Online Appendix
Overreacting and Posturing: How Accountability and Ideology Shape Executive Policies

1 Baseline Model

Proposition 2. *There exist perfect Bayesian equilibria of the model that survive D1 characterized as follows:*

1. *Voting Behavior:*
   (a) *If $x_1 \leqslant \underline{x}$ or $x_1 \geqslant \overline{x}$, then the voter reelects the incumbent.*
   (b) *If $x_1 \in (\underline{x}, \overline{x})$, then the voter kicks out the incumbent.*
2. *Informed Incumbent:*
   (a) *If $\omega_1 \in [0, \omega^*)$, then I* ***overreacts to the right*** *and chooses $x_1 = \overline{x}$.*
   (b) *If $\omega_1 \in [-\omega^*, 0)$, then I* ***overreacts to the left*** *and chooses $x_1 = \underline{x}$.*
   (c) *If $\omega_1 \leqslant -\omega^*$ or $\omega_1 \geqslant \omega^*$, then I chooses the* ***first-best*** *policy $x_1 = \omega_1$*
3. *Uninformed Incumbent:*
   (a) *With any probability $\overline{\pi} \in [0, \min\{1, \overline{\Pi}\}]$, I* ***postures to the right*** *and chooses $x_1 = \overline{x}$.*
   (b) *With any probability $\underline{\pi} \in [0, \min\{1 - \overline{\pi}, \underline{\Pi}\}]$, I* ***postures to the left*** *and chooses $x_1 = \underline{x}$.*
   (c) *With probability $1 - \underline{\pi} - \overline{\pi}$, I chooses the* ***first-best*** *policy $x = 0$.*

Off the path of play assume the voter believes the incumbent is uninformed with probability 1.

Proposition 3. *The equilibria characterized by Proposition 2 survive D1.*

Proposition 4. *Increasing office benefit increases $\overline{x}$, $\frac{\partial \overline{x}}{\partial \beta} > 0$, and decreases $\underline{x}$, $\frac{\partial \underline{x}}{\partial \beta} < 0$. Furthermore, if $\beta \to \infty$, then $\underline{x} \to -\infty$ and $\overline{x} \to \infty$.*

Proofs of Propositions 2, 3 and 4. To start, recall that $\overline{x}$ and $\underline{x}$ solve

$$-\sigma^2 + \beta - (1 - q_C)\sigma^2 = -x^2 - 2\sigma^2 + 2\beta \tag{1}$$

Solving equation (1) yields explicit solutions $\overline{x} = \sqrt{\beta - q_C\sigma^2}$ and $\underline{x} = -\sqrt{\beta - q_C\sigma^2}$. From this, Proposition 2 immediately follows.

I split the proof into two parts. I first prove that if equilibrium strategies are characterized as in Proposition 1, then D1 forces the voter to believe that deviations off the path come from the low quality type. Next, I prove that the characterizations given in Proposition 1 yield perfect Bayesian equilibria.

Part 1. *Assume there is a perfect Bayesian equilibrium characterized by the strategies in Proposition 1. D*1 *requires the voter to believe the incumbent is low quality with probability* 1 *following any off path policy choice.*

An arbitrary incumbent type is given by $\tau \in \mathbb{R} \cup \{\phi\}$. Define $R_\sigma(\tau, x)$ as the set of reelection probabilities for which the τ type strictly prefers choosing policy x and getting reelected with probability ρ, over getting their equilibrium payoff in a PBE σ. Similarly, define $R^0_\sigma(\tau, x)$ as those reelection probabilities that make τ indifferent. If $\hat{x}$ is off the path of play, then $D1$ requires putting probability 0 on a type τ, if there exists a type τ' such that $R_\sigma(\tau, \hat{x}) \cup R^0_\sigma(\tau, \hat{x}) \subseteq R_\sigma(\tau', \hat{x})$. This implies that the voter should not believe the deviation came from type τ if there is another type who is willing to deviate to $\hat{x}$ and win reelection with a lower probability.

I first show that if the incumbent is informed, then the $\omega_1 = 0$ type has the strongest incentive to choose an off path action, thus, the voter should not believe that the deviation came from any type $\omega_1 \in \mathbb{R}/\{0\}$. Second, I eliminate that the deviation should come from the $\omega_1 = 0$ type by showing that the uninformed type is willing to deviate for a larger set of reelection probabilities than the $\omega_1 = 0$ type.

Clearly the voter should never believe that a deviation came from a type such that $\omega_1 \geqslant \overline{x}$ or $\omega_1 \leqslant \underline{x}$, as these types obtain their highest possible payoff and would not deviate to $\hat{x} \in (\underline{x}, \overline{x})$ for any reelection probability. Next, consider a type $\omega_1 \in [0, \overline{x}]$. In this case, she chooses $x_1 = \overline{x}$ in equilibrium, and her equilibrium payoff is

$$-(\overline{x} - \omega_1)^2 + 2\beta.$$

If she deviates to $x_1 = \hat{x}$, then her payoff, given reelection probability $\rho_{\hat{x}}$, is

$$-(\hat{x} - \omega_1)^2 + \beta + \rho_{\hat{x}}\beta - (1 - \rho_{\hat{x}})\Big((1 - q_C)\sigma^2\Big).$$

Comparing these payoffs and rearranging, we get that for any $\hat{x}$, the ω_1 type has an incentive to deviate from choosing $\overline{x}$ if

$$\rho_{\hat{x}} > \frac{(\hat{x}-\omega_1)^2 - (\overline{x}-\omega_1)^2 + \beta + (1-q_C)\sigma^2}{\beta + (1-q_C)\sigma^2}. \tag{2}$$

Differentiating the RHS of (2) with respect to the type ω_1 yields

$$\frac{\partial RHS(2)}{\partial \omega_1} = \frac{2(\overline{x}-\hat{x})}{\beta + (1-q_C)\sigma^2} > 0.$$

Therefore, the RHS of (2) is minimized at $\omega_1 = 0$, and D1 requires putting probability 0 on the deviation coming from any type $\omega_1 \in (0, \overline{x})$, as the set of reelection probabilities for which these types strictly prefer or are indifferent to deviating to $\hat{x}$ is a subset of the reelection probabilities for which the $\omega_1 = 0$ type will deviate.

Now consider the incentive for a type $\omega_1 \in (\underline{x}, 0)$ to deviate to an off path action $\hat{x} \in (0, \overline{x})$. In this case, she is willing to deviate if

$$\rho_{\hat{x}} > \frac{(\hat{x}-\omega_1)^2 - (\underline{x}-\omega_1)^2 + \beta + (1-q_C)\sigma^2}{\beta + (1-q_C)\sigma^2}. \tag{3}$$

Now, differentiating (3) with respect to ω_1 yields

$$\frac{\partial(3)}{\partial \omega_1} = \frac{2(\underline{x}-\hat{x})}{\beta + (1-q_C)\sigma^2} < 0,$$

where the inequality follows from $\underline{x} < 0$. Thus, increasing $\omega_1 \in (\underline{x}, 0)$ decreases the RHS of (3). Letting $\omega_1 \to 0$, this converges to the $\omega_1 = 0$ type's payoff. Thus, by D1 we must place probability 0 on a deviation to $\hat{x} \in (0, \overline{x})$ coming from any type $\omega_1 \in (\underline{x}, 0)$. Analogous arguments show that for a deviation to $\hat{x} \in (\underline{x}, 0)$ D1 places probability 0 on it coming from any informed type $\omega_1 \in (\underline{x}, \overline{x})/\{0\}$.

Finally, consider the uninformed type's incentive to choose $\hat{x} \in (0, \overline{x})$. Her equilibrium payoff is equivalent to choosing $\overline{x}$ and being reelected, i.e.,

$$-\overline{x}^2 - 2\sigma^2 + 2\beta.$$

Choosing $x_1 = \hat{x}$ and getting reelected with probability $\rho_{\hat{x}}$ gives an expected payoff

$$-\hat{x}^2 - \sigma^2 + \beta + \rho_{\hat{x}}(\beta - \sigma^2) - (1-\rho_{\hat{x}})\Big((1-q_C)\sigma^2\Big).$$

Comparing these payoffs and rearranging, we get that the uninformed type will deviate to $\hat{x}$

for any $\rho_{\hat{x}}$ such that

$$\rho_{\hat{x}} > \frac{\hat{x}^2 - \overline{x}^2 + \beta - q_C\sigma^2}{\beta - q_C)\sigma^2 + (L-R)^2}. \tag{4}$$

We need to show that the lower bound on the reelection probabilities for which the uninformed type deviates is lower than the lower bound for which the $\omega_1 = 0$ type deviates. Setting $\omega_1 = 0$ in equation (2) and comparing to (4) yields

$$\frac{\hat{x}^2 - \overline{x}^2 + \beta + (1-q_C)\sigma^2}{\beta + (1-q_C)\sigma^2} > \frac{\hat{x}^2 - \overline{x}^2 + \beta - q_C\sigma^2}{\beta - q_C\sigma^2} \tag{5}$$

$$\Leftrightarrow \left(\beta - q_C\sigma^2\right)\left(y + \beta + (1-q_C)\sigma^2\right) >$$
$$\left(\beta + (1-q_C)\sigma^2\right)\left(y + \beta - q_C\sigma^2\right) \tag{6}$$

$$\Leftrightarrow -\sigma^2(\hat{x}^2 - \overline{x}^2) > (\beta + (1-q_C)\sigma^2)(\beta - q_C\sigma^2)$$
$$-(\beta - q_C\sigma^2)(\beta + (1-q_C)\sigma^2) \tag{7}$$

$$\Leftrightarrow \sigma^2(\overline{x}^2 - \hat{x}^2) > 0. \tag{8}$$

Equation (5) is the condition that must hold. Equations (6) - (8) follow from manipulating the previous equation. Finally, (8) holds by $\overline{x} > \hat{x}$. Analogous arguments show that a similar relationship holds for an off path action $\hat{x} \in (\underline{x}, 0)$. Therefore, if the voter puts probability 0 on an off path policy choice $\hat{x} \in (\underline{x}, \overline{x})$ coming from an informed type, these equilibria survive D1.

Part 2. *The strategies and beliefs given in Propostion 2 form a perfect Bayesian equilibrium.*

The expected first-period policy utility to an uninformed incumbent for policy x is $-x^2 - \sigma^2$. Hence, choosing $x_1 \in (\underline{x}, \overline{x})/\{0\}$ and getting kicked out is strictly worse than choosing $x_1 = 0$. Likewise, choosing $x_1 > \overline{x}$ or $x_1 < \underline{x}$ and getting reelected is strictly worse than choosing $\overline{x}$ or $\underline{x}$ and getting reelected. By construction, $\underline{x}$ and $\overline{x}$ make an uninformed incumbent indifferent between choosing $x_1 = \underline{x}$, $x_1 = \overline{x}$, and $x_1 = 0$. Therefore, an uninformed incumbent will not deviate from mixing over $\underline{x}$, $\overline{x}$, and 0.

