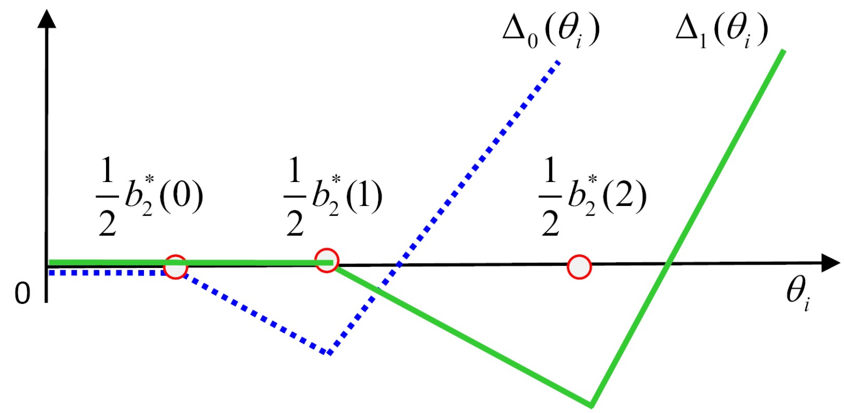


**Figure 1. A voter's conditional payoff gains $\Delta_1(\theta_i)$ and $\Delta_0(\theta_i)$**

(ii) when $\frac{1}{2}b_2^*(0) \leq \theta_i < \frac{1}{2}b_2^*(1)$, $V_{diff}^S(\theta_i; k^*) = -2F(k^*)C_0\theta_i + F(k_S^*)D_0$;

(iii) when $\frac{1}{2}b_2^*(1) \leq \theta_i < \frac{1}{2}b_2^*(2)$, $V_{diff}^S(\theta_i; k^*) = [1-F(k^*)][-2b_2^*(1)\theta_i + b_2^*(1)^2] + F(k^*)[2A_0\theta_i - B_0]$; and

(iv) when $\theta_i \geq \frac{1}{2}b_2^*(2)$, $V_{diff}^S(\theta_i; k_S^*) = [1 - F(k^*)][2A_1\theta_i - B_1] + F(k^*)[2A_0\theta_i - B_0]$.

Based on the above characterizations, we can simply verify that: $V_{diff}^S(\theta_i; k^*) \leq 0$ when $\theta_i < \frac{1}{2}b_2^*(1)$; and $V_{diff}^S(\theta_i; k^*)$ is strictly increasing in $\theta_i$ when $\theta_i \geq \frac{1}{2}b_2^*(2)$.

If $V_{diff}^S(\theta_i = \frac{1}{2}b_2^*(2); k^*) < 0$ as shown in Figure 2, then we have: $V_{diff}^S(\theta_i; k^*) \leq 0$ when $\theta_i < \frac{1}{2}b_2^*(2)$; and $V_{diff}^S(\theta_i; k^*)$ is strictly increasing in $\theta_i$ when $\theta_i \geq \frac{1}{2}b_2^*(2)$. The indifferent type $k^*$ must be higher than $\frac{1}{2}b_2^*(2)$, and no other type has an incentive to deviate.

If $V_{diff}^S(\theta_i = \frac{1}{2}b_2^*(2); k^*) \geq 0$ as shown in Figure 3, then we have: $V_{diff}^S(\theta_i; k^*) \leq 0$ when $\theta_i < \frac{1}{2}b_2^*(1)$; and $V_{diff}^S(\theta_i; k^*)$ is strictly increasing in $\theta_i$ when $\theta_i \geq \frac{1}{2}b_2^*(1)$. The indifferent type $k^*$ must be weakly lower than $\frac{1}{2}b_2^*(2)$, and no other type has an incentive to deviate.

(2) We now establish the existence of the indifference type.

(2.1) We show that $V_{diff}^S(k; k) > 0$ as $k$ is sufficiently close to $\overline{\theta}$.

$$\lim_{k \to \overline{\theta}} V_{diff}^S(k; k)$$
$$= [1 - F(\tfrac{1}{2}\beta(1; \overline{\theta}))][2\overline{\theta}\beta(1; \overline{\theta}) - \beta(1; \overline{\theta})^2] - [1 - F(\tfrac{1}{2}\beta(0; \overline{\theta}))][2\overline{\theta}\beta(0; \overline{\theta}) - \beta(0; \overline{\theta})^2]$$
$$\geq [1 - F(\tfrac{1}{2}\beta(1; \overline{\theta}))][2\theta_A\beta(1; \overline{\theta}) - \beta(1; \overline{\theta})^2] - [1 - F(\tfrac{1}{2}\beta(0; \overline{\theta}))][2\theta_A\beta(0; \overline{\theta}) - \beta(0; \overline{\theta})^2]$$
$$> 0.$$

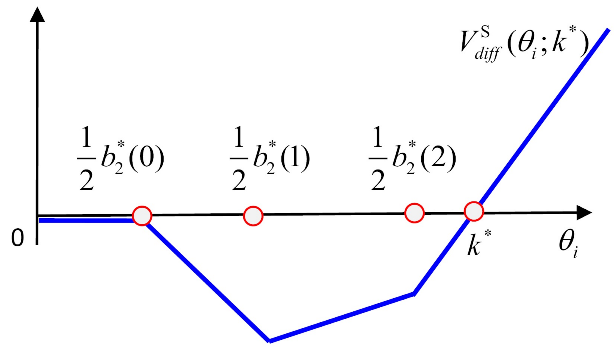


**Figure 2. A voter's payoff gain from making an endorsement** $V^S_{diff}(\theta_i;k^*)$ **with** $V^S_{diff}(\theta_i = \frac{1}{2}b_2^*(2);k^*) < 0$

