On-Line Appendices for "Quitting in Protest: Presidential Policymaking and Civil Service Response"

Charles M. Cameron and John M. de Figueiredo

Princeton University   Duke University and NBER

March 29, 2020 (revised)

A Baseline: No Presidential Centralization

In this version of the game the President has no central capacity for policy formulation, so his proposal effort must be $e^i = 0$.

The following lemma assures that if the Bureaucrat's proposal attempt fails, the President will not choose a new policy at random. Let $F(x)$, with density $f(x)$, be a distribution over the policy space. Assume $F(x)$ is uniform on $[-z, z]$. If the President selects a policy other than q or B's successful proposal, he draws a policy from this distribution. The key element in the lemma is: policies outside the interval $[q, t^i]$ be sufficiently probable.

Lemma 1. *(No Guessing Lemma) If the Bureaucrat's proposal attempt is unsuccessful, the President chooses no policy (retains the status quo), so $x^F = q$.*

Proof. For the President, choosing the status quo $q = 0$ brings policy utility of zero. Suppose an unknowledgeable president selects a policy at random, that is, implements a random draw from $F(x)$, which is uniform on $[-z, z]$, $z > 0$. For an $R-$president the expected utility of a random policy (using the Matthews normalized policy function) is:

$$\int_{-z}^{z} \psi^R(x; r) f(x) dx = \begin{cases} \int_{-z}^{r} \frac{x}{2z} dx + \int_{r}^{z} \frac{2r-x}{2z} dx = -\frac{(r-z)^2}{2z} \text{ if } z > r \\ \int_{-z}^{z} \frac{x}{2z} dx = 0 \text{ if } z \leq r \end{cases}$$

So expected utility must be less than or equal to zero and the random draw cannot be profitable. Similarly for L

$$\int_{-z}^{z} \psi^L(x; \ell) f(x) dx = \begin{cases} \int_{-z}^{\ell} \frac{x-2\ell}{2z} dx + \int_{\ell}^{z} \frac{-x}{2z} dx = -\frac{(\ell+z)^2}{2z} \text{ if } -z < \ell \\ \int_{\ell}^{z} \frac{-x}{2z} dx = 0 \text{ if } \ell < -z \end{cases}$$

which also must be weakly negative for all $-z < \ell$. Hence the President chooses the status quo rather than a random policy.□

No Bureaucrat ever quits absent centralized policymaking. To see this, note that no

Bureaucrat will generate a proposal worse for itself than the status quo, which has a utility value of 0 (the proposals a successful Bureaucrat will proffer are detailed in the next Proposition). In light of the No Guessing Lemma, the President will never select a policy at random, i.e., if the Bureaucrat fails to generate a policy. Hence, the value to the Bureaucrat of x^F cannot be lower than 0. The value of quitting is 0. Hence, staying must (weakly) dominate exiting in both rounds of play. Moreover, because no period 1 Bureaucrat exits, the two rounds of play simply involve repetition of the same situation.

The following is a subgame perfect set of policy proposals and final policies; this Proposition is almost identical to the central result in Romer and Rosenthal 1978 but accounts for costly proposal development.

Proposition 2. *(Agency Proposal and Presidential Policy Choice). The President sets final policy*

$$x^F = \begin{cases} x^B \text{ if } B \text{ was successful and and } x^B \in [\min\{q, t^i\}, \max\{q, t^i\}] \\ q \text{ otherwise} \end{cases}$$

The bureaucrat's policy proposal in each period is

(A1) $$x^B = \begin{cases} b \text{ if } B \text{ was successful, } i = R \text{ and } b < t^R \\ t^R \text{ if } B \text{ was successful, } i = R \text{ and } b \geq t^R \\ q \text{ if } B \text{ was not successful or } i = L \end{cases}$$

