Online Appendix For: The Rank Effect in Multimember District Elections

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A Summary Statistics

Panel A: Korean Municipal Legislative Election						
Mean Std Dev Min Max						
Win at t	0.391	0.488	0.000	1.000	18,983	
First-Place	0.161	0.367	0.000	1.000	18,983	
Second-Place	0.161	0.367	0.000	1.000	18,983	
Candidate Vote Share	0.161	0.106	0.001	0.608	18,983	
Run at $t+1$	0.367	0.482	0.000	1.000	18,983	
Win at $t+1$	0.207	0.405	0.000	1.000	18,983	
Run for Higher Office at $t+1$	0.061	0.239	0.000	1.000	18,983	
Win Higher Office at $t+1$	0.024	0.152	0.000	1.000	18,983	
Being Place First the Ballot	0.261	0.439	0.000	1.000	14,764	
Share of Party Money	0.027	0.091	0.000	1.000	$10,\!955$	
Panel B: Spanish S	enatorial	Election				
	Mean	Std Dev	Min	Max	Ν	
Win at t	0.141	0.348	0.000	1.000	16,170	
First-Place	0.040	0.196	0.000	1.000	16,170	
Second-Place	0.040	0.196	0.000	1.000	16,150	
Candidate Vote Share	0.107	0.165	0.000	0.775	16,170	
Run at $t+1$	0.109	0.312	0.000	1.000	16,170	
Win at $t+1$	0.048	0.213	0.000	1.000	$16,\!170$	
Panel C: Japanese House o	of Represe	entative Elec	ction			
	Mean	Std Dev	Min	Max	Ν	
Win at t	0.109	0.311	0.000	1.000	17,000	
First-Place	0.122	0.327	0.000	1.000	17,000	
Second-Place	0.122	0.327	0.000	1.000	17,000	
Candidate Vote Share	0.122	0.078	0.000	0.621	17,000	
Run at $t+1$	0.672	0.470	0.000	1.000	17,000	
Win at $t+1$	0.416	0.493	0.000	1.000	17,000	

Table A.1. Summary Statistics

Panel D: Korean Provincial Legislative Election

	Mean	Std Dev	Min	Max	Ν
Win at t	0.395	0.489	0.000	1.000	5,369
First-Place Winner (Lower Office) at $t-1$	0.055	0.229	0.000	1.000	5,369
Non-First-Place Winner (Lower Office) at $t-1$	0.063	0.242	0.000	1.000	5,369
Candidate Vote Share	0.381	0.188	0.006	0.874	5,248
Incumbent	0.192	0.394	0.000	1.000	5,369

B Incumbency Effects

This section presents the results from the tests of whether incumbency increases the probabilities of running in and winning the subsequent election for the same office. The RD analysis of the incumbency effect is similar to that of the first-rank effect. Now, the treatment T_{id} is an indicator variable for whether candidate *i* won a seat in a multimember district *d*. It should be noted, however, that there are multiple winners in a multimember district. Accordingly, I restrict the sample to those who barely won an election and who barely lost it. For instance, in a three-member district, I focus on the third- and fourth-place candidates, where these two candidates belong to the treatment and the control groups, respectively. The running variable VM_{id} is defined as the vote margin of the candidates who are in the sample. For instance, for a third-place candidate in a three-member district, it is his/her vote share minus the vote share of the fourth-place candidate. Under the same identifying assumption of the RD design—candidates do not have a full control over the outcome of close races—we can estimate the effect of incumbency on the outcome of the subsequent elections using an RD design.¹

Figures B.1 shows the results graphically. As indicated by the "jumps" at the cutoffs in Graphs (a), (c), and (e) of Figure B.1, the results suggest a positive effect of incumbency on the probability of running in the subsequent election in all three cases. However, while incumbency has a positive effect on the probability of winning the next election in the Korean and Spanish cases (Graphs b and d), it has no effect on the probability of winning in the Japanese case (Graph f). These results are consistent with previous studies. ? find evidence of an incumbency advantage in Korea's other local elections (mayoral and provincial legislative elections) using an RD design, and ?, using regression analyses, reports a positive effect of incumbency in the Spanish Senatorial elections. Finally, ? finds no evidence of an incumbency advantage in the Japanese House of Representatives elections during the study period.