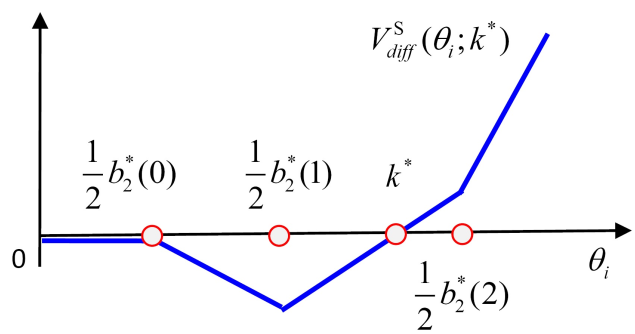


**Figure 3. A voter's payoff gain from making an endorsement** $V^S_{diff}(\theta_i;k^*)$ **with** $V^S_{diff}(\theta_i = \frac{1}{2}b_2^*(2);k^*) \geq 0$

The first inequality is due to the fact that $[1-F(\frac{1}{2}\beta(1;k))]\beta(1;k) > [1-F(\frac{1}{2}\beta(1;k))]\beta(1;k)$. The second inequality is due to the definition of $\beta(1;k)$.

(2.2) We now show that $V^S_{diff}(k;k) < 0$ as $k$ is sufficiently close to 0. When $k$ is close to 0, we have

$$\frac{V^S_{diff}(k;k)}{2\beta(1;k)k} = \left\{ \begin{array}{c} F(k)\{[1-\widetilde{F}(\frac{1}{2}\beta(1;k);k)][1-\frac{\beta(1;k)^2}{2\beta(1;k)k}] - [1-\widetilde{F}(\frac{1}{2}\beta(0;k);k)][\frac{\beta(0;k)}{\beta(1;k)} - \frac{\beta(0;k)}{\beta(1;k)}\frac{\beta(0;k)^2}{2k\beta(0;k)}]\} \\ +(1-F(k))[-1+\frac{\beta(1;k)}{2k}] \end{array} \right. . \tag{A18}$$

Because $\frac{\beta(1;k)^2}{2\beta(1;k)k}, \frac{\beta(0;k)}{\beta(1;k)}, \frac{\beta(0;k)^2}{2k\beta(0;k)}$ are bounded, $\lim_{k\to 0}\frac{V_{diff}(k;k)}{2\beta(1;k)k} = \lim_{k\to 0}\frac{\beta(1;k)}{2k} - 1$. Recall that $\beta(1;k)$ is pinned down by $\frac{F(k)-F(\frac{1}{2}b)}{f(\frac{1}{2}b)} = \frac{1}{2}\frac{2\theta_A b - b^2}{2(\theta_A - b)}$. This first order condition implies a monotonic relationship between $k$ and $b = \beta(1;k)$. As $k \to 0$, we have $b = \beta(1;k) \to 0$ because $\beta(1;k) \in (0, 2k)$. The first order condition of the revised proposal $\frac{F(k)-F(\frac{1}{2}b)}{f(\frac{1}{2}b)} =$

$\frac{1}{2}\frac{2\theta_A b - b^2}{2(\theta_A - b)}$ can be rewritten as $F(k) = \frac{1}{2}\frac{2\theta_A b - b^2}{2(\theta_A - b)} f(\frac{1}{2}b) + F(\frac{1}{2}b)$. Therefore, we have $\lim_{k\to 0}\frac{\beta(1;k)}{2k} = \lim_{b\to 0^+}\frac{b}{2F^{-1}[\frac{1}{2}\frac{2\theta_A b - b^2}{2(\theta_A - b)} f(\frac{1}{2}b) + F(\frac{1}{2}b)]}$. Since $\lim_{b\to 0} b = \lim_{b\to 0} F^{-1}[\frac{1}{2}\frac{2\theta_A b - b^2}{2(\theta_A - b)} f(\frac{1}{2}b) + F(\frac{1}{2}b)] = 0$, we need to apply L'Hospital's rule and get $\lim_{k\to 0}\frac{\beta(1;k)}{2k} = \frac{1}{2}\lim_{b\to 0}\frac{f(0)}{[(\frac{1}{2}\frac{2\theta_A b - b^2}{2(\theta_A - b)})' f(\frac{1}{2}b) + (\frac{1}{2}\frac{2\theta_A b - b^2}{2(\theta_A - b)}) f'(\frac{1}{2}b)\frac{1}{2} + f(\frac{1}{2}b)\frac{1}{2}]} = \frac{1}{2}\frac{f(0)}{\frac{1}{2}f(0) + f(0)\frac{1}{2}} = \frac{1}{2}$. As a result, we have $\lim_{k\to 0}\frac{\beta(1,k)}{2k} < 1$ and $\lim_{k\to 0}\frac{V_{diff}(k;k)}{2\beta(1;k)k} < 0$. Thus, $\exists k_0 > 0$ such that $V_{diff}(k_0; k_0) < 0$.

(2.3) Because $V^S_{diff}(k;k)$ is a continuous function of $k$, $\exists k^* \in (k_0, \overline{\theta})$ such that $V^S_{diff}(k^*; k^*) = 0$. ■

# Supplementary Appendix for *Communication in Collective Bargaining*

Jidong Chen*

This Supplementary Appendix supplements *Communication in Collective Bargaining*. We provide detailed proofs for some technical results and extensions of the model. We do not cover the results that are proven directly in the paper or in the Online Appendix. We first characterize the properties of the second-period proposal in Lemma 1. Lemma 3 and Lemma 4 offer some properties of order statistics and serve as preparations for the proof of Lemma 1. Second, we show an example of a unique equilibrium under straw poll with two voters and simple majority rule. Third, we offer a proof for Proposition 6, showing a necessary equilibrium condition ($k^* \geq \frac{1}{2}\theta_A$) with an arbitrary number of voters and an arbitrary voting rule. In the end, we offer a proof for Proposition 9, a robustness result when we allow a general message space.