Now consider an informed incumbent. If $\omega_1 \leqslant \underline{x}$ or $\omega_1 \geqslant \overline{x}$, then choosing $x_1 = \omega_1$ and getting reelected with certainty is clearly optimal. Next, assume $\omega_1 \in [0, \overline{x})$. The best policy payoff for choosing an x_1 that leads to reelection is $x_1 = \overline{x}$. The incumbent's greatest policy utility from a policy that leads to removal from office is $x_1 = \omega_1$. The expected utility for choosing $x_1 = \omega_1$ and being removed from office is $\beta - (1-q_C)\sigma^2$, while the expected utility for choosing $x_1 = \overline{x}$ and being reelected is $-(\overline{x} - \omega_1)^2 + 2\beta$. As the expected utility for choosing $\overline{x}$ is strictly decreasing in ω_1, if the $\omega_1 = 0$ type prefers $\overline{x}$ over choosing $x_1 = 0$ then every type $\omega_1 \in (0, \overline{x})$ will also prefer to choose $x_1 = \overline{x}$. This yields

$$-\overline{x}^2 + 2\beta > \beta - (1 - q_C)\sigma^2 \tag{9}$$

$$(1 - q_C)\sigma^2 + \beta > \overline{x}^2 \tag{10}$$

$$(1 - q_C)\sigma^2 + \beta > \beta - q_C\sigma^2 \tag{11}$$

$$\sigma^2 > 0. \tag{12}$$

Where (9) follows from rearranging the inequality (10). Inequality (11) follows from substituting in $\overline{x}$, and (12) from reducing (11). An analogous argument yields the optimality of choosing $\underline{x}$ if $\omega_1 \in (\underline{x}, 0)$.

Finally, given the strategy of the incumbent, the voter must be willing to reelect the incumbent following $x_1 \geqslant \overline{x}$ or $x_1 \leqslant \underline{x}$, and be willing to elect the challenger following $x_1 \in (\underline{x}, \overline{x})$. Policies $x_1 \in (\underline{x}, \overline{x})/\{0\}$ are off the path of play, thus, assigning any belief $\tilde{q}_I(x_1) \leqslant q_C$ it is optimal for the voter to elect the challenger. By Part 1 of the proof, anticipating the demands of $D1$, moving forward assume $\tilde{q}_I(x_1) = 0$ for $x_1 \in (\underline{x}, \overline{x})/\{0\}$. As only the uninformed type ever chooses $x = 0$ the voter updates that $\tilde{q}_I(0) = 0 < q_C$ and kicks out the incumbent as required. On the other hand, policies such that $x_1 > \overline{x}$ or $x_1 < \underline{x}$ are only ever chosen by the informed type. In this case, $\tilde{q}_I(x_1) = 1 > q_C$ and the voter reelects as required. If $x_1 = \overline{x}$, then for it to be optimal for the voter to reelect the incumbent requires

$$\tilde{q}_I(\overline{x}) \geqslant q_C \tag{13}$$

$$\frac{q_I\Big(F(\overline{x}) - F(0)\Big)}{q_I\Big(F(\overline{x}) - F(0)\Big) + (1 - q_I)\overline{\Pi}} \geqslant q_C \tag{14}$$

$$q_I\Big(F(\overline{x}) - F(0)\Big) \geqslant q_C\Big(q_I\Big(F(\overline{x}) - F(0)\Big) + (1 - q)\overline{\Pi}\Big) \tag{15}$$

$$\frac{q_I(1 - q_C)}{(1 - q_I)q_C}\Big(F(\omega^*) - F(0)\Big) \geqslant \overline{\Pi}. \tag{16}$$

Where (13) is the optimality requirement. (14) follows by using Bayes rule to find $\tilde{q}_I$. (15) rearranges (14), and (16) rearranges (15). Finally, (16) holds from the definition of $\overline{\Pi}$. Similarly, it is optimal for the voter to reelect the incumbent following $x_1 = \underline{x}$.

Proposition A1. *Assume $\underline{\Pi} + \overline{\Pi} \leqslant 1$. If an equilibrium survives $D1$, then it must be characterized by the strategies in Proposition 1.*

Proof of Proposition A1. To start, assume there exists an equilibrium such that $\hat{x} > \overline{x}$ is off the path of play. By definition of $\overline{x}$, if $\hat{x} > \overline{x}$, then for any reelection probability the uninformed type strictly prefers to choose $x = 0$ and get reelected with any probability ρ. In any equilibrium, the uninformed type's payoff must be at least as good as choosing $x_1 = 0$ and getting kicked out of office. Thus, $R_\sigma(\phi, \hat{x}) = \varnothing$, and so the voter must put probability 0 on the deviation coming from the uninformed type, e.g., at a minimum the $\omega_1 = \hat{x}$ type would certainly deviate for $\rho = 1$. Hence, in a perfect Bayesian equilibrium that survives D1, the voter must reelect the incumbent following an off path action $\hat{x}$. This implies, however, that there cannot be an equilibrium that survives D1 and has an off path action $\hat{x} > \overline{x}$, because the $\omega_1 = \hat{x}$ type would always strictly prefer to deviate from her equilibrium action in order to choose $x_1 = \hat{x}$, get reelected, and get her highest policy payoff. Similarly, there are no equilibria that survive D1 with off path actions $\hat{x} < \underline{x}$. Consequently, in every equilibrium that survives D1 it must be for $\omega_1 > \overline{x}$ and $\omega_1 < \underline{x}$ an informed incumbent chooses $x_1 = \omega_1$ and the voter reelects with probability 1.

Additionally, in an equilibrium, the voter must also reelect with probability 1 following $x_1 = \overline{x}$ and $x_1 = \underline{x}$. If not, the $\omega_1 = \overline{x}$ type would have a best response problem.

Let Σ^* be the set of policies in $[\underline{x}, \overline{x}]$ which the uninformed type chooses with positive probability in equilibrium. It must be that, for $x' \in \Sigma^*$, if $\rho(x') = 0$, then $x' = 0$. If $\rho(x') = 0$ and $x' \neq 0$ then the uninformed type can choose $x = 0$, obtain a higher expected policy utility and be reelected with weakly greater probability, contradicting that $x' \in \Sigma^*$.

Assume $\overline{\Pi} + \underline{\Pi} \leqslant 1$. Note, this always holds for $q_C \geqslant q_I$. I show that the uninformed type cannot only be choosing policies that lead to a positive probability of reelection. Assume otherwise. That is, assume $\rho(x) > 0$ for all $x \in \Sigma^*$. Thus, after observing $x \in \Sigma^*$, by Bayes' rule the voter believes that the incumbent is high quality with probability:

$$Pr(H|x \in \Sigma^*) = \frac{Pr(x \in \Sigma^*|H)Pr(H)}{Pr(x \in \Sigma^*)}$$
$$= \frac{Pr(x \in \Sigma^*|H)Pr(H)}{Pr(x \in \Sigma^*|H)Pr(H) + Pr(x \in \Sigma^*|L)Pr(L)}.$$

For the voter to reelect the incumbent with positive probability he must believe the incumbent is at least as likely to be high quality as the challenger. Note that $Pr(x \in \Sigma^*|L) = 1$, since the low quality type is only choosing policies in Σ^* and these all lead to a positive probability of reelection. Thus, the following sequence of expressions must hold:

$$\begin{aligned}
&\frac{Pr(x \in \Sigma^*|H)Pr(H)}{Pr(x \in \Sigma^*|H)Pr(H) + Pr(x \in \Sigma^*|L)Pr(L)} \geqslant q_C \\
&\Leftrightarrow \frac{Pr(x \in \Sigma^*|H)q_I}{q_I Pr(x \in \Sigma^*|H) + (1-q_I)} \geqslant q_C \\
&\Leftrightarrow Pr(x \in \Sigma^*|H) \geqslant \frac{(1-q_I)q_C}{q_I(1-q_C)}.
\end{aligned}$$

However, from our earlier argument, we know that for all ω such that $\omega > |\omega^*|$ the informed type chooses $x_1 = \omega$. Thus, $Pr(x \in \Sigma^*|H) \leqslant F(\omega^*) - F(-\omega^*) \leqslant \frac{(1-q_I)q_C}{q_I(1-q_C)}$, where the second inequality holds by $\overline{\Pi} + \underline{\Pi} \leqslant 1$. Hence, this contradicts that in equilibrium the low quality type is reelected with positive probability following every policy choice.

Thus, for some $x \in \Sigma^*$ it must be that $\rho(x) = 0$. However, from our earlier argument, this can only hold for $x = 0$. Therefore, in equilibrium, the uninformed type must be choosing $x_1 = 0$ with positive probability and losing reelection.

As the uninformed type must be indifferent over policies in Σ^* to be willing to mix, we have that for any $x' \in \Sigma^*$, such that $x' \neq 0$, it must be that

$$\begin{aligned}
&-(x')^2 + \rho(x')(\beta - \sigma^2) - (1-\rho(x'))\Big((1-q_C)\sigma^2\Big) = -(1-q_C)\sigma^2 \\
&\Rightarrow \rho(x') = \frac{(x')^2}{\beta - q_C\sigma^2}. \qquad (17)
\end{aligned}$$

Now I show that for $x' \in \Sigma^*$, it must be that $x' \in \{\underline{x}, 0, \overline{x}\}$. Assume not. Let $x' > 0$. Consider $\omega' \in [0, \overline{x}]$. The expected utility to the ω' type for choosing x' is

$$-(x'-\omega')^2 + \rho(x')\beta - (1-\rho(x'))\Big((1-q_C)\sigma^2\Big),$$

while her expected utility for choosing $x = \overline{x}$ is

$$-(\overline{x}-\omega')^2 + \beta.$$

I now show that ω' strictly prefers choosing $\overline{x}$. This holds if

$$-(\overline{x}-\omega')^2+\beta > -(x'-\omega')^2+\rho(x')\beta-(1-\rho(x'))\Big((1-q_C)\sigma^2\Big) \tag{18}$$

$$\Leftrightarrow (x'-\omega')^2-(\overline{x}-\omega')^2+\beta+(1-q_C)\sigma^2 > \rho(x')(\beta+(1-q_C)\sigma^2) \tag{19}$$

$$\Leftrightarrow 2\omega'(\overline{x}-x')-\overline{x}^2+(x')^2+\beta+(1-q_C)\sigma^2+ >$$

$$\frac{(x')^2}{\beta-q_C\sigma^2}(\beta+(1-q_C)\sigma^2) \tag{20}$$

$$\Leftrightarrow 1-\frac{\overline{x}^2-(x')^2}{\beta+(1-q_C)\sigma^2} > \frac{(x')^2}{\beta-q_C\sigma^2} \tag{21}$$

$$\Leftrightarrow \overline{x}^2 > (x')^2 \tag{22}$$

Where (18) is the incentive condition that must hold. (19) follows from rearranging the first line. Inequality (20) is derived by further rearranging and substituting in for $\rho(x')$. (21) follows from noting that for $\omega' \geqslant 0$ the LHS side of the inequality is minimized as $\omega' = 0$. Thus, this is a sufficient condition for the original inequality to hold. The final line follows from substituting in for $\overline{x}^2$ and then expanding and cancelling terms. Finally, note that (22) holds by the assumption that $x' < \overline{x}$.