Proof. Part 1, Presidential policy choice. If the Bureaucrat's proposal effort was successful, the President is placed in the position of the receiver in a Romer-Rosenthal take-it-or-leave-it (TILI) game: he accepts any policy that is as good or better than the status quo (that is, where $\psi^i(x^B) \geq 0$) and rejects all others. (Recall: if the Bureaucrat succeeds, the utility value of her recommendation is verifiable for the President). The set $[\min\{q, t^i\}, \max\{q, t^i\}]$ indicates all the policies that are weakly better for the President than the status quo. From the No Guessing Lemma, if B's attempt was unsuccessful the President will not choose a final policy at random so the status quo q again continues. Part 2, the Bureaucrat's policy proposal. Given the President's policy choice strategy in Part 1, a successful zealous Bureaucrat is able to make a proposal as if she were the proposer in a Romer-Rosenthal TILI game. That is, a successful zealous Bureaucrat offers the proposal that maximizes $\psi^B(x)$ among those proposals that the President will accept, namely the set of policies $[\min\{q, t^i\}, \max\{q, t^i\}]$. The indicated proposals follow immediately (see Romer and Rosenthal 1978). If the Bureaucrat's proposal effort was unsuccessful, the President will not accept any proposal from the Bureaucrat other than q so the Bureaucrat may as well offer q (no successful proposal is equivalent to recommending q). Note that if P is an L-President, there is no proposal other than q that the Bureaucrat could recommend that L would accept so B might as well recommend q. If B is a slacker she does not care about policies and may as well follow the indicated strategy; of course, if the slacker undertook no proposal effort, she can only offer q (which is equivalent to no proposal).□

The proposal strategy is effectively unique in the following sense. Unsuccessful Bureaucrats (which will include all slackers in equilibrium) could propose a random policy knowing that their offer will be rejected by the President who will understand that it is a random policy; but a random policy is thus equivalent to recommending q.

In light of the above, the expected utility of the Bureaucrat after the election but prior to undertaking effort is:

$$Eu^B(e^B; i, b, \theta) = \begin{cases} \theta e^B b - \left(e_2^B\right)^2 & \text{if } i = R \text{ and } b \text{ in Regions 1-3} \\ \theta e^B 2r - \left(e_2^B\right)^2 & \text{if } i = R \text{ and } b \text{ in Region 4} \\ -\left(e_2^B\right)^2 & \text{if } i = L \end{cases}$$

where $\theta = 1$ denotes a zealot and $\theta = 0$ denotes a slacker. Using these expected utilities one may straightforwardly derive optimal effort for B:

(A2)
$$e^B(b,r,\theta)^* = \begin{cases} \frac{b}{2} \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ in Regions 1, 2, or 3} \\ r \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ is in Region 4} \\ 0 \text{ if } \theta = 0 \text{ or } i = L \end{cases}$$

Note that these values require $0 \leq b < 2$ and $0 \leq r \leq 1$ in order to restrict e^{B*} in $[0, 1]$.

Given the Bureaucrat's optimal effort strategy and proposal strategy and the President's acceptance strategy, expected final policy is simply $0(1 - e^B) + x^B e^B$, to wit:

(A3)
$$Ex^F = \begin{cases} \frac{b^2}{2} \text{ if } i = R,\ \theta = 1, \text{and } b \text{ in Regions 1-3} \\ 2r^2 \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ is in Region 4} \\ 0 \text{ if } \theta = 0 \text{ or } i = L \end{cases}$$

Finally, the per-period expected utility of the President at the beginning of a round of play is $\psi^i(q)(1 - e^B) + \psi^i(x^B)e^{B*}$, to wit:

(A4)
$$Eu^i(e^B, x^B; i, b, \theta) = \begin{cases} \frac{b^2}{2} \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ in Regions 1 or 2} \\ r - \frac{b^2}{2} \text{ if } i = R,\ \theta = 1, \text{and } b \text{ in Region 3} \\ 0 \text{ otherwise} \end{cases}$$

B The Game in Period 2

The following describes the Bureaucrat's stay/go strategy in Period 2.

Proposition 3. *(Stay/go Period 2) The Bureaucrat's stay/go strategy in Period 2 is:*

$$g_2(x_2^F; b) = \begin{cases} 1 \text{ if } \psi^B(x_2^F; b) < 0 \\ 0 \text{ otherwise} \end{cases}$$

Proof. B's effort costs are sunk when deciding to stay or go, hence only the policy impact of x^F matters. If B quits ($g = 1$) her utility is 0. If she stays ($g = 0$) she receives $\psi^B(x_2^F)$. The comparison of these two utility values determines the strategy. □

The following lemma extends the No Guessing Lemma to centralized policymaking. It assures that, if proposal development fails, the president will not choose a new policy at random. Let $F(x)$, with density $f(x)$, be a uniform distribution of policies over the policy space. For simplicity assume $F(x)$ is uniform.