## Characterizing the Second-Period Proposal

**Lemma 1** *(Basic Properties of Order Statistics) Suppose $F_{x,y}$ is the distribution function representing the $y$ th smallest random variable among the $x$ i.i.d. random variables with distribution function $F(\cdot)$ and probability density function $f(\cdot), (x, y \in \mathbb{Z}^+, x \geq y)$ then we have:*

*(1)* $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} = \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)}$;

*(2)* $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ *is decreasing in $\theta$ provided $F(\cdot)$ satisfies increasing hazard rate property; when $y \geq 2$,* $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ *is strictly decreasing in $\theta$;*

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*(3)* $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} > \frac{1-F_{x+1,y}(\theta)}{f_{x+1,y}(\theta)}$; *and*

*(4)* $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x,y+1}(\theta)}{f_{x,y+1}(\theta)}$ *and* $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)}$.

**Proof of Lemma 1.**

(1) The distribution function $F_{x,y}(\theta)$ and the probability density function $f_{x,y}(\theta)$ of the $y$ th smallest order statistics from $x$ i.i.d. random variables are given by:

$$1 - F_{x,y}(\theta) = \sum_{i=0}^{y-1} \binom{x}{i} F^i (1-F)^{x-i}, \tag{S1}$$

$$f_{x,y}(\theta) = \frac{x!}{(y-1)!(x-y)!} F^{y-1}(1-F)^{x-y} f. \tag{S2}$$

By calculation, we get

$$\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} = \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)}. \tag{S3}$$

(2) From the expression above, we know that $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ is strictly decreasing in $\theta$, provided $F(\cdot)$ satisfies increasing hazard rate property and $y \geq 2$. When $y = 1$, we have $\frac{1-F_{x,1}(\theta)}{f_{x,1}(\theta)} = \frac{1}{x}\frac{1-F(\theta)}{f(\theta)}$. Therefore, $\frac{1-F_{x,1}(\theta)}{f_{x,1}(\theta)}$ is weakly decreasing in $\theta$.

(3) We now show that $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} > \frac{1-F_{x+1,y}(\theta)}{f_{x+1,y}(\theta)}$ is decreasing in $x$. Specifically, we have

$\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} > \frac{1-F_{x+1,y}(\theta)}{f_{x+1,y}(\theta)}$

$\Leftrightarrow \sum_{i=0}^{y-1} \frac{(y-1)!(x+1-y)!}{(x+1-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)} < \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)}$

$\Leftrightarrow \sum_{i=0}^{y-1} \frac{(x+1-y)}{(x+1-i)(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} < \sum_{i=0}^{y-1} \frac{1}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1}$

$\Leftarrow \frac{(x+1-y)}{(x+1-i)(x-i)!(i)!} < \frac{1}{(x-i)!(i)!} \forall 0 \leqslant i \leqslant y-1$

$\Leftrightarrow x+1-y < x+1-i, \forall 0 \leqslant i \leqslant y-1$

$\Leftrightarrow i < y, \forall 0 \leqslant i \leqslant y-1.$

(4) To show $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ is increasing in $y$, we only need to show $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)}$ because $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)} < \frac{1-F_{x,y+1}(\theta)}{f_{x,y+1}(\theta)}$. We have

$$\frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)} = \sum_{i=0}^{y} \frac{y!(x-y)!}{(x+1-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i}\frac{1-F(\theta)}{f(\theta)}$$
$$= \sum_{i=1}^{y} \frac{y!(x-y)!}{(x+1-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i}\frac{1-F(\theta)}{f(\theta)} + \frac{y!(x-y)!}{(x+1)!}(\frac{1-F(\theta)}{F(\theta)})^{y}\frac{1-F(\theta)}{f(\theta)}$$
$$> \sum_{i=1}^{y} \frac{y!(x-y)!}{(x+1-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i}\frac{1-F(\theta)}{f(\theta)}$$
$$= \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i-1}\frac{1-F(\theta)}{f(\theta)}\frac{y}{(i+1)}$$
$$\geq \frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}.$$

As a result, $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ is increasing in $y$. ■

We use Lemma 1 to show the following lemma.

**Lemma 2** *Suppose* $\widetilde{F}(x) \triangleq \min\{\frac{F(x)}{F(k)}, 1\}$, $\widehat{F}(x) \triangleq \max\{\frac{F(x)-F(k)}{1-F(k)}, 0\}$ . $\widetilde{F}_{n-y,n-q+1}$ *is the (n-q+1)th smallest order statistics among the* $(n-y)$ *(with* $y \leq q-1$*) i.i.d. random variables with distribution* $\widetilde{F}(x)$. $\widehat{F}_{y,y-q+1}$ *is the* $(y-q+1)$*th smallest order statistics among the* $y$ *(with* $y \geq q$*) i.i.d. random variables with distribution* $\widehat{F}(x)$. *The following statements are true under Assumption 1:*

*(1) provided that* $\widetilde{f}_{n-y,n-q+1}(\theta;k) > 0, \frac{1-\widetilde{F}_{n-y,n-q+1}(\theta;k)}{\widetilde{f}_{n-y,n-q+1}(\theta;k)}$ *is strictly increasing in* $y$, *strictly decreasing in* $\theta$, *strictly increasing and continuously differentiable in* $k$; *and*

*(2) provided that* $\widehat{f}_{y,y-q+1}(\theta;k) > 0$, $\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)}$ *is strictly increasing in* $y$, *decreasing in* $\theta$,[1] *and is independent of* $k$ *whenever* $y = q$.