The RD estimates are reported in Table B.1. These estimates are consistent with the graphical analyses. In the Korean case, incumbency increases the probability of running in the next election by 22.8 percentage points and that of winning by 10.3 percentage points. The size of the incumbency advantage in the municipal legislative elections is comparable to that in other local elections. According to ?, the size of the incumbency effect on winning is 14.9 percentage points in the mayoral and 9.8 percentage points in the provincial legislative elections, respectively.

In the Spanish case, close winners are 23.4 percent more likely to run in and 19.1 percent more likely to win the next election than close losers. The size of the incumbency effect on winning is somewhat smaller than that shown by ?, at 24.8 percentage points. However, it should be noted that using regression analysis, ? estimated the incumbency advantage plus the average quality difference between incumbents and their non-incumbent co-partisan candidates, while here, my RD analysis measures the *unconditional* incumbency effect.

Finally, in the Japanese case, the effect of incumbency on running again is approximately

¹Appendix C presents the results of the robustness and validity checks of the RD design.

15 percentage points higher than what is reported in ?, at 10 percentage points. This difference is due to the sample restriction of ?. The RD sample in ? is restricted to the Liberal Democratic Party's (LDP) candidates who ran in districts with at least one LDP incumbent and one LDP non-incumbent. When I restrict the sample to these candidates, I obtain almost identical results.² Consistent with Figure B.1, the RD estimate of the effect of incumbency on winning is small and not statistically significant.

²The results are not reported but available upon request.





Panel A: South Korea

Notes: The dots are local averages of the outcome variables, calculated within 0.002-wide bins of vote margin. The solid lines are second-order polynomial fits.

.05

.025

.8

.2 0

-.05

-.025

0 Vote margin at t

Prob. of running at t+1

.8

.2

0

-.05

-.025

0 Vote margin at t

.05

.025

Prob. of winning at t+1 .e

	Dependent Variable				
	Run	at $t+1$	Win at $t+1$		
	Estimate	Estimate Bandwidth E		Bandwidth	
	(1)	(2)	(3)	(4)	
Korea	$0.228 \\ (0.028)$	0.105 [N= 3,882]	$0.103 \\ (0.025)$	0.134 [N= 4,360]	
95% CI Dep. Var. Mean Dep. Var. St. Dev.	$[0.172, \ 0.284] \\ 0.466 \\ 0.499]$		$[0.053, 0.152] \\ 0.273 \\ 0.446]$		
Spain	$0.234 \\ (0.050)$	0.051 [N= 1,038]	$0.191 \\ (0.045)$	0.046 [N= 1,014]	
95% CI Dep. Var. Mean Dep. Var. St. Dev.	$[0.136, \ 0.331] \\ 0.243 \\ 0.429 $		$[\begin{matrix} 0.102, 0.281 \\ 0.167 \\ 0.373 \end{matrix}]$		
Japan	$0.158 \\ (0.025)$	0.058 [N= 3,096]	$0.000 \\ (0.034)$	0.052 [N= 2,960]	
95% CI Dep. Var. Mean Dep. Var. St. Dev.	$[0.108, 0.207] \\ 0.772 \\ 0.420$		$[-0.066, \ 0.066] \\ 0.522 \\ 0.500$		

Table B.1. The Effects of Incumbency

Standard errors in parentheses are calculated according to Calonico, Cattaneo and Titiunik (2014). The number of observations are in brackets. Estimates are from local linear regressions with triangular kernel using the Calonico, Cattaneo and Titiunik (2014) (CCT) optimal bandwidth.

C Robustness Checks

In this section, I test the validity of my RD design. First, I check for sorting around the threshold using the lagged dependent variables and a dummy variable for winning an election with the most votes at t - 1. For the Korean case, I was able to obtain information about candidates' characteristics and thus added the following variables: dummy variables indicating whether candidates are in their 20s or 30s, whether they are over 60, whether they are female, and whether they belong to one of the two major parties. For the Spanish and Japanese cases, I use an indicator variable for female as an additional covariate.