Lemma 4. *(No Guessing Lemma [centralized policymaking]) If both the Bureaucrat's and the President's innovation attempt fails, the President chooses no policy (so $x^F = q$).*

Proof. For the President, choosing the status quo $q = 0$ brings policy utility of zero. If the proposal effort of both actors has failed, opting for a policy change results in the implementation of a random draw from $F(x)$, which is uniform on $[-z, z]$. For R the expected utility of a random policy (using the Matthews normalized policy function) is: $\int_{-z}^{z} \psi^R(x; r) f(x) dx = \int_{-z}^{r} \frac{x}{2z} dx + \int_{r}^{z} \frac{2r-x}{2z} dx = -\frac{(r-z)^2}{2z}$ which must be negative for all $z > r$. For L $\int_{-z}^{z} \psi^L(x; \ell) f(x) dx = \int_{-z}^{\ell} \frac{x-2\ell}{2z} dx + \int_{\ell}^{z} \frac{-x}{2z} dx = -\frac{(\ell+z)^2}{2z}$ which also must be negative for all $-z < \ell$. Hence the President chooses the status quo rather than a random policy. □

The following is a subgame perfect set of policy choices and proposals in Period 2.

Proposition 5. *(Bureaucrat Proposal and Presidential Policy Choice in Period 2). The President sets final policy*

$$x_2^F = \begin{cases} p \text{ if } i\text{' succeeded} \\ x^B \text{ if } i\text{' failed, } B \text{ succeeded, and } x^B \in [\min\{q, t^i\}, \max\{q, t^i\}] \\ q \text{ otherwise} \end{cases}$$

The bureaucrat's policy proposal is

$$x_2^B = \begin{cases} b \text{ if } B \text{ succeeded, } i = R \text{ and } b < t^R \\ t^R \text{ if } B \text{ succeeded, } i = R \text{ and } b \geq t^R \\ q \text{ if } B \text{ failed or } i = L \end{cases}$$

Proof. Part 1, Presidential choice. If the President's proposal effort succeeded, he can act as the Dictator in a Dictator game. Accordingly, he orders the implementation of his own ideal policy, r if $i = R$ and ℓ if $i = L$. If the President's proposal effort failed but the Bureaucrat's succeeded, the President is in the position of the receiver in a Romer-Rosenthal take-it-or-leave-it (TILI) game: he accepts any policy proposal that is as good or better than the status quo ($\psi^i(x^B) \geq 0$. The set $[\min\{q, t^i\}, \max\{q, t^i\}]$ indicates all those policies. If the President failed and Bureaucrat succeeded but $\psi^i(x^B) < 0$ the President rejects the proposal so that q prevails. From the No Guessing Lemma, if neither proposal attempt succeeded the President will not choose a final policy at random so the status quo q again continues. Part 2, the Bureaucrat's policy proposal. The Bureaucrat makes her proposal before knowing whether the President's proposal effort succeeded. And, if the Bureaucrat succeeds, the utility value of her proposal is verifiable for the President. Given these facts and the President's final policy choice strategy in the prior Proposition, an successful zealous Bureaucrat has a weakly dominant strategy to make a proposal as if she were the proposer in a Romer-Rosenthal TILI game (the strategy is strictly dominant when $e^R < 1$). That is, a successful zealous Bureaucrat offers the proposal that maximizes $\psi^B(x)$ among those proposals that the President will accept if his proposal effort failed but B's succeeded, namely the set of policies $[\min\{q, t^i\}, \max\{q, t^i\}]$. The indicated offers follow (see Romer and Rosenthal 1978). If the Bureaucrat's proposal effort failed the President will not accept any proposal from the Bureaucrat other than q so the Bureaucrat may as well propose q (no proposal is equivalent to recommending q). Note that if P is an L-President, there is no proposal other than q that the Bureaucrat could recommend that L would accept so B might as well propose

q. If B is a slacker she does not care about policies and may as well follow the indicated strategy; if the slacker undertakes no effort, she can only propose q (which is equivalent to no recommendation). □

The proposal strategy is effectively unique in the following sense. Unsuccessful bureaucrats (which will include all slackers in equilibrium) could propose a random policy knowing that their proposal will be rejected by the President who will understand that it is a random proposal; but a random proposal is thus equivalent to proposing q.