**Proof of Lemma 2.**

(1) According to Lemma 1, $\frac{1-\widetilde{F}_{n-y,n-q+1}(\theta;k)}{\widetilde{f}_{n-y,n-q+1}(\theta;k)}$ is strictly strictly increasing in $y$. We know that

$$\frac{1-\widetilde{F}_{n-y,n+1-q}(\theta;k)}{\widetilde{f}_{n-y,n+1-q}(\theta;k)} = \sum_{i=0}^{n-q} \frac{(n-q)![n-y-(n+1-q)]!}{(n-y-i)!(i)!}(\frac{F(k)-F(\theta)}{F(\theta)})^{n-q-i}\frac{F(k)-F(\theta)}{f(\theta)}. \tag{S4}$$

This equation implies that $\frac{1-\widetilde{F}_{n-y,n-q+1}(\theta;k)}{\widetilde{f}_{n-y,n-q+1}(\theta;k)}$ is strictly increasing and continuously differentiable in $k$. In the following we show that $\frac{A-F(x)}{f(x)}$ is strictly decreasing for $F(x) < A$ with

[1] It is strictly decreasing in $\theta$ whenever $y > q$ or $\frac{1-F(\theta)}{f(\theta)}$ is strictly decreasing.

$0 < A < 1$. Notice that $(\frac{1-F}{f})' = \frac{-f^2-(1-F)f'}{f^2} \leq 0$. Then we have $f' \geqslant -\frac{f^2}{1-F}$. The condition $0 < A < 1$ implies that $0 < A - F < 1 - F$. Therefore, we have $-\frac{1}{1-F} > -\frac{1}{A-F}$, so that $f' \geqslant -\frac{f^2}{1-F} > -\frac{f^2}{A-F}$. So we have $(\frac{A-F}{f})' = \frac{-f^2-(A-F)f'}{f^2} < 0$. Thus, $\frac{1-\widetilde{F}_{n-y,n+1-q}(\theta;k)}{\widetilde{f}_{n-y,n+1-q}(\theta;k)}$ is strictly decreasing in $\theta$.

(2) Similar as in (1), we can use Lemma 1 to show the property that $\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)}$ is strictly decreasing in $r$. Notice that $\frac{1-\widehat{F}(x)}{\widehat{f}(x)} = \frac{1-F(x)}{f(k)}$ when $x > k$. The detailed expression of $\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)}$ is given by

$$\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)} = \sum_{i=0}^{y-q} \frac{(y-q)!(q-1)!}{(y-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)-F(k)})^{y-q-i} \frac{1-F(\theta)}{f(\theta)}. \tag{S5}$$

Because the functions $\frac{1-F(\theta)}{F(\theta)-F(k)}$ and $\frac{1-F(\theta)}{f(\theta)}$ are both decreasing functions, $\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)}$ is decreasing in $\theta$. The above equation also implies that $\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)}$ is independent of $k$ whenever $y = q$. ■

**Proof of Lemma 1.**

For convenience, we use $E(\widetilde{u}_A)$ to represent setter's expected utility at the beginning of the second period.

(1) The proof consists of three steps. In the first step, we rule out the possibility that optimal $\frac{1}{2}b$ is outside of the support of the distribution. Based on this, we can then take the derivative and show that the objective function of the setter is inversely U-shaped. Third, we use the sign of derivative at the boundary to pin down whether the optimal proposal is a corner solution or not.

(1.1) For any $b \in (0, 2\theta_A) \cap (-\infty, 2k)$, we have $0 < \frac{1}{2}b < \theta_A$ and $\frac{1}{2}b < k$. Thus $E(\widetilde{u}_A) > 0$. This implies that any proposal lies in $[2k, +\infty) \cup \{0\}$ (i.e., that offers the setter a non-positive expected payoff) is strictly dominated by the proposals in $(0, 2\theta_A) \cap (-\infty, 2k)$. If $2k \leq \theta_A$, the optimal proposal $b < 2k$. If $\theta_A < 2k$, any proposal $b$ in $(\theta_A, 2k]$ is strictly dominated by $b - \epsilon$ for some small positive $\epsilon$. Therefore $\beta(y;k) \leq \theta_A < 2k$. As a summary of the above discussions, we have $\beta(y;k) \in (0, 2k)$ and $\beta(y;k) \leq \theta_A$.

(1.2) We have

$$\frac{dE(\widetilde{u}_A)}{db} = [1 - \widetilde{F}_{n-y,n-q+1}(\tfrac{1}{2}b)]\widetilde{u}'_A(b) - \widetilde{f}_{n-y,n-q+1}(\tfrac{1}{2}b)\tfrac{1}{2}\widetilde{u}_A(b)$$
$$= u'_A(b)\widetilde{f}_{n-y,n-q+1}(\tfrac{1}{2}b)[\frac{1-\widetilde{F}_{n-y,n-q+1}(\frac{1}{2}b)}{\widetilde{f}_{n-y,n-q+1}(\frac{1}{2}b)} - \frac{1}{2}\frac{\widetilde{u}_A(b)}{\widetilde{u}'_A(b)}].$$

By Lemma 3, we know that $\frac{1-\widetilde{F}_{n-y,n-q+1}(\frac{1}{2}b)}{\widetilde{f}_{n-y,n-q+1}(\frac{1}{2}b)}$ is decreasing. Notice that $\frac{1}{2}\frac{\widetilde{u}_A(b)}{\widetilde{u}'_A(b)}$ is increasing and $\widetilde{u}'_A(b)\widetilde{f}_{n-y,n-q+1}(\frac{1}{2}b) > 0$. Thus, as $b$ increases, the sign of$\frac{dE(\widetilde{u}_A)}{db}$ never becomes positive once it reaches a negative value. As a result, $E(\widetilde{u}_A) = [1 - \widetilde{F}_{n-y,n-q+1}(\frac{1}{2}b)]\widetilde{u}_A(b)$ must be inversely U-shaped.