Because the original dataset for the Spanish elections does not contain information about candidates' gender, I use the list of the most frequent Spanish first names from the Instituto Nacional de Estadística.³. If a candidate's name only appears on the list of female (male) first names, I classify her (him) as a female (male). For the candidates whose first names appear both on the female and male first name lists,⁴ I classify the gender based on how frequently the name is used be males and females. For instance, while both females and males use the first name "Angel," it is more frequently used by males. Accordingly, I classify the candidates whose first names are "Angel" as males.

Tables C.1 and C.2 show the results for the Korean case and the other two cases (the Spanish and Japanese), respectively. As shown in the tables, none of the estimates is statistically significant, which suggests that there is no evidence of sorting around the threshold.

Second, I show that the results are not driven by the choice of a specific bandwidth. In order to examine the robustness of findings across a wide range of bandwidths, I estimate the effect of incumbency and first rank using 21 different bandwidths, which increase from 0.05 to 0.25 by 0.01. Figure C.1 shows the results for the rank effects (Table 1), and Figure C.2 shows the results for the incumbency effects (Table B.1). The results are mostly consistent with the main findings, except for the Japanese case. Figure C.1 (graph e) indicates that for larger bandwidths, the effect of first rank on the probability of running in the next election can be positive. It should be noted, however, that for very close elections, the first rank effect on running is not statistically significant. Therefore, whereas Figure C.1 provides some suggestive evidence of a positive effect of first rank on running in the Japanese case, more close races are needed to confirm this.

The results in this paper and that of ? suggest that incumbency had no effect on the probability of winning the subsequent election in the Japanese House of Representatives elections when they were held under an MMD system. In contrast, Figure C.2 shows that for bandwidths greater than 0.1, incumbency is shown to increase the chances of winning the next election. However, this does not provide evidence for an incumbency advantage in the Japanese case, because for very close elections, the winners' advantage is not statistically significant.

Figure C.3 shows the results for the effects of incumbency and first rank on races for higher offices races. The results are robust to a wide range of bandwidths for the first-rank effect.

³Source: https://www.ine.es/en/daco/daco42/nombyapel/nombres_por_edad_media_en.xls (Last accessed: 31 July 2021). I thank an anonymous reviewer for this suggestion.

⁴Such instances account for about 18.6 percent of the sample.

Although the effect of incumbency is significant for bandwidths greater than 0.15, in very close races, winners are not more likely to run for or win higher office.

Finally, I test whether there is a discontinuity in the density of the running variable around the cutoff. Figure C.4 shows the distributions of the running variables for all three cases examined in this paper.⁵ The graphs show no sign of manipulation around the cutoffs. I also test for manipulation using the method suggested by ?. The test for manipulation around the threshold fails to reject the null hypothesis of no manipulation for all the running variables, with the test statistic (T) being close to 0 in all cases. I also conducted McCrary's test (?). The results, not reported but available upon request, also show no sign of manipulation around the threshold in any of the cases.

⁵It should be noted that the RD samples are restricted to the bare winners and losers in the incumbency case and the two most voted candidates for the first-rank case. Accordingly, the distributions of the running variables look symmetric.





Notes: This figure shows the replications of the results reported in Table 1 using various bandwidths. The hollow circle and graph indicate the estimates and 95% confidence intervals.





Notes: This figure shows the replications of the results reported in Table B.1 using various bandwidths. The hollow circle and graph indicate the estimates and 95% confidence intervals, respectively.

Figure C.3. Robustness to Bandwidth Choices: The Effects of Incumbency and First Rank on Advancing to Higher Office (Korea)



Panel A: Incumbency

Notes: This figure shows the replications of the results reported in Table 4 using various bandwidths. The hollow circle and graph indicate the estimates and 95% confidence intervals.



Figure C.4. Distribution of Running Variables around the Thresholds

Notes: The vertical line indicates the threshold. The width of each bin is 0.002. The test for the manipulation around the threshold suggested by ? fails to reject the null hypothesis of no manipulation for all the running variables, with the test statistic (T) being close to 0 in all cases.