The following Corollary indicates the path of play with respect to exits.

Corollary 6. *(Equilibrium exits in Period 2) If P's proposal effort fails, B does not exit. If L's proposal effort succeeds, B exits. If R's proposal effort succeeds, Region 1 B's exit but Region 2-4 B's do not.*

Proof. Follows from the Stay/go Proposition and the Policy Choice Proposition. That is, if L's effort succeeds, $x^F = \ell$ and $\psi^B(\ell) = \ell < 0$ while exiting brings B a utility of 0; if R's effort succeeds $x^F = r$ and $\psi^B(r) = 2b - r < 0$ for Regime 1 B, but $\psi^B(r) = 2b - r > 0$ for Regime 3 B and $\psi^B(r) = r > 0$ for Regimes 3 and 4 B. If President's effort fails then either B's search fails and $x^F = q$ and $\psi^B(q) = 0$ for all B (so don't exit), or B's effort succeeds and $x^F = b$ with $\psi^B(b) = b > 0$ for all B. □

Reaction Functions in Effort in Period 2.—In light of the above results the expected utility of a zealous B after the election but prior to undertaking proposal effort is:

(B1)

$$Eu_2^B(e_2^B; e_2^i, p, b, \theta = 1) = \begin{cases} (1 - e_2^R)\left(e_2^B b\right) - \left(e_2^B\right)^2 \text{ if } i = R \text{ and } b \text{ in Region 1} \\ e_2^R(2b - r) + (1 - e_2^R)\left(e_2^B b\right) - \left(e_2^B\right)^2 \text{ if } i = R \text{ and } b \text{ in Region 2} \\ e_2^R r + (1 - e_2^R) e_2^B b - \left(e_2^B\right)^2 \text{ if } i = R \text{ and } b \text{ in Region 3} \\ e_2^R r + (1 - e_2^R) e^B 2r - \left(e_2^B\right)^2 \text{ if } i = R \text{ and } b \text{ in Region 4} \\ -\left(e_2^B\right)^2 \text{ if } i = L \end{cases}$$

The similar expected utility of a slacker B is:

$$Eu_2^B(e_2^B;\theta=0) = -\left(e_2^B\right)^2$$

The expected utility of R is:

$$Eu_2^R(e_2^R;e_2^B,r,b) = \begin{cases} e_2^R r + (1-e_2^R)e_2^B b - \left(e_2^R\right)^2 \text{ if } b \text{ in Regions 1 or 2} \\ e_2^R r + (1-e_2^R)e_2^B(2r-b) - \left(e_2^R\right)^2 \text{ if } b \text{ in Region 3} \\ e_2^R r - \left(e_2^R\right)^2 \text{ if } b \text{ in Region 4} \end{cases}$$

The expected utility of L is

$$Eu_2^L(e_2^L;\ell) = e_2^L|\ell| - \left(e_2^L\right)^2$$

Using these expected utilities one may straightforwardly derive reaction functions in effort for the actors. These are:

(B2) $$e_2^B(e_2^i;b,r) = \begin{cases} \frac{(1-e_2^R)b}{2} \text{ if } i=R,\ \theta=1, \text{ and Regions 1, 2, or 3 } B \\ (1-e_2^R)r \text{ if } i=R,\ \theta=1, \text{ and Region 4 } B \\ 0 \text{ otherwise} \end{cases}$$

(B3) $$e_2^R(e_2^B;r,b) = \begin{cases} \frac{r-e_2^B b}{2} \text{ if } \theta=1 \text{ and } b \text{ in Regions 1 and 2} \\ \frac{r-e_2^B(2r-b)}{2} \text{ if } \theta=1 \text{ and } b \text{ in Region 3 } B \\ \frac{r}{2} \text{ if } \theta=0 \text{ or } \theta=1 \text{ and } b \text{ in Region 4 } B \end{cases}$$