A similar argument shows that any $\omega' < 0$ type prefers to choose $\underline{x}$ rather than x'. Furthermore, an analogous argument shows that no informed type will choose $x' \in (\underline{x}, 0)$ for $x' \in \Sigma^*$. As $\rho(x') > 0$ for these policies, this is a contradiction.

Consequently, in any equilibrium that survives D1 it must be that the uninformed type only chooses policies in $\{\underline{x}, 0, \overline{x}\}$. When $\omega_1 \in (\underline{x}, \overline{x})$, the high quality type also cannot choose policies other than these in equilibrium, otherwise the voter would reelect with probability 1 following this choice, and the uninformed type could profitably deviate to this policy. From Part 1 of the proof, D1 dictates that off the path the voter believes the deviation came from the low type, and, thus, elects the challenger. As such, all equilibria that survive D1 have the characterization in Proposition 1. Our earlier argument showed that these do, in fact, constitute an equilibria, completing the proof.

Proposition 2A. *Assume $\underline{\Pi}+\overline{\Pi} > 1$. If an equilibrium survives D1 then it is characterized by Proposition 2 or by cut-points $\underline{x}'$ and $\overline{x}'$ such that $\underline{x} < \underline{x}' < 0 < \overline{x}' < \overline{x}$. In the second case, if the incumbent is uninformed she chooses $\overline{x}'$ with probability $\overline{\pi}'$ and chooses $x_1 = \underline{x}$ with probability $\underline{\pi}'$, where $\overline{\pi}' + \underline{\pi}' = 1$. When the incumbent is informed, if $\omega \notin (\underline{x}', \overline{x}')$ she chooses $x_1 = \omega$, if $\omega \in (\underline{x}', 0)$ she chooses $x = \underline{x}'$, if $\omega \in [0, \overline{x}')$ she chooses $x_1 = \overline{x}'$.*

Proof of Proposition 2A. The earlier parts demonstrate that the characterization given in Proposition 1 yields a PBE that survives D1. Next, I show that the only other possible

PBE that survive D1 are characterized by $\underline{x}'$ and $\overline{x}'$ as described.

From the previous parts, we know that for all $x \notin (\underline{x}, \overline{x})$ the voter must reelect the incumbent with probability 1. Furthermore, the uninformed type never chooses $x \notin [\underline{x}, \overline{x}]$. Let Σ^* be the set of policies chosen by the uninformed type with positive probability in an equilibrium that survives D1. If the uninformed type chooses $\overline{x}$ or $\underline{x}$ with positive probability then, from the previous arguments, it is immediate that the equilibrium must be characterized by Proposition 2. Thus, assume $\Sigma^* \subset [\underline{x}, \overline{x}]$. Note we must have $\rho(x) > 0$ for all $x \in \Sigma^*$. Otherwise, if there is a policy z with $\rho(z) = 0$ it must be that $z = 0$ and, again, the previous arguments imply the equilibrium is characterized by Proposition 2.

I now show that there is at most two policies in Σ^*. Assume not, so there exist policies $a < b < c$. For any $x, y \in \Sigma^*$ the informed type when the state is ω prefers x over y if and only if

$$\begin{aligned} &-(x-\omega)^2 + \rho(x)\beta - (1-\rho(x))(1-q_C)\sigma^2 > -(y-\omega)^2 + \rho(y)\beta - (1-\rho(y))(1-q_C)\sigma^2 \\ &\Leftrightarrow [\rho(x) - \rho(y)][\beta - q_C\sigma^2 + \sigma^2] > (x-\omega)^2 - (y-\omega)^2. \end{aligned}$$

The uninformed type must be indifferent between all $x, y \in \Sigma^*$, which implies

$$\rho(x) = \frac{x^2 - y^2 + \rho(y)\overline{x}^2}{\overline{x}^2}.$$

Substituting this into the previous inequality and simplifying yields that the ω type prefers x over y if and only if

$$2\omega(x-y) > -(x^2 - y^2)\frac{\sigma^2}{\overline{x}^2}.$$

I now show that no informed type would choose policy b, contradicting that the voter reelects with positive probability following $x_1 = b$. For the ω type to choose policy b requires the following two inequalities to hold:

$$\begin{aligned} 2\omega(b-a) &> -(b^2 - a^2)\frac{\sigma^2}{\overline{x}^2} \\ 2\omega(c-b) &< -(c^2 - b^2)\frac{\sigma^2}{\overline{x}^2}. \end{aligned}$$

If $0 < a < b < c$ or $a < 0 < b < c$ then $b - a > 0$ and $c - b > 0$. Therefore, there exists an ω

such that the above inequalities hold if and only if

$$-(b+a)\frac{\sigma^2}{2\overline{x}^2} < -(c+b)\frac{\sigma^2}{2\overline{x}^2}$$
$$\Leftrightarrow c < a,$$

which never holds, by assumption. Analogous arguments hold for the cases where $a < b < 0 < c$ and $a < b < c < 0$. Thus, there are at most two policies in Σ^*. Let S be the policy in Σ^* with greatest absolute value.

Consider $x' \in (S, \overline{x}]$, I show that the voter must reelect the incumbent with probability 1. If x' is chosen by an informed type then clearly the voter reelects following $x_1 = x'$. Next, assume x' is off the path of play. For a contradiction assume the voter does not reelect with probability 1. In equilibrium, the $\omega = 0$ type must be choosing either S or the policy closest 0 that wins with probability 1.

First, assume that the $\omega = 0$ type chooses $x = S$ in equilibrium. Consider an off path deviation to $x' \in (S, \overline{x})$. The $\omega = 0$ type is willing to deviate if and only if

$$-(x')^2 + \rho(x')\beta - (1-\rho(x'))(1-q_C)\sigma^2 > -(S)^2 + \rho(S)\beta - (1-\rho(S))(1-q_C)\sigma^2$$
$$\Leftrightarrow \rho(x') > \frac{(x')^2 - (S)^2 + \rho(S)(\overline{x}^2 + \sigma^2)}{\overline{x}^2 + \sigma^2}.$$

The uninformed type is willing to deviate if and only if

$$-(x')^2 + \rho(x')(\beta - \sigma^2) - (1-\rho(x'))(1-q_C)\sigma^2 > -(S)^2 + \rho(S)(\beta - \sigma^2) - (1-\rho(S))(1-q_C)\sigma^2$$
$$\Leftrightarrow \rho(x') > \frac{(x')^2 - (S)^2 + \rho(S)\overline{x}^2}{\overline{x}^2}.$$

Note, $\frac{(x')^2-(S)^2+\rho(S)\overline{x}^2}{\overline{x}^2} > \frac{(x')^2-(S)^2+\rho(S)(\overline{x}^2+\sigma^2)}{\overline{x}^2+\sigma^2}$, by $x' > S$. Thus, the $\omega = 0$ type is willing to deviate to x' for a larger set of reelection probabilities than the uniformed type. Consequently, D1 requires putting probability 0 that the deviation to policy $x = x'$ is from the uninformed type. However, this implies that the voter reelects with probability 1. Thus, off the path, the voter must reelect with probability 1 for all $x \in (S, \overline{x})$. For there to not be a best response problem requires $\rho(S) = 1$ as well. Furthermore, this implies the voter must reelect with probability 1 for all $z < -S$ if $S > 0$ (and for all $z > -S$ if $S < 0$). Clearly the $\omega = z$ is willing to choose $x_1 = z$ for $\rho(z) < 1$, while the uninformed type would never strictly prefer to deviate from choosing $x_1 = S$ and winning with probability 1 to any $z \leqslant S$. Furthermore, there cannot be a policy choice in Σ^* such that $-S < z < S$. This would imply that the uninformed type is indifferent between z and S. However, the previous arguments imply that all types $\omega < 0$ would strictly prefer $x_1 = -S$ over choosing $x_1 = z$ and for all $\geqslant 0$ the informed type strictly prefers to choose $x_1 = S$, contradicting that the voter reelects with positive probability following $x_1 = z$.

Next, assume that the $\omega = 0$ type chooses the policy closest to 0 that wins with probability 1. Denote this policy as z. Note, we must have $z > S$, otherwise the uninformed type would deviate to z. If $z > 0$ this implies that all types $\omega \in (0, z]$ also choose $x_1 = z$ in equilibrium. Since the uninformed type must prefer policy S and winning with probability $\rho(S)$ over choosing policy z and winning with probability 1, this also implies that the uninformed type would never deviate to policy $-z$. Therefore, if $-z$ is off the path then, by D1, the voter must believe that a deviation to $-z$ came from an informed type. Or $-z$ is on the path, which again implies that the voter reelects with probability 1. Therefore, all types $\omega < 0$ strictly prefer $x = -S$, contradicting that the voter reelects with positive probability less than 1 following $x = S$.

Thus, if there is a PBE that survives D1 that is not characterized by Proposition 2 it must be characterized by cut-points $\overline{x}'$ and $\underline{x}' = -\overline{x}'$, and the voter must reelect with probability 1 for $x \notin (\underline{x}', \overline{x}')$ and probability 0 for $x \in (\underline{x}', \overline{x}')$. This implies that the uninformed type only chooses $x_1 \in \{\underline{x}', \overline{x}'\}$, because, by construction of $\overline{x}$, the uninformed type prefers this over choosing $x = 0$ and losing. Similarly, all $\omega \in (0, x')$ choose $x_1 = x'$ and all $\omega \in (-x', 0)$ choose $-x'$.

2 Executive Constraints

Proposition 5. *Increasing q_C increases voter welfare. If β is sufficiently high, then increasing q_I decreases voter welfare.*

Proof of Proposition 5. Voter welfare is

$$q_I \Big(\int_{-\omega*}^{0} -(\underline{x} - \omega)^2 f(\omega) d\omega + \int_{0}^{\omega*} -(\overline{x} - \omega)^2 f(\omega) d\omega \Big) + (1 - q_I)(-\sigma^2 - (1 - q_C)\sigma^2)$$

First, increasing q_C decreases $\overline{x}$ and $\omega*$, which decreases the probability and extent of overreacting. Note, it also decreases $\overline{\Pi}$ and $\underline{\Pi}$, which decreases the maximum amount of posturing that can be supported in equilibrium. Thus, increasing q_C increases voter welfare.