	Running Variable =			
	Vote Margin Between Winner vs. Loser		1 Vote Margin Betw First vs. Second-F Candidates	
	Estimate	Bandwidth	Estimate	Bandwidth
	(1)	(2)	(3)	(4)
Run at $t-1$	0.053 (0.029)	0.103 [N= 3,844]	-0.029 (0.033)	0.138 [N= 3,506]
Win at $t-1$	$0.043 \\ (0.025)$	0.140 [N= 4,446]	$0.002 \\ (0.030)$	0.181 [N= 4,100]
First-Place Winner at $t-1$	$0.026 \\ (0.022)$	0.127 [N= 4,250]	-0.001 (0.027)	0.147 [N= 3,640]
Age: 20–39	-0.003 (0.014)	0.102 [N= 3,832]	$0.003 \\ (0.013)$	0.149 [N= 3,658]
Age: Over 60	0.044 (0.023)	0.097 [N= 3,726]	$0.010 \\ (0.022)$	0.148 [N= 3,656]
Female	-0.010 (0.014)	0.137 [N= 4,410]	$0.033 \\ (0.021)$	0.143 [N= 3,584]
Major Party	-0.024 (0.030)	0.099 [N= 3,778]	$0.002 \\ (0.031)$	0.115 [N= 3,100]

Table C.1. Placebo Tests (Korea)

Standard errors in parentheses are calculated according to Calonico, Cattaneo, and Titiunik (2014). The numbers of observations are in brackets. Estimates are from local linear regressions with a triangular kernel using the Calonico, Cattaneo, and Titiunik (2014) (CCT) optimal bandwidth.

	Running Variable =				
	Vote Margin Between Winner vs. Loser		Vote Margin Between First vs. Second-Plac Candidates		
	Estimate	Bandwidth	Estimate	Bandwidth	
	(1)	(2)	(3)	(4)	
Panel A: Spain					
Run at $t-1$	-0.072 (0.059)	0.031 [N= 902]	$0.087 \\ (0.085)$	0.026 [N= 794]	
Win at $t-1$	-0.092 (0.055)	0.028 [N= 864]	$0.091 \\ (0.087)$	0.025 [N= 766]	
First-Place Winner at $t-1$	$0.008 \\ (0.023)$	0.061 [N= 1,096]	-0.011 (0.058)	0.022 [N= 672]	
Female	-0.054 (0.047)	0.040 [N= 865]	$0.045 \\ (0.083)$	0.021 [N= 517]	
Panel B: Japan					
Run at $t-1$	-0.023 (0.025)	0.070 [N= 3,314]	$0.028 \\ (0.028)$	0.089 [N= 2,978]	
Win at $t-1$	-0.013 (0.033)	0.056 [N= 3,046]	$0.102 \\ (0.037)$	0.071 [N= 2,672]	
First-Place Winner at $t-1$	-0.013 (0.021)	0.052 [N= 2,962]	$0.087 \\ (0.029)$	0.084 [N= 2,908]	
Female	-0.006 (0.011)	0.064 [N= 3,406]	$0.000 \\ (0.007)$	0.093 [N= 3,258]	

Table C.2. Placebo Tests (Spain and Japan)

Standard errors in parentheses are calculated according to Calonico, Cattaneo and Titiunik (2014). The number of observations are in brackets. Estimates are from local linear regressions with triangular kernel using the Calonico, Cattaneo and Titiunik (2014) (CCT) optimal bandwidth.

D Bounds on the Conditional Effects

In this section, I estimate the conditional effect of first rank and incumbency on winning the next election, following ? and ?. The discussion in this section closely follows ?. Let Tbe the treatment variables indicating whether a candidate received the most votes. R_0 and R_1 are the potential outcome variables that indicate whether a candidate runs again when T = 0 and T = 1, respectively. The potential outcome variables for winning, W_0 and W_1 , are defined similarly. Therefore, the *observed* outcome variables are $R = TR_1 + (1 - T)R_0$ and $W = R[TW_1 + (1 - T)W_0]$. The conditional effect of first rank is $E[W_1 - W_0|R_1 = 1]$.⁶

We can define the following four groups based on compliance status: "always-takers," those who always run again; "never-takers," those who never run again; "compliers," those who would run again only if they receive the most votes; and "defiers," those who would run again only if they do not receive the most votes. We can estimate the bounds under the assumption that there are no defiers.

It should be noted, however, that this assumption is likely to be violated in the South Korean case, where candidates can pursue higher office. Because the municipal legislature is considered the lowest-level political office in South Korea, some incumbents may aspire to move on to higher office only when they come in first in an election. The presence of these "defiers" makes the bounds analysis infeasible for the South Korean case. Accordingly, I conduct the bound analysis only for the Spanish and Japanese cases.