(B4) $$e_2^L(\ell) = \frac{|\ell|}{2}$$

The reaction functions $e_2^B(e_2^i)$ and $e_2^i(e_2^B)$ (Equations B2, B3, and B4) may be solved simultaneously to derive the equilibrium proposal efforts:

$$\text{(B5)} \qquad \left(e_2^{i*}, e_2^{B*}\right) = \begin{cases} \left(\frac{2r-b^2}{4-b^2}, \frac{b(2-r)}{4-b^2}\right) \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ in Regions 1 and 2} \\ \left(\frac{2r+b^2-2br}{4+b^2-2br}, \frac{b(2-r)}{4+b^2-2br}\right) \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ in Region 3} \\ \left(\frac{r}{2}, \frac{r(2-r)}{2}\right) \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ in Region 4} \\ \left(\frac{p}{2}, 0\right) \text{ otherwise } (i = L \text{ and/or } \theta = 0) \end{cases}$$

Note that these values require $0 \leq b < 2$ and $0 \leq r \leq 1$. The former is the duopoly stability condition (see e.g.,Dixit 1986). The latter is necessary to restrict e_2^{B*} in $[0, 1]$.

C The Game in Period 1

We first consider B's expected utility conditional on the outcome of the Period 2 election and the expenditure of efforts $\left(e_2^{i*}, e_2^{B*}\right)$. Call this expected utility $Eu_2^{B*}|P$. First, if L is elected $Eu_2^{B*}|L = 0$ since B will quit if L's proposal effort succeeds and only q can prevail if L's proposal effort fails (since B will not have expended effort, reflecting the fact that L will not accept any proposal B prefers to q). Second, if B is a slacker then her expected utility is also 0 since she receives no utility for policy and will not exert proposal effort. Third, if R is elected and B is a zealot, then B's expected utility varies by region as shown in Equation B1. Substituting Period 2 equilibrium efforts (Equation B5) in the appropriate portions of Equation B1 yields a zealous B's expected utility conditional on the election of R. Via algebra $Eu_2^{B*}|R$ are: Region 1: $\frac{b^2(2-r)^2}{(4-b^2)^2}$; Region 2: $\frac{2b^5-br^4-4b^3(2+r)+b^2(4+3r^2)+16br-8r^2}{(4-b^2)^2}$; Region 3: $\frac{b^4r-4b^3r^2+b^2\left(4+3r^2+4r^3\right)-4br^2-4br^2(2+r)+8r^2}{(4+b^2-2br)^2}$; and Region 4: $\frac{r^2\left(6-4r+r^2\right)}{4}$.

We now consider the continuation value to B at the end of Period 1. The continuation value of the game to B at the end of Period 1 depends on her stay/go decision in Period 1 (g_1). If she goes ($g_1 = 1$), then her continuation value $V(1) = 0$. Similarly, if she is a slacker she stays ($g_1 = 1$) but her continuation value $V(0) = 0$. However if she is a zealot who stays in Period 1, her continuation value $V(0) = \pi\left(Eu_2^{B*}|R\right) + (1-\pi)Eu_2^{B*}|L$ (recall that π is

B's Location	B's Continuation Value from Staying (V(0))
Region 1	$\frac{b^2(2-r)^2}{(4-b^2)^2}\pi$
Region 2	$\frac{2b^5-br^4-4b^3(2+r)+b^2(4+3r^2)+16br-8r^2}{(4-b^2)^2}\pi$
Region 3	$\frac{b^4r-4b^3r^2+b^2\left(4+3r^2+4r^3\right)-4br^2-4br^2(2+r)+8r^2}{(4+b^2-2br)^2}\pi$
Region 4	$\frac{r^2\left(6-4r+r^2\right)}{4}\pi$

Table 1: Continuation Values to Bureaucrat From Remaining in Government Employment

the probability an L-president is elected). As noted immediately above, $Eu_2^{B*}|L = 0$ hence $V(0) = \pi\left(Eu_2^{B*}|R\right)$ .The continuation values $V(0)$ of the game for zealous B are shown in Table 1.

Remark 7. *In Table 1,* $V(0) \geq 0$.