(1.3) If $\theta_A < 2k$, we get $\frac{dE(\widetilde{u}_A)}{db}\ |_{b=\theta_A} = -\widetilde{f}_{n-y,n-q+1}(\frac{1}{2}\theta_A)\frac{1}{2}\widetilde{u}_A(\theta_A) < 0$. As a result, $\beta(y;k) \in (0, \min\{2k, \theta_A\})$ and is uniquely determined by the first order condition.

(2.1) Suppose $0 < \frac{1}{2}\theta_A \leq k < \overline{\theta}$. For any $\theta_i \geq k$, we have $\theta_i \geq \frac{1}{2}\theta_A$ so that $\psi_i(|\theta_i - \theta_A|) \geq \psi_i(|0 - \theta_A|)$. Thus, the voter with ideal point $\theta_i$ weakly prefers the setter's ideal point to the status quo. When $y \geq q$, the "pivotal" ideal point must be higher than the cut-point, therefore $\beta(y;k) = \theta_A$.

(2.2) When $0 < k < \frac{1}{2}\theta_A$, we can make the proof in similar ways as in (1). The setter's expected payoff is $E(\widetilde{u}_A) = [1 - \widehat{F}_{y,y-q+1}(\frac{1}{2}b)]\widetilde{u}_A(b)$. Observe that any proposal $b \geq 2\overline{\theta}$ is strictly dominated by $b \in (0, 2\theta_A)$. Furthermore, any $b \in (\theta_A, 2\overline{\theta})$ is strictly dominated by $b - \epsilon$ with a sufficiently small $\epsilon > 0$. In addition, $b < 2k$ is strictly dominated by $b = 2k$. Hence, we must have $\beta(y;k) \in [2k, \theta_A]$. Because $\frac{dE(\widetilde{u}_A)}{db}\ |_{b=\theta_A} = -\widehat{f}_{y,y-q+1}(\frac{1}{2}\theta_A)\frac{1}{2}\widetilde{u}_A(\theta_A) < 0$, we have $\beta(y;k) \in [2k, \theta_A)$. We can also check that $\frac{dE(\widetilde{u}_A)}{db}\ |_{b=2k} = \widetilde{u}'_A(2k) - \frac{1}{2}\widehat{f}_{y,y-q+1}(k)\widetilde{u}_A(2k)$. Because $0 < 2k < \theta_A$, we have $\widetilde{u}'_A(2k) > 0$. Also notice that

$$\widehat{f}_{y,y-q+1}(k) = \begin{cases} 0 & \text{if} \quad q < y \\ q\frac{f(k)}{1-F(k)} & \text{if} \quad q = y \end{cases}. \tag{S6}$$

Thus, only when $y = q$ and $\frac{1}{q}\frac{1-F(k)}{f(k)} \leq \frac{1}{2}\frac{\widetilde{u}_A(2k)}{\widetilde{u}'_A(2k)}$, we get $\beta(y;k) = 2k$; otherwise, $\beta(y;k) \in (2k, \theta_A)$ and is uniquely determined by the first order condition $\frac{1-F_{y,y-q+1}(\frac{1}{2}b;k)}{F_{y,y-q+1}(\frac{1}{2}b;k)} = \frac{1}{2}\frac{\widetilde{u}_A(b)}{\widetilde{u}'_A(b)}$. ■

## Uniqueness of the Straw-Poll Equilibrium under Simple Majority Rule

**Example 1** *Consider the straw-poll game with a committee of a setter and two voters under simple majority rule. Suppose each voter's ideal point follows a uniform distribution, i.e.,* $\theta_i \tilde{} U(0, \overline{\theta})$. *We can verify that the equilibrium cut-point is uniquely determined by* $k^* = \frac{1}{2}\beta(2, k^*) + \frac{\theta_A}{2}$.

**Proof of Example 1.**

According to Lemma 1, $\beta(0;k)$ is determined by $\frac{4k^2-\beta(0;k)^2}{\beta(0;k)} = \frac{2\theta_A\beta(0;k)-\beta(0;k)^2}{\theta_A-\beta(0;k)}$. Combining it with the cut-point condition, i.e., $k = \frac{1}{2}\beta(0;k) + \frac{\theta_A}{2}$, we have $\frac{4k^2-(2k-\theta_A)^2}{(2k-\theta_A)} = (2k-\theta_A)\frac{2\theta_A-(2k-\theta_A)}{\theta_A-(2k-\theta_A)}$. This is equivalent to $k^3 - \frac{7}{2}\theta_A k^2 + 3\theta_A^2 k - \frac{5}{8}\theta_A^3 = 0$.