? show that we can write the conditional effect (effect of first-rank conditional on being always-taker/complier) as follows under the assumption of no defiers:⁷

Note that (i) is $\lim_{x\downarrow 0} E[R|VM = x]$, (ii) is the RD effect on W, (iii) is the RD effect on R, and (iv) is the probability that a complier who failed to receive the most votes would win the subsequent election. Because compliers never run when they do not receive the most votes (by definition), (iv) is unobservable. An upper bound can be obtained by assuming that $E(W_0|R_1 > R_0) = 0$. Similarly, we can obtain a lower bound by assuming $E(W_0|R_1 > R_0) = 1$. To obtain a tighter lower bound, I assume that, following ?, the probability of compliers with T = 0 to win an election had they chosen to run again is at most equal to the probability that close second-place candidates who chose to run again would win that year. We can also estimate the bounds of the conditional incumbency effects in the same manner.

The results of bound analyses are presented in Table D.1. The results are, in general, consistent with the findings in previous sections. Most importantly, none of the bounds

⁶More specifically, it is $E[W_{1i} - W_{0i}|R_{1i} = 1, VM_i = 0]$. I suppress the subscript *i* and conditioning on VM = 0 for simplicity.

⁷See ? for proof.

for the rank effects are statistically significant. The conditional effect of incumbency on winning is positive for the Spanish case, although the lower bound is marginally significant. In the Japanese case, the results indicate that incumbency does not increase a candidate's probability of winning the next election. Whereas the lower bound indicates that incumbency can have a negative effect on winning the next election conditional on running, the upper bound suggests that incumbency may produce no greater chance of winning for those who actually run in the next election.⁸

	Incumbency		First Rank		
	Lower Bound Upper Bound		Lower Bound	Upper Bound	
	(1)	(2)	(3)	(4)	
Spain	$0.081 \\ (0.042)$	$0.613 \\ (0.075)$	-0.016 (0.252)	$0.057 \\ (0.126)$	
Japan	-0.109 (0.027)	$0.014 \\ (0.036)$	-0.027 (0.026)	-0.006 (0.029)	

Table D.1. Bounds on The Conditional Effects of Incumbency and First Rank

Standard errors are in parentheses. The bounds are calculated following Anagol and Fujiwara (2016). the Calonico, Cattaneo and Titiunik (2014) (CCT) optimal bandwidth.

⁸? suggests that a positive effect of incumbency on running and a null effect on winning unconditional on running imply an incumbency disadvantage. However, even if incumbency has no effect on winning for those who re-run, it can still increase the probability of running without affecting the probability of winning unconditional on running. For instance, a narrow win may have a strong effect on running especially for low-quality candidates who are not likely to win again.

E Multinomial Logit

Let Y be the outcome variable that belongs to the three mutually exclusive categories, $Y_i \in \{0, 1, 2\}$. Y_i is 0 for those who retire (loses/retires), 1 for those who run for higher office (win higher office), and 2 for those who run for the same office (win the same office). $Y_i = 0$ is the baseline category. The treatment effect of interest, τ_j , is defined as follows:

$$\tau_{j} \equiv \operatorname{Prob}(Y_{i}(1) = j | VM_{id} = 0) - \operatorname{Prob}(Y_{i}(0) = j | VM_{id} = 0)$$

=
$$\lim_{x \downarrow 0} \operatorname{Prob}(Y_{i} = j | VM_{id} = x) - \lim_{x \uparrow 0} \operatorname{Prob}(Y_{i} = j | VM_{id} = x),$$
(2)

for j = 1, 2. The treatment effect τ_j is the change in the probability of belonging to category j due to the treatment conditional on $VM_{id} = 0$. We can estimate τ_j using the multinomial logit model (MNL) for choice probability at the cutoff (?). The estimation of the treatment effects, bandwidth selection, and standard errors are calculated according to ?.

Table E.1 presents the results of the local MNL estimates of the first-rank effects. The estimates of the first-rank effects are consistent with those in Table 1 and Table 4. As in Table 1, first rank has no effect on running in nor on winning the same office. However, the estimated effects of first rank on advancing to higher office are still statistically significant both for running and winning, although their sizes are somewhat smaller than those in Table 4.