Proof. $V(0)$ reflects optimal stay/go and work decisions by a zealous Bureaucrat in Period 2. B can always assure herself zero net utility in Period 2 by not working and quitting for any election realization or equilibrium policy effort by R or L. Hence, any equilibrium choices in Period 2 by B must afford B expected net utility of at least 0 prior to Period 2. □

Proposition 8. *(Stay/go strategy in Period 1). The Bureaucrat's stay/go strategy in Period 1 is:*

$$g_1(x_1^F;b) = \begin{cases} 1 \text{ if } \psi^B(x_1^F;b) + V(0) < 0 \\ 0 \text{ otherwise} \end{cases}$$

Proof. B's Period 1 effort costs are sunk at the stay/go decision, hence only the policy impact of x_1^F and the continuation value matters. If B quits ($g_1 = 1$) her policy utility is 0 and her continuation value $V(1) = 0$. If she stays ($g_1 = 0$) she receives $\psi^B(x_1^F) + V(0)$. The comparison of these two utility values determines the strategy. □

Proposition 9. *(Policy Choice and Recommendation in Period 1) The President's final policy selection strategy and the Bureaucrat's policy proposal strategy in Period 1 are the same as in Period 2.*

Proof. Given a future-is-now president, the President's final policy choice in Period 1 must be the same as in Period 2. In addition, no deviation from B's Period 2 proposal strategy could be profitable for B in Period 1, as B recommends the most profitable policy that an unsuccessful P will accept. Hence the earlier Proposition also describes Presidential final policy choice and Bureaucrat's policy proposal strategies in Period 1. □

Given the two previous propositions and the fact that $V(0) > 0$, the following corollary is straightforward.

Corollary 10. *(Actual Stay/Go in Period 1). In Period 1*

$$g_1 = \begin{cases} 1 \text{ if } \begin{cases} L \text{ is president, } L \text{ succeeded and } |\ell| > V(0) \\ R \text{ is president, } R \text{ succeeded, } b \text{ lies in Region 1 and } 2b - r + V(0) < 0 \end{cases} \\ 0 \text{ if } \begin{cases} L \text{ is president and } \begin{cases} L\text{' failed} \\ L \text{ succeeded but } |\ell| \leq V(0) \end{cases} \\ R \text{ is president and } \begin{cases} R \text{ failed} \\ R \text{ succeeded but } b \text{ lies in Regions 2-4} \\ R \text{ succeeded, } b \text{ lies in Region 1 but } 2b - r + V(0) \geq 0 \end{cases} \end{cases} \end{cases}$$

Proof. Recall that B will exit if and only if $\psi^B(x_1^F) + V(0) < 0$. Recall as well that $V(0) > 0$. If L is president and L's proposal effort failed, $x_1^F = q$ so $\psi^B(x_1^F) = 0$ and thus $\psi^B(x_1^F) + V(0) > 0$. If L's proposal effort succeeded, $x_1^F = \ell$ and $\psi^B(x_1^F) = \ell < 0$ for all B. So B stays or goes as $|\ell| \leq> V(0)$. If R is president and R's proposal effort failed, either $x_1^F = q$ (when B's effort failed), $x_1^F = b$ for a successful Region 1-3 B, or $x_1^F = 2r$ for a successful Region 4 B. In all these cases $\psi^B(x_2^F) \geq 0$ so $\psi^B(x_2^F) + V(0) > 0$ so B stays. If R is president and R's proposal effort succeeded, $x_1^F = r$. By construction $\psi^B(r) \geq 0$ for all B in Regions 2-4 so for such B $\psi^B(x_1^F) + V(0) > 0$ and they stay. And, by construction, $\psi^B(r) = 2b - r < 0$ for all B in Region 1. Region 1 B then stays or goes as $2b - r + V(0) \geq< 0$. These exhaust all the cases. □

The corollary identifies two situations in which Period 1 zealous B might quit: 1) when L is president, L's proposal effort succeeded, and $\ell + V(0) < 0$, and 2) when R is president, R's effort succeeded, b lies in Region 1, and $2b - r + V(0) < 0$.The comparative static results in the text on when quitting is "more likely" consider the effects of changes in exogenous variables on the magnitudes of $\ell + V(0)$ and $2b - r + V(0)$, respectively.

Remark 11. *(WTO) For b in all four regions, $\ell + V(0)$ is increasing in π and decreasing in $|\ell|$.*

Proof. From inspection of Table 1, $V(0)$ is increasing in π in all four regions. $V(0)$ is not a function of ℓ and $\ell < 0$ so $\ell + V(0)$ is decreasing for b in all four regions.