The derivative with respect to q_I is

$$\int_{-\omega*}^{0} -(\underline{x} - \omega)^2 f(\omega) d\omega + \int_{0}^{\omega*} -(\overline{x} - \omega)^2 f(\omega) d\omega + (2 - q_C)\sigma^2.$$

This term is negative if $\sqrt{\beta - q_C \sigma^2}$ is sufficiently, thus, it is negative if β is sufficiently large, as required.

Proposition 6. *Assume constraints are strong,* $\Psi < \overline{x}$.

1. *Suppose the incumbent is popular,* $q_I > q_C$.

   - *Informed Incumbent: If* $\omega \geqslant 0$ *then* I *chooses* $x_1 = \Psi$*. If* $\omega < 0$ *then* I *chooses* $x_1 = -\Psi$.
   - *Uninformed Incumbent:* I *chooses* $x = \Psi$ *with probability* $1 - F(0)$ *and* $x = -\Psi$ *with probability* $F(0)$.
   - *The voter always reelects the incumbent on the path of play.*

2. *Suppose the incumbent is unpopular,* $q_I < q_C$.

   - *Informed Incumbent: If* $\omega \geqslant 0$ *then* I *chooses* $x_1 = \Psi$*. If* $\omega < 0$ *then* I *chooses* $x_1 = -\Psi$.
   - *Uninformed Incumbent:* I *chooses* $x = \Psi$ *with probability* $\frac{q_I(1-q_C)}{q_C(1-q_I)}\Big(1 - F(0)\Big)$ *and* $x = -\Psi$ *with probability* $\frac{q_I(1-q_C)}{q_C(1-q_I)}F(0)$.
   - *Following* $x_1 = \Psi$ *or* $x_1 = -\Psi$ *the voter reelects the incumbent with probability* $\rho(\Psi) = \rho(-\Psi) = \frac{\Psi^2}{\beta - \sigma^2 - V_C(\Psi)}$.

Proof of Proposition 6. Assume $q_C \leqslant q_I$. Given the strategy of the incumbent, after seeing $x_1 = C$ the voter's updated belief that the incumbent is high quality is

$$\tilde{q}_I(\Psi) = \frac{q_I(1 - F(0))}{q_I(1 - F(0)) + (1 - q_I)(1 - F(0)} = q_I \geqslant q_C.$$

Thus, the voter reelects the incumbent, as needed. A similar argument holds for $x_1 = -\Psi$.

Consider the uninformed type of the incumbent. Any $x \in (-\Psi, \Psi)$ is off-the-path, in which case assume the voter believes the incumbent is the low quality type and kicks out the incumbent. By definition of $\overline{x}$, the uninformed type strictly prefers $x_1 = \Psi$ and winning reelection over $x = 0$ and losing. Furthermore, the uninformed type is indifferent over Ψ and $-\Psi$, as they are equidistant from 0. Thus, she is willing to mix over the two, as required. Finally, clearly the informed type chooses the closest bound for $\omega < -\Psi$ or $\omega > \Psi$; and by definition of $\underline{x}$ and $\overline{x}$, if $\omega \in (-\Psi, \Psi)$, then she strictly prefers to choose the closest bound and win, over choosing $x \in (-\Psi, \Psi)$ and losing.

Now assume $q_C > q_I$. After observing $x_1 = \Psi$ the voter's updated belief that the incumbent is the informed type is

$$\tilde{q}_I(\Psi) = \frac{q_I(1 - F(0))}{q_I(1 - F(0)) + (1 - q_I)\tilde{\Pi}},$$

where $\tilde{\Pi}$ is the probability that the uninformed type chooses $x = \Psi$. For the voter to be willing to mix after seeing $x_1 = \Psi$ requires $\tilde{q}_I(\Psi) = q_C$. This gives the following condition:

$$\frac{q_I(1-F(0))}{q_I(1-F(0)) + (1-q_I)\tilde{\Pi}} = q_C$$
$$\Leftrightarrow \tilde{\Pi} = \frac{q_I(1-q_C)}{(1-q_I)q_C}\Big(1-F(0)\Big),$$

where the second line follows by rearranging the first equality and the second equality holds by assumption that the uninformed type chooses $x_1 = \Psi$ with probability $\frac{q_I(1-q_C)}{(1-q_I)q_C}\Big(1 - F(0)\Big)$. A similar derivation shows that the voter is indifferent following $x = -\Psi$, given the conjectured strategy for the uninformed type. Let $\underset{\sim}{\Pi}$ be the probability with which the uninformed type chooses $x = -\Psi$.

As the uninformed type is the only type that chooses $x_1 = 0$, under the conjectured strategy profile, we have $\tilde{q}_I(0) = 0$. Therefore, the voter kicks out the incumbent following $x_1 = 0$. Define the expected utility from electing the challenger as

$$V_C(\Psi) = q_C\Big(\int_{-\infty}^{-\Psi} -(-\Psi-\omega)^2 f(\omega)d\omega + \int_{\Psi}^{\infty} -(\Psi-\omega)^2 f(\omega)d\omega\Big) - (1-q_C)\sigma^2.$$

Mixing requires the uninformed type to be indifferent over $x_1 = \Psi$ and $x_1 = 0$. This yields the equality

$$-\sigma^2 + \beta - (1-q_C)V_C(\Psi) = -\sigma^2 + \beta - \Psi^2 + \rho(\Psi)(\beta - \sigma^2) + (1-\rho())V_C(\Psi)$$
$$\Leftrightarrow \rho(\Psi) = \frac{\Psi^2}{\beta - \sigma^2 - V_C(\Psi)}.$$

Where the second equality follows from rearranging the first, and holds by the assumed strategy for the voter. Furthermore, as Ψ and $-\Psi$ are equidistant from 0 and $\rho(-\Psi) = \rho(\Psi)$, the uninformed type is also indifferent between $x = -\Psi$ and $x = \Psi$. Finally, by similar arguments as before, given that the uninformed type is indifferent over choosing $x = \Psi$ and winning reelection, or choosing her ex ante optimal policy and losing, any informed type with $\omega \in (-\Psi, \Psi)$ strictly prefers to choose the closest bound and win with probability $\rho(\Psi)$; and if $|\omega| > \Psi$ the informed type prefers to choose the closest bound over any policy in the interior.

Proposition 7. *There exists $\hat{\beta}$ such that the voter's optimal constraint is $\Psi^* \in (0, \overline{x})$ if and only if $\beta > \hat{\beta}$.*

Proof of Proposition 7. First, assume $q_C \leqslant q_I$. In this case, voter welfare is:

$$\begin{aligned} W(\Psi) =& q_I\Big(\int_{-\infty}^{0} -(-\Psi-\omega)^2 f(\omega)d\omega + \int_{0}^{\infty} -(\Psi-\omega)^2 f(\omega)d\omega + \int_{-\infty}^{-\Psi} -(-\Psi-\omega)^2 f(\omega)d\omega \\ &+ \int_{\Psi}^{\infty} -(\Psi-\omega)^2 f(\omega)d\omega\Big) - (1-q_I)(\Psi^2+2\sigma^2). \end{aligned}$$

The derivative with respect to Ψ is:

$$\begin{aligned} & q_I\Big(\int_{-\infty}^{0} 2(-\Psi-\omega)f(\omega)d\omega - \int_{0}^{\infty} 2(\Psi-\omega)f(\omega)d\omega + (\Psi-(-\Psi))^2 f(\Psi) + \int_{-\infty}^{-\Psi} 2(-\Psi-\omega)f(\omega)d\omega \\ & + (\Psi-\Psi)^2 f(\Psi) + \int_{\Psi}^{\infty} -2(\Psi-\omega)f(\omega)d\omega\Big) - 2(1-q_I)\Psi \qquad (23) \\ & = q_I\Big(-2[1-F(0)]\Psi - 2\int_{-\infty}^{0} \omega f(\omega)d\omega - 2F(0)\Psi - 2\int_{-\infty}^{-\Psi} \omega f(\omega)d\omega \\ & + 2\int_{\Psi}^{\infty} \omega f(\omega)d\omega - 2\Psi F(-\Psi) - (1-F(\Psi))2\Psi\Big) - 2(1-q_I)\Psi \qquad (24) \end{aligned}$$

By our earlier argument about weak constraints it must be that $\Psi^* < \overline{x}$. Next, note

$$\lim_{\Psi\to 0} \frac{\partial W}{\partial \Psi} = 4q_I\Big(\int_{0}^{\infty} \omega f(\omega)d\omega - \int_{-\infty}^{0} \omega f(\omega)d\omega\Big) > 0.$$

Thus, $\Psi^* > 0$.

By Proposition 2, we have that welfare under no constraints is strictly decreasing in β, goes to $-\infty$ as $\beta \to \infty$ and goes to the first best as $\beta \to q_C\sigma^2$. As $W(\Psi^*)$ is not a function of β, there exists $\overline{\beta} > q_C\sigma^2$ such that if $\beta > \overline{\beta}$, then constraint Ψ^* is optimal. Otherwise, if $\beta < \overline{\beta}$ then no constraints is optimal.

Now consider $q_C > q_I$. The expression for voter welfare is more complicated in this case. Specifically,

$$\begin{aligned} W(\Psi) = q_I\Bigg[& \int_{-\infty}^{0} -(-\Psi-\omega)^2 f(\omega)d\omega + \int_{0}^{\infty} -(\Psi-\omega)^2 f(\omega)d\omega \\ & + \rho(\Psi)\Big(\int_{-\infty}^{-\Psi} -(-\Psi-\omega)^2 f(\omega)d\omega + \int_{\Psi}^{\infty} -(\Psi-\omega)^2 f(\omega)d\omega\Big) + (1-\rho(\Psi))V_C(\Psi)\Bigg] \\ & + (1-q_I)\Bigg[\Big(\tilde{\Pi} + \underset{\sim}{\Pi}\Big)\Big(-\Psi^2 - \sigma^2 - \rho(\Psi)\sigma^2 + (1-\rho(\Psi))V_C(\Psi)\Big) \\ & + \Big(1 - \tilde{\Pi} - \underset{\sim}{\Pi}\Big)\Big(-\sigma^2 + V_C(\Psi)\Big)\Bigg] \end{aligned}$$

As $W(\Psi)$ is continuous over a compact set $[0,\overline{x}]$ there exists a maximizer $\Psi^* \in [0,\overline{x}]$.