Define $\varphi(x) \triangleq x^3 - \frac{7}{2}\theta_A x^2 + 3\theta_A^2 x - \frac{5}{8}\theta_A^3$ for $x \in [\frac{\theta_A}{2}, \theta_A]$. We have $\varphi'(x) = 3x^2 - 7\theta_A x + 3\theta_A^2$ for $x \in [\frac{\theta_A}{2}, \theta_A]$. Specifically, we have: $\varphi'(\frac{\theta_A}{2}) = \frac{1}{4}\theta_A^2 > 0$, $\varphi'(\theta_A) = -\theta_A^2 < 0$, $\varphi(\frac{\theta_A}{2}) = \frac{1}{8}\theta_A^3 > 0$ and $\varphi(\theta_A) = -\frac{1}{8}\theta_A^3 < 0$. As a result, $\varphi(x)$ is strictly increasing from $\frac{\theta_A}{2}$ to some $\widehat{x} < \theta_A$ and is strictly decreasing from $\widehat{x}$ to $\theta_A$. Thus $\varphi(x)$ has a unique root, which is on $(\widehat{x}, \theta_A)$. ■

## Generalized Results

**Proof of Proposition 6.**

We prove the result by contradiction. Suppose the result does not hold. Then we have at least one equilibrium cut-point $k^* < \frac{1}{2}\theta_A$. By Lemma 1 and Lemma 2, in this equilibrium, we have $\theta_A > b_2^*(n) > ... > b_2^*(q) \geq 2k^* > b_2^*(q-1)... > b_2^*(1) > b_2^*(0)$. By the definition of the cut-point equilibrium, we must have $V_{diff}^S(\theta_i = \frac{1}{2}b_2^*(n); k^*) \geq 0$. We will then derive an implication that contradicts with this inequality.

We first show the following condition holds

$$\binom{n-1}{j} F(k^*)^{n-j-1}[1-F(k^*)]^j V(\theta_i, b_2^*(j+1), j) < \binom{n-1}{j+1} F(k^*)^{n-j-2}[1-F(k^*)]^{j+1} V(\theta_i, b_2^*(j+1), j+1), \tag{S7}$$

for all $j \leq n-2$ when $\theta_i = \frac{1}{2}b_2^*(n)$. Summing up these inequalities will then imply $V^S_{diff}(\theta_i = \frac{1}{2}b_2^*(n); k^*) < 0$. In the following, we make the proof by three steps.

To save the notation in the following analysis, let's simply use $\widetilde{F}_{m_1,m_2}(t)$ to denote $\widetilde{F}_{m_1,m_2}(t;k^*)$, and use $\widehat{F}_{m_1,m_2}(t)$ to denote $\widehat{F}_{m_1,m_2}(t;k^*)$.

(1) In the first step, we show that $V(\theta_i, b_2^*(j+1), j+1) > V(\theta_i, b_2^*(j+1), j)$ for all $j$ when $\theta_i = \frac{1}{2}b_2^*(n)$.

(1.1) Consider the case with $j \leq q-1$. Because $\theta_i = \frac{1}{2}b_2^*(n) \geq \frac{1}{2}b_2^*(j+1)$, we have $V(\theta_i, b_2^*(j+1), j+1) = [1 - \widetilde{F}_{n-2-j,n-q+1}(\frac{1}{2}b_2^*(j+1))][2\theta_i b_2^*(j+1) - b_2^*(j+1)^2] > 0$ and $V(\theta_i, b_2^*(j+1), j) = [1 - \widetilde{F}_{n-1-j,n-q+1}(\frac{1}{2}b_2^*(j+1))][2\theta_i b_2^*(j+1) - b_2^*(j+1)^2] > 0$. Lemma 3 implies that $\frac{1-\widetilde{F}_{n-1-j,n-q+1}(\frac{1}{2}b_2^*(j+1))}{\widetilde{f}_{n-1-j,n-q+1}(\frac{1}{2}b_2^*(j+1))} < \frac{1-\widetilde{F}_{n-2-j,n-q+1}(\frac{1}{2}b_2^*(j+1))}{\widetilde{f}_{n-2-j,n-q+1}(\frac{1}{2}b_2^*(j+1))}$.

For a cumulative distribution function $\widehat{G}(\theta)$ with probability density function $\widehat{g}(\theta)$, we always have $\widehat{G}(\theta) = 1 - \exp(-\int_a^\theta \frac{g(s)}{1-G(s)}ds)$. So we get $\widetilde{F}_{n-1-j,n-q+1}(\frac{1}{2}b_2^*(j+1)) > \widetilde{F}_{n-2-j,n-q+1}(\frac{1}{2}b_2^*(j+1))$, or $[1 - \widetilde{F}_{n-2-j,n-q+1}(\frac{1}{2}b_2^*(j+1))] > [1 - \widetilde{F}_{n-1-j,n-q+1}(\frac{1}{2}b_2^*(j+1))]$. This implies that $V(\theta_i, b_2^*(j+1), j+1) > V(\theta_i, b_2^*(j+1), j)$.

(1.2) Similarly, consider the case when $q \leq j \leq n-1$. Because $\theta_i = \frac{1}{2}b_2^*(n) \geq \frac{1}{2}b_2^*(j+1)$, we have $V(\theta_i, b_2^*(j+1), j) = [1 - \widehat{F}_{j,j-q+2}(\frac{1}{2}b_2^*(j+1))][2\theta_i b_2^*(j+1) - b_2^*(j+1)^2] \geq 0$, and $V(\theta_i, b_2^*(j+1), j+1) = [1 - \widehat{F}_{j+1,j-q+3}(\frac{1}{2}b_2^*(j+1))][2\theta_i b_2^*(j+1) - b_2^*(j+1)^2] \geq 0$. Based on Lemma 3, we have $\frac{1-\widehat{F}_{j,j-q+2}(\frac{1}{2}b_2^*(j+1))}{\widehat{f}_{j,j-q+2}(\frac{1}{2}b_2^*(j+1))} < \frac{[1-\widehat{F}_{j+1,j-q+3}(\frac{1}{2}b_2^*(j+1))]}{\widehat{f}_{j+1,j-q+3}(\frac{1}{2}b_2^*(j+1))}$. Thus, $[1 - \widehat{F}_{j,j-q+2}(\frac{1}{2}b_2^*(j+1))] < [1 - \widehat{F}_{j+1,j-q+3}(\frac{1}{2}b_2^*(j+1))]$. Therefore, we have $V(\theta_i, b_2^*(j+1), j+1) > V(\theta_i, b_2^*(j+1), j)$.