Remark 12. *(IMD) A small group of bureaucrats in Region 1 do not exit when R's proposal effort succeeds.*

Proof. In Region 1 the stay condition after a successful R imposes $x_1^F = r$ is $2b - r + V(0) \geq 0$.Recall that in Region 1 $V(0) = \frac{b^2(2-r)^2}{(4-b^2)^2}\pi$. Note that $\lim_{b \to \frac{r}{2}} \left(2b - r + \frac{b^2(2-r)^2}{(4-b^2)^2}\pi\right) = \frac{4(2-r^2)r^2}{(16-r^2)^2}\pi > 0$, so as b approaches the upper bound of Region 1 $(r/2)$ there is a group of bureaucrats who do not exit. A closed form solution for b such that $b - r + V(0) = 0$ is intractable but numerical solutions indicate that for plausible parameter values the range of staying bureaucrats is very small. □

Remark 13. *(IMD) For b in Region 1, $2b - r + V(0)$ is increasing in π, increasing in b, and decreasing in r.*

Proof. Recall that $0 \leq b < 2$, $0 \leq r \leq 1$, and $V(0) = \frac{b^2(2-r)^2}{(4-b^2)^2}\pi$.Hence $\frac{\partial}{\partial \pi}(2b - r + \frac{b^2(2-r)^2}{(4-b^2)^2}\pi) = \frac{b^2(2-r)^2}{(4-b^2)^2} \geq 0$; $\frac{\partial}{\partial b}(2b - r + \frac{b^2(2-r)^2}{(4-b^2)^2}\pi) = 2 + \frac{2b(4+b^2)(2-r)^2}{(4-b^2)^3}\pi > 0$; and $\frac{\partial}{\partial r}(2b - r + \frac{b^2(2-r)^2}{(4-b^2)^2}\pi) = -1 - \frac{2b^2(2-r)}{(4-b^2)^2}\pi < 0$.□

We now consider expected utilities in Period 1 in order to derive reaction functions. Assume an R president. Recall we assume a zealous B in Period 1. From above, B in

Regions 2-4 will not quit in Period 1. Hence, prior to undertaking effort, the expected utility for B in Regions 2-4 is:

$$Eu_1^B(e_1^B; e_1^R, r, b) = \begin{cases} e_1^R(2b-r) + \left(1-e_1^R\right) e_1^B b + V(0) - \left(e_1^B\right)^2 \text{ if } b \text{ is in Region 2} \\ e_1^R r + \left(1-e_1^R\right) e_1^B b + V(0) - \left(e_1^B\right)^2 \text{ if } b \text{ is in Region 3} \\ e_1^R r + \left(1-e_1^R\right) e_1^B 2r + V(0) - \left(e_1^B\right)^2 \text{ if } b \text{ is in Region 4} \end{cases}$$

For B in Region 1 there are two possibilities: 1) If R is successful, B exits; 2) If R is successful, B stays. Hence:

$$Eu_1^B(e_1^B; e_1^R, r, b) = \begin{cases} e_1^R(2b-r) + \left(1-e_1^R\right) e_1^B b + V(0) - \left(e_1^R\right)^2 \text{ if Region 1 } B \text{ stays when } R \text{ succeeds} \\ \left(1-e_1^R\right)\left(e_1^B b + V(0)\right) - \left(e_1^R\right)^2 \text{ if Region 1 } B \text{ quits when } R \text{ succeeds} \end{cases}$$

For R:

$$Eu_1^R(e_1^R; e_1^B, r, b) = \begin{cases} e_1^R r + \left(1-e_1^R\right) e_1^B b - \left(e_1^R\right)^2 \text{ if } b \text{ is in Region 1 or 2} \\ e_1^R r + \left(1-e_1^R\right) e_1^B (2r-b) - \left(e_1^R\right)^2 \text{ if } b \text{ is in Region 3} \\ e_1^R r - \left(e_1^R\right)^2 \text{ if } b \text{ is in Region 4} \end{cases}$$

Assume an $L-$President. For B there are two possibilities: 1) If L is successful, B exits; 2) If L is successful, B stays. Hence:

$$Eu_1^B(e_1^B; e_1^L, \ell, b) = \begin{cases} e_1^L \ell + V(0) - \left(e_1^B\right)^2 \text{ if } b \text{ stays when } L \text{ succeeds} \\ \left(1-e_1^L\right) V(0) - \left(e_1^B\right)^2 \text{ if } b \text{ quits when } L \text{ succeeds} \end{cases}$$