Since $W(\Psi^*)$ is not a function of β, by Proposition 2, we have that welfare under no constraints is strictly decreasing in β, goes to $-\infty$ as $\beta \to \infty$ and goes to the first best as $\beta \to q_C\sigma^2$. Again, there exists a cut-point in β, such that above this cut-point voter welfare is maximized by any constraint Ψ^*, and if β is below this cut-point then having no constraints is optimal.

I now show that any Ψ^* is strictly greater than 0. To start, differentiate V_C and ρ with respect to Ψ. This yields:

$$\begin{aligned}\frac{\partial V_C}{\partial \Psi} &= 4q_C\Big(\int_{\Psi}^{\infty}\omega f(\omega)d\omega - F(-\Psi)\Psi\Big),\\ \frac{\partial \rho}{\partial \Psi} &= \frac{(\beta-\sigma^2-V_C(\Psi))2\Psi + \frac{\partial V_C}{\partial \Psi}\Psi^2}{(\beta-\sigma^2-V_C(\Psi))^2}.\end{aligned}$$

Differentiating $W(\Psi)$ with respect to Ψ we obtain:

$$\begin{aligned}\frac{\partial W}{\partial \Psi} &= q_I\Bigg[\int_{-\infty}^{0}2(-\Psi-\omega)f(\omega)d\omega - \int_{0}^{\infty}2(\Psi-\omega)f(\omega)d\omega\\ &+ \frac{\partial\rho(\Psi)}{\partial\Psi}\Big(\int_{-\infty}^{-\Psi}-(-\Psi-\omega)^2f(\omega)d\omega + \int_{\Psi}^{\infty}-(\Psi-\omega)^2f(\omega)d\omega\Big) + \rho(\Psi)\Big((\Psi-(-\Psi))^2f(\Psi)\\ &+ \int_{-\infty}^{-\Psi}2(-\Psi-\omega)f(\omega)d\omega + (\Psi-\Psi)^2f(\Psi) + \int_{\Psi}^{\infty}-2(\Psi-\omega)f(\omega)d\omega\Big)\\ &+ (1-\rho(\Psi))\frac{\partial V_C(\Psi)}{\partial\Psi} - \frac{\partial\rho(\Psi)}{\partial\Psi}V_C(\Psi)\Bigg] + (1-q_I)\Bigg[\Big(\underset{\sim}{\Pi}+\tilde{\Pi}\Big)\Big(-2\Psi - \frac{\partial\rho(\Psi)}{\partial\Psi}\sigma^2 - \frac{\partial\rho(\Psi)}{\partial\Psi}V_C(\Psi)\\ &+ (1-\rho(\Psi))\frac{\partial V_C(\Psi)}{\partial\Psi}\Big) + \Big(1-\tilde{\Pi}-\underset{\sim}{\Pi}\Big)\frac{\partial V_C(\Psi)}{\partial\Psi}\Bigg]\\ &= q_I\Bigg[-2\Psi + 4\int_{0}^{\infty}\omega f(\omega)d\omega + \frac{\partial\rho(\Psi)}{\partial\Psi}\Big(\int_{-\infty}^{-\Psi}-(-\Psi-\omega)^2f(\omega)d\omega + \int_{\Psi}^{\infty}-(\Psi-\omega)^2f(\omega)d\omega\Big)\\ &+ 4\rho(\Psi)\Big(\int_{\Psi}^{\infty}\omega f(\omega)d\omega - F(-\Psi)\Psi\Big) + (1-\rho(\Psi))\frac{\partial V_C(\Psi)}{\partial\Psi} - \frac{\partial\rho(\Psi)}{\partial\Psi}V_C(\Psi)\Bigg]\\ &+ (1-q_I)\Bigg[\Big(\underset{\sim}{\Pi}+\tilde{\Pi}\Big)\Big(-2\Psi - \frac{\partial\rho(\Psi)}{\partial\Psi}\Big(\sigma^2+V_C(\Psi)\Big) + (1-\rho(\Psi))\frac{\partial V_C(\Psi)}{\partial\Psi}\Big) + \Big(1-\tilde{\Pi}-\underset{\sim}{\Pi}\Big)\frac{\partial V_C(\Psi)}{\partial\Psi}\Bigg]\end{aligned}$$

Letting $\Psi \to 0$ we have

$$\lim_{\Psi\to 0} \rho(\Psi) = 0,$$
$$\lim_{\Psi\to 0} \frac{\partial \rho(\Psi)}{\partial \Psi} = 0,$$
$$\lim_{\Psi\to 0} V_C(\Psi) = -\sigma^2,$$
$$\lim_{\Psi\to 0} \frac{\partial V_C(\Psi)}{\partial \Psi} = 4q_C \int_{\Psi}^{\infty} \omega f(\omega) d\omega.$$

Thus,

$$\lim_{\Psi\to 0} \frac{\partial W}{\partial \Psi} = q_I \left(4 \int_0^{\infty} \omega f(\omega)\omega + 4q_C \int_0^{\infty} \omega f(\omega) d\omega \right) + (1-q_I)q_C \int_0^{\infty} \omega f(\omega) d\omega > 0.$$

Consequently, it must be that if Ψ^* is optimal then $\Psi^* > 0$.

3 Ideological Model

Proposition 8. *Assume the election is lopsided. If the incumbent is high quality, then $x_1 = R + \omega$. If the incumbent is low quality, then $x_1 = R$. The voter always reelects the incumbent when she is advantaged. By contrast, the voter always elects the challenger when the incumbent is disadvantaged.*

Proposition 9. *Assume the election is competitive.*

1. *Substituting in $\omega_R^*, \overline{x}_R, \underline{x}_R, \overline{\Pi}_R$, and $\underline{\Pi}_R$, equilibrium behavior is characterized analogously to Proposition 2.*
2. *The cutoff ω_R^* is increasing in polarization.*

Proofs of Propositions 8 and 9. For Proposition 8, because the voter always kicks out or always elects the incumbent, the incumbent maximizes her policy payoff by choosing $x_1 = R + \omega_1$ if informed, and $x_1 = R$ if uninformed.

Under the characterization in Proposition 9, if the incumbent is uninformed her expected utility from choosing R is $-\sigma^2 + \beta - (L-R)^2 - (1-q)\sigma^2$. Her expected utility for choosing $\overline{x}$ is $-(\overline{x} - R)^2 - 2\sigma^2 + 2\beta$. Similarly, her expected utility for $\underline{x}$ is $-(\underline{x} - R)^2 - 2\sigma^2 + 2\beta$. From the definitions of $\overline{x}$ and $\underline{x}$, we have that the uninformed type is indifferent between choosing $\overline{x}$, $\underline{x}$, or R. Using analogous arguments as before it is clear that the uninformed type will not deviate from mixing over these policies.

If the incumbent is informed and learns $\omega_1 \notin (-\omega_R^*, \omega_R^*)$, then choosing $x_1 = \omega_1 + R$ yields her highest policy payoff and she gets reelected. Thus, there is not a profitable deviation. If the incumbent is informed and she learns $\omega_1 \in (0, \omega^*)$, then her equilibrium payoff from choosing $\overline{x}_R$ is

$$-(\omega^* - \omega_1)^2 + 2\beta.$$

Her most profitable deviation is to instead choose $x_1 = R + \omega_1$, and be removed from office. This yields

$$\beta - (1-q)\sigma^2 - (L-R)^2.$$

Comparing expected utilities, we have that the incumbent will not deviate from $\overline{x}_R$ if

$$-(\omega^* - \omega_1)^2 + 2\beta \geqslant \beta - (1-q)\sigma^2 - (L-R)^2, \tag{25}$$

$$\beta + (1-q)\sigma^2 + (L-R)^2 \geqslant (\omega^* - \omega_1)^2, \tag{26}$$

$$\beta + (1-q)\sigma^2 + (L-R)^2 \geqslant (\omega^*)^2. \tag{27}$$

Where (25) is the incentive constraint, and (26) follows from manipulating (25). Line (27) follows from noting that, because $\omega_1 \in (0, \omega^*)$, if (27) holds then (26) will hold as well. Finally, note that the last inequality holds by the definition of ω^*. Therefore, the $\omega_1 \in (0, \omega^*)$ type incumbent does not want to deviate from her equilibrium action. Similarly, neither does a type such that $\omega_1 \in (-\omega^*, 0)$.

After observing x_1 and updating his belief, the voter's expected utility for reelecting the incumbent is

$$-R^2 - (1 - \tilde{q}(x_1))\sigma^2.$$

On the other hand, if the voter elects the challenger, then his expected utility is

$$-L^2 - (1-q)\sigma^2.$$

Comparing, we get that the voter reelects the incumbent if

$$-R^2 - (1 - \tilde{q}(x_1))\sigma^2 \geqslant -L^2 - (1-q)\sigma^2 \tag{28}$$

$$\Leftrightarrow \tilde{q}(x_1) \geqslant q + \frac{R^2 - L^2}{\sigma^2}. \tag{29}$$

Because the election is competitive, the RHS of (29) is strictly less than 1 and greater than 0. If (29) holds with equality, then the voter can reelect with any probability, and if the inequality is reversed, then he must elect the challenger.

As only high quality types choose $x_1 \notin (\underline{x}_R, \overline{x}_R)$, the voter's belief following such a policy is $q(x_1) = 1$. Hence, he reelects as required. As only the low quality type ever chooses $x_1 = R$, $q(R) = 0$ and electing the challenger is optimal. If $x_1 \in (\underline{x}_R, \overline{x}_R)$, this is off the path of play. Assuming for x_1 off the path of play we have $q(x_1) = 0$, then the voter will kick out the incumbent. Finally, if $x_1 = \overline{x}_R$ the voter's updated belief that the incumbent is high quality is

$$\tilde{q}(\overline{x}_R) = \frac{q\Big(F(\omega_R^*) - F(0)\Big)}{q\Big(F(\omega_R^*) - F(0)\Big) + (1-q)\overline{\Pi}_R}. \tag{30}$$

Substituting (30) into equation (29), the voter will reelect the incumbent if

$$\frac{q\Big(F(\omega_R^*) - F(0)\Big)}{q\Big(F(\omega_R^*) - F(0)\Big) + (1-q)\overline{\Pi}_R} \geqslant q + \frac{R^2 - L^2}{\sigma^2}, \tag{31}$$

$$\Leftrightarrow \Big(\frac{q}{1-q}\frac{1-q-\frac{R^2-L^2}{\sigma^2}}{q+\frac{R^2-L^2}{\sigma^2}}\Big)\Big(F(\omega_R^*) - F(0)\Big) \geqslant \overline{\Pi}_R. \tag{32}$$

where (32) simply rearranges (31). Inequality (32) is the definition of $\overline{\pi}_R$ and, thus, the voter is willing to reelect following $x_1 = \overline{x}_1$, as well as for $x_1 = \underline{x}_R$.