	Run at $t+1$	Win at $t+1$
	(1)	(2)
Same Office	-0.013	-0.011
Higher Office	(0.025) 0.036 (0.015)	(0.020) (0.029) (0.009)
Bandwidth	0.103	0.144

Table E.1. The Local MNL Estimates of First Rank Effects (Korea)

Standard errors are in parenthesis. Estimation, standard error, and bandwidths are calculated according to Xu (2017).

F Second-Rank Effect

If first rank has a positive effect on candidates' advancement to higher office, either because it sends a positive signal to party leaders or because parties incentivize top candidates by advancing them to higher office (?), second rank is likely to have similar effects on candidates. To test this, I repeat the analyses reported in Tables 1 and 4, but this time using second rank as a treatment variable. More specifically, my RD design compares close second-place and third-place winners. For this analysis, I restrict my sample to districts with a magnitude greater than two to isolate the second-rank effect from that of incumbency. As such, under the identifying assumptions of the RD design, close second- and third-place candidates are of similar quality, and they are both winners of an election in an MMD.

Table F.1 presents the results. The three panels in the table show the results for the Korean, Spanish, and Japanese cases. For the dependent variables of running for and winning higher office, the sample is restricted to the Korean case as before. As in Table 1, I find that second rank does not increase candidates' probabilities of running in and winning the same office in any of the three cases: none of the RD estimates is statistically significant. In contrast, second-place winners are 4.6 percent more likely to run for and 3.7 percent more likely to win higher office. The effects of second rank on running for and winning higher office in the Korean case are very similar to those of first rank shown in Table 4: 4.6 and 3.8 percentage points, respectively. However, the RD estimates of the second-rank effects are only marginally significant (with *p*-values of 0.072 and 0.073), possibly due to the smaller sample size and larger standard errors.⁹

The results in this section provide evidence that candidates' ranks in the previous election impact the probability of their advancing to higher office, but this has no effect on running in nor on winning the same office.

 $^{^{9}\}mathrm{As}$ I mentioned earlier, the sample size is smaller because the districts with a magnitude of less than three are excluded.

	Same Office		Highe	er Office
	Estimate	Bandwidth	Estimate	Bandwidth
	(1)	(2)	(3)	(4)
Panel A: Korea				
Run at $t + 1$	-0.016 (0.049)	0.071 [N= 1,468]	$0.046 \\ (0.026)$	0.079 [N= 1,548]
Dep. Var. Mean Dep. Var. St. Dev.	$\begin{array}{c} 0.610\\ 0.488\end{array}$		$0.099 \\ 0.299$	
Win at $t + 1$	$0.082 \\ (0.057)$	0.055 [N= 1,242]	$\begin{array}{c} 0.037 \ (0.021) \end{array}$	0.078 [N= 1,542]
Dep. Var. Mean Dep. Var. St. Dev.	$0.390 \\ 0.488$		$0.047 \\ 0.211$	
Panel B: Spain				
Run at $t+1$	-0.057 (0.058)	0.032 [N= 940]		
Dep. Var. Mean Dep. Var. St. Dev.	$\begin{array}{c} 0.373 \\ 0.484 \end{array}$			
Win at $t + 1$	-0.016 (0.054)	0.036 [N= 962]		
Dep. Var. Mean Dep. Var. St. Dev.	$0.298 \\ 0.458$			
Panel C: Japan				
Run at $t+1$	-0.002 (0.020)	0.045 [N= 2,880]		
Dep. Var. Mean Dep. Var. St. Dev.	$0.915 \\ 0.278$			
Win at $t+1$	-0.043 (0.034)	0.044 [N= 2,824]		
Dep. Var. Mean Dep. Var. St. Dev.	$\begin{array}{c} 0.695 \\ 0.460 \end{array}$			

Table F.1. The Rank Effect on Running in and Winning the Subsequent Election, Second vs. Third-Place Incmbents

Standard errors in parentheses are calculated according to Calonico, Cattaneo and Titiunik (2014). The number of observations are in brackets. Estimates are from local linear regressions with triangular kernel using the Calonico, Cattaneo and Titiunik (2014) (CCT) optimal bandwidth.