For L:

$$Eu_1^L(e_1^L; e_1^B, \ell, b) = e_1^L \left|\ell\right| - \left(e_1^L\right)^2$$

Using these expected utilities one may straightforwardly derive reaction functions in effort for the actors. These are:

$$\text{(C1)} \qquad e_1^B(e_1^i; b, r) = \begin{cases} \frac{(1-e_1^R)b}{2} \text{ if } i = R \text{ and } b \text{ is in Regions 1, 2, or 3} \\ (1-e_1^R)r \text{ if } i = R \text{ and } b \text{ is in Region 4} \\ 0 \text{ if } i = L \end{cases}$$

$$\text{(C2)} \qquad e_1^R(e_1^B; r, b) = \begin{cases} \frac{r-e_1^B b}{2} \text{ if } b \text{ is in Regions 1 and 2} \\ \frac{r-e_1^B(2r-b)}{2} \text{ if } b \text{ is in Region 3} \\ \frac{r}{2} \text{ if } b \text{ is in Region 4} \end{cases}$$

$$\text{(C3)} \qquad e_1^L(\ell) = \frac{|\ell|}{2}$$

The reaction functions $e^i(e^B)$ and $e^B(e^i)$ (Equations C1 , C2 and C3) may be solved simultaneously to derive the equilibrium policymaking efforts:

$$\text{(C4)} \qquad \left(e^{i*}, e^{B*}\right) = \begin{cases} \left(\frac{2r-b^2}{4-b^2}, \frac{b(2-r)}{4-b^2}\right) \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ is in Regions 1 and 2} \\ \left(\frac{2r+b^2-2br}{4+b^2-2br}, \frac{b(2-r)}{4+b^2-2br}\right) \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ is in Region 3} \\ \left(\frac{r}{2}, \frac{r(2-r)}{2}\right) \text{ if } i = R,\ \theta = 1, \text{ and } b \text{ is in Region 4} \\ \left(\frac{p}{2}, 0\right) \text{ otherwise } (i = L \text{ and/or } \theta = 0) \end{cases}$$

As in Period 2, these values require $0 \leq b < 2$ and $0 \leq r \leq 1$.

Expected policy in a period is simply $e^{i*}p + (1-e^{i*})e^{B*}x^B$ and may readily be calculated using the above results.

Finally, consider the expected utility of an $R-$President at the beginning of Period 1. As noted in the text this is $Eu_1^R(e_1^{R*}, e_1^{B*}; b) = \psi^R(r)(e^{R*}) + \psi^i(x^{B*})\left(1-e^{R*}\right)e^{B*} - \left(e^{R*}\right)^2$. Using the definition of policy utility and optimal policy recommendations and choices, this is:

$$Eu_1^R(e_1^{R*}, e_1^{B*}; b) = \begin{cases} re^{R*} - \left(e^{R*}\right)^2 \text{ if } b < 0 \ (L - \text{side Bureaucrat}) \\ re^{R*} + b\left(1 - e^{R*}\right)e^{B*} - \left(e^{R*}\right)^2 \text{ if } 0 < b < r \text{ (Regions 1 and 2)} \\ re^{R*} + (2r - b)\left(1 - e^{R*}\right)e^{B*} - \left(e^{R*}\right)^2 \text{ if } r \leq b \leq 2r \text{ (Region 3)} \\ re^{R*} - \left(e^{R*}\right)^2 \text{ if } b > 2r \text{ (Region 4)} \end{cases}$$

and using Equation C4

(C5)

$$Eu_1^R(e_1^{R*}, e_1^{B*}; b) = \begin{cases} \frac{r^2}{4} \text{ if } b < 0 \ (L - \text{side Bureaucrat}) \\ \frac{8b^2(1-r) - b^4(1-r) + 4r^2}{\left(b^2-4\right)^2} \text{ if } 0 < b < r \text{ (Regions 1 and 2)} \\ \frac{-b^4(1-r) + 16b(1-r)r + 4b^3(1-r)r + 4r^2 + 4b^2\left(-2 = 2r - r^2 + r^3\right)}{\left(4 + b^2 - 4br\right)^2} \text{ if } r \leq b \leq 2r \text{ (Region 3)} \\ \frac{r^2}{4} \text{ if } b > 2r \text{ (Region 4)} \end{cases}$$

Equation C5 is displayed by the heavy line in the left-hand panel of Figure 6.