Part 2 of Proposition 9 follows by differentiating ω_R^* with respect to $R - L$.

Proposition 10. *(Symmetric Polarization)*
Suppose the challenger and incumbent have biases that are equally distant from the median voter. Symmetrically increasing polarization weakly increases the probability that the incumbent wins reelection.

Proposition 11. *(Challenger Driven Polarization)*
Increasing the challenger's ideological bias weakly increases the probability that the incumbent wins reelection.

Proposition 12. *(Incumbent Driven Polarization)*
Assume F is log-concave, twice differentiable, and f is symmetric about 0. Suppose the incumbent and challenger are initially unbiased. There exists a threshold on office benefit, $\beta^* > q\sigma^2$, such that if $\beta \in (q\sigma^2, \beta^*)$, then increasing the incumbent's ideological bias weakly increases the probability the incumbent wins reelection.

Proofs of Propositions 10, 11, and 12. In the equilibrium with maximum posturing, the probability of reelection is given by

$$q + (1-q)(\overline{\Pi}_R + \underline{\Pi}_R).$$

Expanding, this can be written as

$$q + (1-q)\left(\frac{q}{1-q}\frac{1-q-\frac{R^2-L^2}{\sigma^2}}{q+\frac{R^2-L^2}{\sigma^2}}\right)(F(\omega_R^*) - F(-\omega_R^*)). \tag{33}$$

To prove Part 1 of the proposition, set $R = -L$. This simplifies the probability of reelection to

$$q + (1-q)(F(\omega_R^*) - F(-\omega_R^*)), \tag{34}$$

where $\omega_R^* = \sqrt{\beta - q\sigma^2 + 4R^2}$. Differentiating with respect to R, we get

$$\frac{\partial(34)}{\partial R} = (1-q)(f(\omega_R^*) + f(-\omega_R^*))2\frac{\partial \omega_R^*}{\partial R}.$$

This expression has the same sign as the derivative of ω_R^*. Differentiating, we have

$$\frac{\partial \omega_R^*}{\partial R} = \frac{4R}{\sqrt{\beta - q\sigma^2 + 4R^2}} > 0.$$

Thus, $\frac{\partial(34)}{\partial R} > 0$, as required.

For Part 2 of the proposition, we fix R and consider how equation (33) changes in L. Differentiating, we have

$$\begin{aligned}\frac{\partial(33)}{\partial L} &= (1-q)\left(\frac{2q\sigma^2 L}{(1-q)(R^2-L^2+q\sigma^2)^2}\right)(F(\omega_R^*) - F(-\omega_R^*)) \\ &+(1-q)\left(\frac{q}{1-q}\frac{1-q-\frac{R^2-L^2}{\sigma^2}}{q+\frac{R^2-L^2}{\sigma^2}}\right)(f(\omega_R^*) + f(-\omega_R^*))\frac{\partial \omega_R^*}{\partial L}\end{aligned} \tag{35}$$

Because $L < 0$, the first line of equation (35) is negative. The sign of the second line will have the same sign as $\frac{\partial \omega_R^*}{\partial L}$. Differentiating yields

$$\frac{\partial \omega_R^*}{\partial L} = \frac{-(R-L)}{\sqrt{\beta - q\sigma^2 + (R-L)^2}} < 0.$$

Thus, $\frac{\partial(33)}{\partial L} < 0$.

To prove Part 3 we again differentiate equation (33), this time with respect to R. Doing so yields

$$\begin{aligned}\frac{\partial(33)}{\partial R} &= (1-q)(\frac{-2q\sigma^2 R}{(1-q)(R^2-L^2+q\sigma^2)^2})(F(\omega_R^*)-F(-\omega_R^*)) \\ &\quad +(1-q)(\frac{q}{1-q}\frac{1-q-\frac{R^2-L^2}{\sigma^2}}{q+\frac{R^2-L^2}{\sigma^2}})(f(\omega_R^*)+f(-\omega_R^*))\frac{\partial\omega_R^*}{\partial L}.\end{aligned} \tag{36}$$

Additionally,

$$\frac{\partial\omega_R^*}{\partial R} = \frac{R-L}{\sqrt{\beta-q\sigma^2+(R-L)^2}} > 0.$$

As $R > 0$, the first line of equation (36) is negative. On the other hand, the second line of (36) is positive because $\frac{\partial\omega_R^*}{\partial R} > 0$. Letting $R = 0$, we get $\frac{\partial(33)}{\partial R} = 0$. To complete the proof, I show that if $\beta < \beta^*$, then (33) has a local min at $R = 0$. Otherwise, if $\beta > \beta^*$ then at $R = 0$ (33) is at a local max.

Differentiating again with respect to R, we get

$$\begin{aligned}\frac{\partial^2(33)}{\partial R^2} =&(\frac{\beta-q\sigma^2}{(\beta-q\sigma^2+R^2)^{\frac{3}{2}}}(f(\omega_R^*)+f(-\omega_R^*)) \\ &+\frac{R}{\sqrt{\beta-q\sigma^2+R^2}}(f'(\omega_R^*)-f'(-\omega_R^*))\frac{\partial\omega_R^*}{\partial R})(\frac{1-q-\frac{R^2}{\sigma^2}}{q+\frac{R^2}{\sigma^2}}) \\ &+\frac{R}{\sqrt{\beta-q\sigma^2+R^2}}(f(\omega_R^*)+f(-\omega_R^*))(\frac{-2\sigma^2 R}{(q\sigma^2+R^2)^2}) \\ &-\frac{2\sigma^2(q\sigma^2-3R^2)}{(q\sigma^2+R^2)^3}(F(\omega_R^*)-F(-\omega_R^*)) \\ &-\frac{2\sigma^2 R}{(R^2+q\sigma^2)^2}(f(\omega_R^*)+f(-\omega_R^*))\frac{\partial\omega_R^*}{\partial R}.\end{aligned} \tag{37}$$

Letting $R = 0$, equation (37) simplifies to

$$\frac{\partial^2(33)}{\partial R^2} = (\frac{1-q}{q})(\frac{1}{\sqrt{\beta-q\sigma^2}})(f(\omega_R^*)+f(-\omega_R^*)) - \frac{2((F(\omega_R^*)-F(-\omega_R^*)))}{q^2\sigma^2}.$$

Rearranging, we have $\frac{\partial(33)}{\partial R} > 0$ if and only if

$$\frac{q\sigma^2}{\sqrt{\beta - q\sigma^2}} > \frac{2(F(\omega_R^*) - F(-\omega_R^*))}{f(\omega_R^*) + f(-\omega_R^*)}.$$

Since f is assumed to be symmetric, we can rewrite the above as

$$\frac{q\sigma^2}{\sqrt{\beta - q\sigma^2}} > \frac{F(\omega_R^*) - F(-\omega_R^*)}{f(\omega_R^*)}. \tag{38}$$

The LHS of equation (38) is strictly decreasing in office benefit. Furthermore, $\lim_{\beta \to q\sigma^2} LHS(38) = \infty$ and $\lim_{\beta \to \infty} LHS(38) = 0$. Thus, it suffices to show that the RHS of (38) is strictly increasing in β.

Inspecting ω_R^*, we have that $\frac{\partial \omega_R^*}{\partial \beta} > 0$. Thus, simplifying notation, Part 3 of the proposition holds if

$$\frac{\partial}{\partial z} \frac{F(z) - F(-z)}{f(z)} > 0.$$

Differentiating, this inequality becomes

$$\begin{aligned} \frac{f(z)(f(z) + f(z)) - (F(z) - F(-z))f'(z)}{f(z)^2} &> 0 \\ \Leftrightarrow 2f(z)^2 &> f'(z)(F(z) - F(-z)). \end{aligned} \tag{39}$$

First, since $f(z)^2 > 0$ and $F(z) - F(-z) > 0$, if $f'(z) < 0$ then the equation holds immediately.

Second, assume that $f'(z) > 0$. Note that

$$2f(z)^2 \geqslant f(z)^2 \geqslant f'(z)F(z) \geqslant f'(z)F(z) - f'(z)F(-z).$$

The first inequality holds as $f(z)^2 > 0$. The second inequality holds by log-concavity. Finally, the third inequality holds as $F(-z) > 0$ and we have assumed $f'(z) > 0$.

Proposition 13. *Assume f is symmetric about 0. Suppose the incumbent and challenger have ideological biases equally distant from the voter. Symmetrically increasing polarization decreases voter welfare.*

Proof of Propositions 13. When R and L are equidistant from 0 we can write $L = -R$ and the election is always competitive. In this case, we can write voter welfare as

$$W(R) = q\Big[\Big(1{-}F(\omega_R^*) + F(-\omega_R^*)\Big)\Big(-R^2\Big) + \int_{-\omega_R^*}^{0}\Big(-(\underline{x}_R-\omega)^2\Big)f(\omega)d\omega + \int_0^{\omega_R^*}\Big(-(\overline{x}_R-\omega)^2\Big)f(\omega)d\omega - R^2\Big] + (1-q)\Big[\overline{\Pi}_R\Big(-\overline{x}_R^2-\sigma^2-R^2-\sigma^2\Big) + \underline{\Pi}_R\Big(-\underline{x}_R^2-\sigma^2-R^2-\sigma^2\Big) + (1-\overline{\Pi}_R-\underline{\Pi}_R)\Big(-R^2-\sigma^2-R^2-(1-q)\sigma^2\Big)\Big]. \tag{40}$$

First, consider the welfare effect of R through the informed type. Denote this

$$\hat{W}^I = -\Big(1-F(\omega_R^*)+F(-\omega_R^*)\Big)R^2 - \int_{-\omega_R^*}^{0}(\underline{x}_R-\omega)^2 f(\omega)d\omega - \int_0^{\omega_R^*}(\overline{x}_R-\omega)^2 f(\omega)d\omega - R^2.$$

Applying Leibniz rule, we can differentiate $\hat{W}^I$ with respect to R. This yields

$$\frac{\partial \hat{W}^I}{\partial R} = -2R(1+F(-\omega_R^*)-F(\omega^*)) - R^2(-f(\omega^*)\frac{\partial \omega^*}{\partial R} - f(-\omega^*)\frac{\partial \omega^*}{\partial R}) \tag{41}$$

$$-(\overline{x}_R-\omega_R^*)^2 f(\omega_R^*)\frac{\partial \omega^*}{\partial R} + \int_0^{\omega_R^*} -2\frac{\partial \overline{x}_R}{\partial R}(\overline{x}_R-\omega)f(\omega)d\omega \tag{42}$$

$$+(\underline{x}_R-(-\omega_R^*))^2 f(-\omega_R^*)(-\frac{\partial \omega_R^*}{\partial R}) + \int_{-\omega_R^*}^{0} -2\frac{\partial \underline{x}_R}{\partial R}(\underline{x}_R-\omega)f(\omega)d\omega - 2R. \tag{43}$$