(2) In the second step, we show that $\binom{n-1}{j+1}[1-F(k^*)]^{j+1}F(k^*)^{n-2-j} > \binom{n-1}{j}[1-F(k^*)]^j F(k^*)^{n-1-j}$ for all $j \leq n-2$. Notice that

$$\frac{(n-1)!}{(n-j-2)!(j+1)!}(1-F(k^*)) > \frac{(n-1)!}{(n-j-1)!j!}F(k^*)$$
$$\Leftrightarrow (n-1-j)(1-F(k^*)) > (j+1)F(k^*)$$
$$\Leftrightarrow F(k^*) < \frac{n-j-1}{n}$$
$$\Leftarrow F(k^*) < \frac{1}{n} \text{ (because } j+1 \leq n-1)$$
$$\Leftarrow F(\tfrac{1}{2}\theta_A) < \frac{1}{n}.$$

As long as the setter is sufficiently moderate, i.e., $\theta_A$ is sufficiently small, we must have

$F(\frac{1}{2}\theta_A) < \frac{1}{n}$ and therefore $\binom{n-1}{j+1}[1-F(k^*)]^{j+1}F(k^*)^{n-2-j} > \binom{n-1}{j}[1-F(k^*)]^{j}F(k^*)^{n-1-j}$ for all $j \leq n-2$.

(3) Provided that $\theta_A$ is sufficiently small, Inequality S7 holds for all $j \leq n-2$ when $\theta_i = \frac{1}{2}b_S^*(n)$. By summing up those inequalities, we get $\sum_{j=0}^{n-2}\binom{n-1}{j}F(k^*)^{n-j-1}[1-F(k^*)]^j V(\theta_i, b_2^*(j+1), j) < \sum_{j=0}^{n-2}\binom{n-1}{j+1}F(k^*)^{n-j-2}[1-F(k^*)]^{j+1}V(\theta_i, b_2^*(j+1), j+1)$. The right-hand side is equal to $\sum_{j=1}^{n-1}\binom{n-1}{j}F(k^*)^{n-j-1}[1-F(k^*)]^j V(\theta_i, b_2^*(j), j)$. So we get $\sum_{j=0}^{n-2}\binom{n-1}{j}F(k^*)^{n-j-1}[1-F(k^*)]^j V(\theta_i, b_2^*(j+1), j) < \sum_{j=1}^{n-1}\binom{n-1}{j}F(k^*)^{n-j-1}[1-F(k^*)]^j V(\theta_i, b_2^*(j), j)$ when $\theta_i = \frac{1}{2}b_S^*(n)$. Notice that $V_{diff}^S(\theta_i; k_S^*) = \sum_{j=0}^{n-1}\binom{n-1}{j}F(k^*)^{n-j-1}[1-F(k^*)]^j[V(\theta_i, b_2^*(j+1), j) - V(\theta_i, b_2^*(j+1), j)] = \sum_{j=0}^{n-2}\binom{n-1}{j}F(k^*)^{n-j-1}[1-F(k^*)]^j V(\theta_i, b_2^*(j+1), j) - \sum_{j=1}^{n-1}\binom{n-1}{j}F(k^*)^{n-j-1}[1-F(k^*)]^j V(\theta_i, b_2^*(j), j) + [1-F(k^*)]^{n-1}V(\theta_i, b_2^*(n), n-1) - F(k^*)^{n-1}V(\theta_i, b_2^*(0), 0)$.

Thus, when $\theta_i = \frac{1}{2}b_2^*(n)$, we have $V_{diff}^S(\theta_i; k_S^*) < [1-F(k^*)]^{n-1}V(\theta_i, b_2^*(n), n-1) - F(k^*)^{n-1}V(\theta_i, b_2^*(0), 0)$. Given that $\frac{1}{2}b_S^*(n) > \frac{1}{2}b_2^*(0)$, we have $V(\theta_i, b_2^*(0), 0) \geq 0$. Because $V(\theta_i = \frac{1}{2}b_S^*(n), b_2^*(n), n-1) = 0$, we have $V_{diff}^S(\theta_i; k_S^*) < -F(k^*)^{n-1}V(\theta_i, b_2^*(0), 0) \leq 0$ for $\theta_i = \frac{1}{2}b_2^*(n)$. It is a contradiction. As a result, any equilibrium involves $k^* \geq \frac{1}{2}\theta_A$ provided $\theta_A$ is sufficiently small. ■

**Proof of Proposition 9.**

In the any symmetric monotone equilibrium, there are various indifferent types. It is sufficient to show that the lowest indifferent type $k$ must be weakly greater than $\frac{\theta_A}{2}$.

(1) Under simple majority rule, if $k < \frac{\theta_A}{2}$, we need to check the incentive compatibility for types weakly smaller than but sufficiently close to $k$, i.e., $\theta_i \in (k-\epsilon, k]$ for some small $\epsilon > 0$.