Grouping terms and using the symmetry of F, we can rewrite the above as

$$\frac{\partial \hat{W}^I}{\partial R} = -2R(1+F(-\omega_R^*)-F(\omega^*)) - 2R \tag{44}$$

$$\int_0^{\omega_R^*} -2\frac{\partial \overline{x}_R}{\partial R}(\overline{x}_R-\omega)f(\omega)d\omega + \int_{-\omega_R^*}^{0} -2\frac{\partial \underline{x}_R}{\partial R}(\underline{x}_R-\omega)f(\omega)d\omega \tag{45}$$

$$2R^2 f(\omega^*)\frac{\partial \omega^*}{\partial R} - \frac{\partial \omega_R^*}{\partial R}f(\omega_R^*)((\underline{x}_R+\omega_R^*)^2+(\overline{x}_R-\omega_R^*)^2) \tag{46}$$

Line (44) is clearly negative. From symmetry of F, we have that line (45) will be less than 0 if:

$$
\begin{aligned}
&-\frac{\partial \overline{x}_R}{\partial R}\overline{x}_R - \frac{\partial \underline{x}_R}{\partial R}\underline{x}_R < 0 \\
&\Leftrightarrow -(1-\frac{\partial \omega_R^*}{\partial R})(R-\omega_R^*) < (1+\frac{\partial \omega_R^*}{\partial R})(R+\omega_R^*) \\
&\Leftrightarrow -R+\omega_R^* - \frac{\partial \omega_R^*}{\partial R}R + \frac{\partial \omega_R^*}{\partial R}\omega_R^* < R+\omega_R^* + \frac{\partial \omega_R^*}{\partial R}R + \frac{\partial \omega_R^*}{\partial R}\omega_R^* \\
&\Leftrightarrow 0 < R + \frac{\partial \omega_R^*}{\partial R}
\end{aligned}
$$

where the first derivation expands terms, the second expands the previous, and the last expression eliminates like terms. Finally, the last line holds by $R > 0$ and $\frac{\partial \omega_R^*}{\partial R} > 0$.

Finally, we show that the term on line (46) is equal to zero. For this to hold requires:

$$
\begin{aligned}
&2R^2 f(\omega^*)\frac{\partial \omega^*}{\partial R} - \frac{\partial \omega_R^*}{\partial R} f(\omega_R^*)((\underline{x}_R+\omega_R^*)^2 + (\overline{x}_R-\omega_R^*)^2) = 0 \\
&\Leftrightarrow 2R^2 f(\omega^*)\frac{\partial \omega^*}{\partial R} = \frac{\partial \omega_R^*}{\partial R} f(\omega_R^*)((\underline{x}_R+\omega_R^*)^2 + (\overline{x}_R-\omega_R^*)^2) \\
&\Leftrightarrow R^2 = (\underline{x}_R+\omega^* R)^2 + (\overline{x}_R-\omega_R^*)^2 \\
&\Leftrightarrow R^2 = (R-\omega_R^*+\omega_R^*)^2 + (R+\omega_R^*-\omega_R^*)^2 \\
&\Leftrightarrow R^2 = R^2
\end{aligned}
$$

Thus, welfare is decreasing from the informed type as R increases.

To finish proving the proposition, we need that welfare is decreasing through the low quality type as well. This is given by the term in equation (41) that is multiplied by $1-q$.

From the proof of Part 1 of Proposition 5, we have that $\overline{\Pi}_R$ and $\underline{\Pi}_R$ are increasing in R, while $1-\overline{\Pi}_R-\underline{\Pi}_R$ is decreasing. Thus, inspecting equation (41), to show that the part of voter welfare due to the low quality type is decreasing in R it is sufficient to show that the following inequalities hold

$$R^2 \leqslant \overline{x}_R^2 \tag{47}$$

$$R^2 \leqslant \underline{x}^2. \tag{48}$$

To show inequality (47), we need the following to hold

$$\begin{aligned}
& R^2 \leqslant \overline{x}_R^2 \\
& \Leftrightarrow R^2 \leqslant (R+\omega_R^*)^2 \\
& \Leftrightarrow R^2 \leqslant R^2 + 2R\omega_R^* + \omega_R^{*2} \\
& \Leftrightarrow 0 \leqslant 2R\omega_R^* + \omega_R^{*2},
\end{aligned}$$

which holds by $R > 0$ and $\omega_R^* > 0$. To show inequality (48), note

$$\begin{aligned}
& R^2 \leqslant \underline{x}_R^2 \\
& \Leftrightarrow R^2 \leqslant (R-\omega_R^*)^2 \\
& \Leftrightarrow R^2 \leqslant R^2 - 2R\omega_R^* + \omega_R^* \\
& \Leftrightarrow 2R\omega_R^* \leqslant \omega_R^{*2} \\
& \Leftrightarrow 2R < \omega_R^* \\
& 2R = \sqrt{4R^2} < \sqrt{\beta - q\sigma^2 + 4R^2} = \omega_R^*.
\end{aligned}$$

Therefore, $W(R)$ is decreasing in R.

Proposition 14. *Assume office benefit is sufficiently large. If $L^2 < q\sigma^2$, then voter welfare is maximized at $R = \overline{R} > 0$. Otherwise, if $L^2 > q\sigma^2$, then voter welfare is maximized when the incumbent has a matching ideology, $R = 0$.*

Proof of Proposition 14. To begin, note that if $L > q\sigma^2$, then at $R = 0$ the election is lopsided. As such, the voter's welfare from an incumbent with ideology $R = 0$ is $-(1-q)\sigma^2$, which is his payoff under first-best outcomes and, thus, optimal.

Next, if $R \geqslant \overline{R}$, then the voter always replaces the incumbent and welfare is $W(R \geqslant \overline{R}) = W_{\geqslant}(R)$, given by

$$W_{\geqslant}(R) = -R^2 - (1-q)\sigma^2 - L^2 - (1-q)\sigma^2.$$

As $W_{\geqslant}(R)$ is strictly decreasing in R, it is maximized at $R = \overline{R}$.

If $R < \overline{R}$, then, because $L < q\sigma^2$, the election is always competitive. Here, welfare is more complicated as the voter's first period payoff depends on the realization of the state and he may or may not reelect the incumbent. In this case, voter welfare is $W(R < \overline{R}) = W_<(R)$

$$W_<(R) = q\Big[\Big(1-F(\omega_R^*) + F(-\omega_R^*)\Big)\Big(-2R^2\Big) + \int_{-\omega_R^*}^{0}\Big(-(\underline{x}_R - \omega)^2 - R^2\Big)f(\omega)d\omega$$
$$+\int_{0}^{\omega_R^*}\Big(-(\overline{x}_R - \omega)^2 - R^2\Big)f(\omega)d\omega\Big] + (1-q)\Big[\overline{\Pi}_R\Big(-\overline{x}_R^2 - \sigma^2 - R^2 - \sigma^2\Big)$$
$$+\underline{\Pi}_R\Big(-\underline{x}_R^2 - \sigma^2 - R^2 - \sigma^2\Big) + (1 - \overline{\Pi}_R - \underline{\Pi}_R)\Big(-R^2 - \sigma^2 - L^2 - (1-q)\sigma^2\Big)\Big]$$

If $\beta \to \infty$, then $\omega^* \to \infty$ and $W_< \to -\infty$. As $W_<$ is continuous in β, there exists $\overline{\beta} < \infty$ such that if $\beta > \overline{\beta}$, then $W_<(R) < W_\geqslant(\overline{R})$.

4 Extensions

Voter Learning. Following Canes-Wrone, Herron and Shotts (2001) and Maskin and Tirole (2004), modify the model so with probability r the state ω is revealed before the election and with probability $1-r$ it is never revealed.

Proposition 1. *Assume $q_I = q_C = q$.*

1. *For cut-points $\underline{x}_r$ and $\overline{x}_r$, the incumbent's strategy is characterized analogously to Proposition 1.*

2. *When the state is not revealed, the voter reelects when $x \notin (\underline{x}_r, \overline{x}_R)$. When the state is revealed, the voter reelects if and only if one of the three following outcomes hold:*

   (a) $x = \omega$,

   (b) $x = \overline{x}_r$ and $\omega \in [0, \overline{x}_r)$, or

   (c) $x = \underline{x}_r$ and $\omega \in (\underline{x}_r, 0)$.

3. *For all $\overline{x}_r$ and $\underline{x}_r$, cut-points are ordered as follows: $\underline{x} < \underline{x}_r < \overline{x}_r < \overline{x}$.*

If F is twice differentiable and f is single-peaked at 0, then $\underline{x}_r$ and $\overline{x}_r$ are unique.

The expected utility to the uninformed type of choosing $x = 0$ is still $-\sigma^2 + \beta - (1-q)\sigma^2$. On the other hand, choosing $\overline{x}_r$ yields

$$-\overline{x}^2 - \sigma^2 + \beta + r\Big((F(\overline{x}) - F(0))(\beta - \sigma^2) - (1 - F(\overline{x}) + F(0))(1-q)\sigma^2\Big) + (1-r)\Big(\beta - \sigma^2\Big).$$

Thus, we have that the uninformed type is indifferent if $\overline{x}_r$ solves

$$x^2 = \left(1 - r + r\Big(F(\overline{x}) - F(0)\Big)\right)\left(\beta - q\sigma^2\right). \tag{49}$$

At $x = 0$ the LHS of (49) is equal to 0, while the RHS is $(1-r)(\beta - q\sigma^2) > 0$. On the other hand, letting $x \to \infty$, the LHS goes to ∞ while the RHS goes to $(1 - rF(0))(\beta - q\sigma^2) < \infty$. Thus, by continuity, $\overline{x}_r$ exists. Now suppose that $f(x)$ is single-peaked at 0. We show that if the LHS of (49) intersects the RHS of (49) at some x', then it cannot intersect again for any $x > x'$. The derivative of the LHS of (49) is $2x > 0$ and the second derivative is $2 > 0$. The derivative of the RHS is $rf(x)(\beta - q\sigma^2) > 0$ and the second derivative is $rf'(x)(\beta - q\sigma^2) < 0$, by $x > 0$ and f single-peaked at 0. Thus, for any $x > x'$ the LHS is increasing faster in x than the RHS, and so there cannot be another solution to (49). Analogous arguments yield $\underline{x}_r$.