(1.1) We first characterize the expected payoff of a voter with an ideal $\theta_i$ when the other voter is such that $\theta_j \geq k$. If this voter pretends to be the type above $k$, he induces the setter to propose some $\overline{b} \geq 2k$ so that he gets $\eta(\overline{b})(2\theta_i\overline{b} - \overline{b}^2)$, where $\eta(\overline{b})$ is the probability the other voter accepts $\overline{b}$. If this voter pretends to be the type below $k$, he induces the setter to propose some $\underline{b}$ so that he gets $\eta(\underline{b})(2\theta_i\underline{b}-\underline{b}^2)$. By the same method in Lemma 1 and Lemma 2, we can verify that $\bar{b} > \underline{b}$.

(1.2) We now characterize the expected payoff of the voter with an ideal $\theta_i$ when the other voter is such that $\theta_j < k$. If this voter pretends to be the type above $k$, the setter believes that the pivotal voter has an ideal point greater than $k$ so that the induced proposal is weakly greater than $2k$. He knows that the other voter always rejects this proposal so that he is pivotal. If the proposal is strictly greater than $2k$, he rejects it and gets 0. If the proposal equals $2k$, he gets 0 anyway. As a result, in this case, he always gets a payoff of 0. If $\theta_i$ pretends to be the type below $k$, he induces some proposal strictly less than $2k$. So he always accepts it and gets $2\theta_i b' - b'^2$, where $b'$ is the induced proposal and $b' < 2k$.

(1.3) According to (1.1) and (1.2), the payoff gain from indicating a type above $k$ (rather than indicating a type below $k$) for the voter with $\theta_i$ as his ideal is $(1-F(k))[\eta(\bar{b})(2\theta_i\bar{b} - \bar{b}^2) - \eta(\underline{b})(2\theta_i\underline{b} - \underline{b}^2)] - F(k)[2\theta_i b' - b'^2]$. By the definition of the equilibrium, we must have

$$[1-F(k)][\eta(\bar{b})\bar{b} - \eta(\underline{b})\underline{b}] - F(k)b' \geq 0, \tag{S8}$$

and

$$(1-F(k))[\eta(\bar{b})(2k\bar{b} - \bar{b}^2) - \eta(\underline{b})(2k\underline{b} - \underline{b}^2)] = F(k)[2kb' - b'^2]. \tag{S9}$$

Inequality S8 implies $\eta(\bar{b})\bar{b} - \eta(\underline{b})\underline{b} > 0$. Equation S9 implies $\eta(\bar{b})(2k\bar{b} - \bar{b}^2) \geq \eta(\underline{b})(2k\underline{b} - \underline{b}^2)$, which is equivalent to $k \geq \frac{\eta(\bar{b})\bar{b}^2 - \eta(\underline{b})\underline{b}^2}{2[\eta(\bar{b})\bar{b} - \eta(\underline{b})\underline{b}]}$. This inequality implies that $\frac{\eta(\bar{b})\bar{b}^2 - \eta(\underline{b})\underline{b}^2}{2[\eta(\bar{b})\bar{b} - \eta(\underline{b})\underline{b}]} > \frac{\bar{b}}{2}$, so that $k > \frac{\bar{b}}{2}$. It contradicts the fact that $\bar{b} \geq 2k$. As a result, we have $k \geq \frac{\theta_A}{2}$.

(2) Under unanimity rule, suppose $k < \frac{\theta_A}{2}$. We prove the result by contradiction. In the following, let's examine the indifference condition for type $\theta_i = k$.

(2.1) We characterize the expected payoff of the voter with an ideal $\theta_i$ when the other voter's ideal $\theta_j \geq k$. If $\theta_i$ pretends to be the type above $k$, he induces the setter to propose some policy greater than $2k$ so that he always gets 0. If $\theta_i$ pretends to be the type below $k$, he induces the setter to propose some $b^* < 2k$ so that he gets $2\theta_i b^* - b^{*2}$.

(2.2) We characterize the expected payoff of the voter with an ideal $\theta_i$ when the other voter's ideal $\theta_j < k$. If $\theta_i$ pretends to be the type above $k$, he induces the setter to propose $b^*$ so that he gets $[1 - \frac{F(\frac{1}{2}b^*)}{F(k)}](2\theta_i b^* - b^{*2})$. If $\theta_i$ pretends to be the type above $k$, he induces the setter to propose some $\underline{b}' < 2k$ so that he gets $[1 - \frac{F(\frac{1}{2}\underline{b}')}{F(k)}](2\theta_i \underline{b}' - (\underline{b}')^2)$.

(2.3) According to (2.1) and (2.2), the indifference condition can be written as $-(1 - F(k))(2kb^* - b^{*2}) + [F(k) - F(\frac{1}{2}b^*)](2kb^* - b^{*2}) = [F(k) - F(\frac{1}{2}\underline{b}')](2k\underline{b}' - (\underline{b}')^2)$. The equation implies that $2F(k) - 1 - F(\frac{1}{2}b^*) > 0$. Therefore, we must have $F(k) > \frac{1}{2}$. We have $F(k) < F(\frac{\theta_A}{2})$ given that $k < \frac{\theta_A}{2}$.

When $\theta_A \leq 2F^{-1}(\frac{1}{2})$, we get $F(k) < \frac{1}{2}$, which is a contradiction.

The inequality $2F(k) - 1 - F(\frac{1}{2}b^*) > 0$ also implies that $F(k) > \frac{1}{2}F(\frac{1}{2}b^*) + \frac{1}{2}F(\overline{\theta})$. When $F(\cdot)$ is convex, we have $F(\frac{\theta_A}{2}) > F(k) > F(\frac{1}{2}\frac{1}{2}b^* + \frac{1}{2}\overline{\theta})$, which implies $\theta_A > \overline{\theta}$. This is a contradiction. As a result, when $F(\cdot)$ is convex or $\theta_A \leq 2F^{-1}(\frac{1}{2})$, we always have $k \geq \frac{\theta_A}{2}$. ■