Noisy Signals. In the baseline model I make the stark assumption that low quality executives are no better informed than voters. Additionally, in Canes-Wrone, Herron and Shotts (2001), pandering is driven by low quality types ignoring their signal to choose the ex ante popular policy. Here, I show that politicians are incentivized to choose policies away from the ex ante optimal, even if this assumption is relaxed.

Assume $q_I = q_C$. Let F be the normal distribution. Change the model so that the low quality type observes a signal $s = \omega + \epsilon$, where ϵ is drawn from a normal distribution with mean 0 and variance γ^2. Thus, $s \sim \mathcal{N}(0, \sigma^2 + \gamma^2)$.

Given this structure, we have

$$\hat{\omega} = \mathbb{E}[\omega|s] = \frac{\sigma^2}{\gamma^2 + \sigma^2}s.$$

In this case, ex ante $\omega \sim \mathcal{N}(0, \sigma^2)$ and $\hat{\omega} \sim \mathcal{N}(0, \hat{\sigma}^2)$, where $\hat{\sigma}^2 = \frac{\sigma^4}{\sigma^2+\gamma^2}$. The first-best outcome is for all high quality types to choose $x = \omega$ and all low quality types to choose $x = \hat{\omega}$. Fix the first-best policy choices. Thus, the distribution of high quality policy choices is given by F, the distribution of ω and the distribution of low quality policy choices is given by $\hat{F}$, the distribution of $\hat{\omega}$. The voter reelects if and only if

$$\begin{aligned}
&\tilde{q}(x_1) \geqslant q \\
&\Leftrightarrow \frac{Pr(x_1|H)Pr(H)}{Pr(x_1)} \geqslant q \\
&\Leftrightarrow \frac{qf(x_1)}{qf(x_1) + (1-q)\hat{f}(x_1)} \geqslant q \\
&f(x_1) \geqslant \hat{f}(x_1).
\end{aligned}$$

Since $f(x_1)$ and $\hat{f}(x_1)$ have the same mean and different variances, they intersect at two points. Solving yields that in the first-best the voter reelects if

$$x_1 \geqslant \frac{\sqrt{\frac{4}{\sigma^2\hat{\sigma}^2}(\sigma^2 - \hat{\sigma}^2)\ln\left(\frac{\sigma^2}{\hat{\sigma}^2}\right)}}{2\left(-\frac{1}{\sigma^2} + \frac{1}{\hat{\sigma}^2}\right)} \tag{50}$$

$$\Leftrightarrow x_1 \geqslant \frac{\frac{\sigma^6}{\sigma^2+\gamma^2}\sqrt{\frac{\gamma^2}{\sigma^4}\ln\left(\frac{\sigma^2+\gamma^2}{\sigma^2}\right)}}{\frac{\sigma^2\gamma^2}{\sigma^2+\gamma^2}} \equiv \Gamma. \tag{51}$$

And also reelects if $x_1 \leqslant -\Gamma$. There is a unique signal s such that the low quality type that observes s has $\hat{\omega}(s) = \Gamma$. Let this signal and corresponding optimal policy be given by s_Γ and $\hat{\omega}(s_\Gamma)_\Gamma$, respectively. Define the variance in ω conditional on observing signal s as $\nu^2 = \frac{\sigma^2\gamma^2}{\sigma^2+\gamma^2}$.

Thus, the expected utility to a low type for choosing $\hat{\omega}$ and losing the election is $\beta - \nu^2 - (1-q)\nu^2$. The expected utility for choosing a policy x and winning is $\beta - (x-\hat{\omega})^2 - \nu^2 + \beta - \hat{\sigma}^2$.

Therefore, at $\overline{x}_\gamma = \hat{\omega}_\Gamma + \sqrt{\beta - q\nu^2}$, the s_Γ type is indifferent between choosing policy $\overline{x}_\gamma$ and winning, or $\hat{\omega}_\Gamma$ and losing.

For a high quality type, she prefers $\overline{x}_\gamma$ and winning over her ideal policy and losing if

$$\begin{aligned}
&\beta - (\overline{x}_\gamma - \omega)^2 + \beta \geqslant \beta - (1-q)\nu^2 \\
&\Leftrightarrow \omega \geqslant \overline{x}_\gamma - \sqrt{\beta + (1-q)\nu^2} \\
&\Leftrightarrow \omega \geqslant \hat{\omega}(s_\gamma) - \left(\sqrt{\beta + (1-q)\nu^2} - \sqrt{\beta - q\nu^2}\right).
\end{aligned}$$

Thus, all high quality types that observe $\omega \in (\hat{\omega}_\Gamma, \overline{x}_\gamma)$ overreact and choose $\overline{x}_\gamma$. Additionally, since $\beta + (1-q)\nu^2 > \beta - q\nu^2$, high quality types that see a lower signal also overreact. Specifically, those that see $\omega \in \left[\max\left\{0, \hat{\omega}(s_\gamma) - \left(\sqrt{\beta + (1-q)\nu^2} - \sqrt{\beta - q\nu^2}\right)\right\}, \hat{\omega}(s_\Gamma)\right)$ also overreact and choose $\overline{x}_\gamma$.

Given the indifference condition, all low quality types such that $\hat{\omega}(s) \in (\omega_\Gamma, \overline{x}_\gamma)$ posture and choose $\overline{x}_\gamma$. As $f(z) > \hat{f}(z)$ for $z > \overline{x}_\gamma$, after seeing $\overline{x}_\gamma$ the voter updates that $\tilde{q}(\overline{x}_\gamma) > q$ and reelects the incumbent.

Finally, for any low quality type such that $\hat{\omega}(s) > \overline{x}_\gamma$ she chooses $x = \hat{\omega}(s)$ as this maximizes her expected policy utility and wins reelection. Similarly, for a high quality type such that $\omega > \overline{x}_\gamma$. Again, by the condition on $\overline{x}_\gamma$, the voter is willing to reelect following these policy choices.

Now we can study what happens to $\overline{x}_\gamma$ as γ decreases, i.e., the low quality type's signal becomes more accurate. Differentiating yields $\frac{\partial \overline{x}_\gamma}{\partial \gamma} = \frac{\partial \hat{\omega}_\gamma}{\partial \gamma} - \frac{\sigma^4 \gamma}{(\sigma^2+\gamma^2)^2 \sqrt{\beta - \frac{\sigma^2\gamma^2}{\sigma^2+\gamma^2}}}$, where

$$\frac{\partial \hat{\omega}_\gamma}{\partial \gamma} = \frac{\sigma^2 \Big(\gamma^2 - (\sigma^2+\gamma^2)\ln(1+\gamma^2/\sigma^2)\Big)}{\gamma(\sigma^2+\gamma^2)\sqrt{\gamma^2 \ln(1+\gamma^2/\sigma^2)}} < 0.$$

The inequality holds by $\gamma^2 - (\sigma^2+\gamma^2)\ln\Big(1+\gamma^2/\sigma^2\Big) < 0$. Thus, increasing the accuracy of the low type's signal (decreasing γ) increases $\overline{x}_\gamma$. Note that voter welfare may not be decreasing, however, as the low quality type is getting better information.

Additionally, $\lim_{\gamma \to 0} \overline{x}_\gamma = \sqrt{\sigma^2} + \sqrt{\beta} > \sqrt{\beta - q\sigma^2} = \lim_{\gamma \to \infty} \overline{x}_\gamma$.

Orthogonal Ideological Dimension. Amend the baseline model to include a separate policy dimension driven by ideological differences. Each player has a known ideal policy on this dimension. The voter has an ideal point at 0, the incumbent has ideal point R, and the challenger ideal point L. I assume $L \leqslant 0 \leqslant R$. In each period she holds office, the politician implements her ideal point on this dimension.

Given second period policymaking, the voter's expected utility for electing a high quality politician is $-(\omega_t - \omega_t)^2 - \hat{y}_i^2 = -\hat{y}_i^2$, and his expected utility for a low quality incumbent is $\int_{\mathbb{R}} -\omega^2 dF(\omega) - \hat{y}_i^2 = -\sigma^2 - \hat{y}_i^2$. Therefore, the voter's decision is based on his belief about the incumbent officeholder's ability, as well as the candidates' ideologies. Let $\tilde{q}_I(x_1)$ be the voter's belief that the incumbent is high quality, following policy choice x_1, and this belief is updated according to Bayes' rule whenever possible.

The expected utility to the voter for reelecting the incumbent is: $-(1-\tilde{q}_I(x_1))\sigma^2 - R^2$. On the other hand, the expected utility for electing the challenger is: $-(1-q_C)\sigma^2 - L^2$. Therefore, in equilibrium, if $\tilde{q}_I(x_1) > q_C + \frac{R^2-L^2}{\sigma^2}$, then the voter must reelect the incumbent. If $\tilde{q}_I(x_1) < q_C + \frac{R^2-L^2}{\sigma^2}$, then he must elect the challenger. Finally, if $\tilde{q}(x_1) = q_C + \frac{R^2-L^2}{\sigma^2}$, then the voter is indifferent and, as such, he can reelect the incumbent with any probability $\rho(x_1) \in [0,1]$.

The term $q_C + \frac{R^2 - L^2}{\sigma^2}$ measures the popularity of the challenger relative to the incumbent. If the belief about the challenger's competence increases, the challenger becomes more ideologically moderate, or the incumbent becomes more extreme, then the voter's posterior belief that the incumbent is competent must increase for the incumbent to win reelection. If $q_C + \frac{R^2 - L^2}{\sigma^2} > 1$, then the voter always prefers the challenger, even when certain the incumbent is high quality. Similarly, if $q_C + \frac{R^2 - L^2}{\sigma^2} < 0$, then the voter reelects the incumbent, even when certain the incumbent is low quality. Thus, as in the previously studied ideological model, lopsided elections exist in which the incumbent always wins or always loses.

When the election is not lopsided, arguments similar to the proof of Propositions 1 and 2 imply that there is a PBE characterized $\underline{x}'$ and $\overline{x}'$ which solve an indifference condition for the uninformed type. That is, they solve

$$-\sigma^2 + \beta - (1 - q_C)\sigma^2 - (L - R)^2 = -x^2 - 2\sigma^2 + 2\beta \tag{52}$$

Solving equation (52) yields explicit solutions $\overline{x}' = \sqrt{\beta - q_C\sigma^2 + (L - R)^2}$ and $\underline{x}' = -\sqrt{\beta - q_C\sigma^2 + (L - R)^2}$.

Given the election is competitive, increasing polarization clearly increases overreacting and weakly increases posturing. Additionally, this decreases welfare. Again, creating a lopsided election can increase welfare if β is sufficiently high.

Note, unlike the ideological model in the paper, here the interval is centered around 0 rather than R. Thus, increasing polarization always distorts policy choices away from 0 on the crisis dimension and polarizes policies on the orthogonal dimension.