# Online Appendix

**Endogenous Sources of Compliance with Territorial Agreements**

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This supplemental appendix presents the formal proofs for the equilibria described in the paper. Proposition 1 covers the game with investors, while Proposition 2 covers the game with co-ethnics. Proposition 3 addresses the possibility of an equilibrium in which low cost types of state A sign a treaty.

PROPOSITION 1. If $\rho < \frac{r_f - r_d}{r_f - r_c}$, there exists a perfect Bayesian equilibrium to the game with investors that takes the following form:

S-1. In period 1, state A agrees to a treaty if and only if $a \geq \hat{a}$, and state B agrees to a treaty if and only if $b \geq \hat{b} = 0$.

S-2. If a treaty is signed, the investors invest $m^T$, and if no treaty is signed, they invest $m^{NT}$, with $m^T > m^{NT}$.

S-3. After the shock is realized, state A initiates a crisis if $s > a + \alpha m^T \lambda$ after a treaty has been concluded and if $s > a + \alpha m^{NT} \lambda$ in the absence. Sate B initiates a crisis if and only if $s < -b$ regardless of whether or not a treaty was signed.

On the equilibrium path, beliefs follow Bayes rule. If $\hat{a} = 0$, then investors believe that $a = 0$, in the event that no treaty is signed.

PROOF. The strategies described in S-3 follow from discussions in the text, so we begin with the investors' decision in S-2. To do so, we need to characterize the investors' beliefs about the probability of a crisis with or without a treaty. To establish some preliminary results, first assume that the level of foreign investment, *m*, does not depend on whether or not there is a treaty. Given the states' decision rule in the second period, the probability of a crisis given the investment decision, *m*, and the states' costs of conflict, *a* and *b*, is given by:

$$\Pr(\text{Crisis} \mid m, a, b) = \omega(m, a, b) = 1 - F(a + \alpha m \lambda) + F(-b).$$

Notice that, all else equal, the probability of a crisis is decreasing in *m*. The signing of a treaty means that both *a* and *b* exceed their relevant thresholds, $\hat{a}$ and $\hat{b}$, respectively. It follows that the probability of a crisis following a treaty and allocation *m* is given by

$$\Omega^T(m) = \iint_{\substack{a>\hat{a}, \\ b>\hat{b}}} \omega(t,u,m) g_a(t) g_b(u) dt du$$
$$= \int_{\hat{a}}^{1-p_1} \left[1 - F(t + \alpha m \lambda)\right] g_a^T(t) dt + \int_{\hat{b}}^{p_1} F(-u) g_b^T(u) du \qquad \text{A.1}$$

where $g_a^T(t) = \dfrac{g_a(t)}{1 - G_a(\hat{a})}$ and $g_b^T(t) = \dfrac{g_b(t)}{1 - G_b(\hat{b})}$ denote the posterior probability distributions of *a* and *b*, respectively, when truncated from below. Note that the upper bounds on the integrals arise because the shocks are bounded, so the highest cost types will never initiate a crisis.

The expected probability of a crisis in the absence of a treaty is more complicated, since the investors do not observe the individual states' preferences for or against a treaty, only whether or not a treaty is signed. There are three conditions that can bring about no treaty: state A wanted a treaty but state B did not ($a \geq \hat{a}$ and $b < \hat{b}$), state B wanted a treaty by state A did not ($a < \hat{a}$ and $b < \hat{b}$), and neither wanted a treaty ($a < \hat{a}$ and $b < \hat{b}$). Let $\Omega(m) = \iint \omega(m,t,u) g_a(t) g_b(u) dt du$ denote the ex ante probability of a crisis—that is, before the treaty decision is made – and let $\sigma = [1 - G_a(\hat{a})][1 - G_b(\hat{b})]$ denote the ex ante probability that both states would want a treaty. Then

$$\Omega(m) = \sigma \Omega^T(m) + (1-\sigma)\Omega^{NT}(m), \text{ so}$$

$$\Omega^{NT}(m) = \frac{\Omega(m) - \sigma \Omega^T(m)}{1-\sigma}.$$

Because $\Omega(m) > \Omega^T(m)$, it follows immediately that $\Omega^{NT}(m) > \Omega^T(m)$ for all *m*: that is, for any given *m*, the probability of a crisis is higher if there is no treaty.

While this is true for any $\hat{b}$, the math becomes simpler when we accept the conjecture in S-1 that $\hat{b}=0$. In this case, the probability that B will start a crisis, with or without a treaty, is $\rho=\int_0^{p_1} F(-u)g_b(u)du$. Then

$$\Omega^T(m)=\rho+\int_{\hat{a}}^{1-p_1}\left[1-F(t+\alpha m\lambda)\right]\frac{g_a(t)}{1-G_a(\hat{a})}dt \text{, and}$$

$$\Omega^{NT}(m)=\rho+\int_0^{\hat{a}}\left[1-F(t+\alpha m\lambda)\right]\frac{g_a(t)}{G_a(\hat{a})}dt$$

Since $F$ is increasing in $t$, the fact the ranges of the integrals do not overlap ensures that $\Omega^{NT}(m)>\Omega^T(m)$ for all $m$.

Each individual investor makes its own investment decision, taking into account the decisions of all other investors and the states' incentives. Let $m^\tau$, with $\tau\in[T,NT]$, denote the optimal level of foreign investment with and without a treaty, respectively. Now consider a generic investor who invests $m$. Then $M=\frac{(N-1)m^\tau+m}{N}$ denotes the total amount of foreign investment if all of the investors but one allocate the optimal amount, and remaining individual allocates $m$. Thus, assuming all other investors invest the optimal amount, each individual's expected utility from investing $m$, conditional on $\tau$, is

$$EU_i(m\mid\tau)=\Omega^\tau(M^\tau)v\left((1-m)r_d+mr_c\right)+\left[1-\Omega^\tau(M^\tau)\right]v\left((1-m)r_d+mr_f\right). \qquad \text{A.2}$$

The first term captures the investor's utility in the event of a crisis, while the second term captures the utility in the event of peace. Notice that the probability of a second-period crisis depends on the total investment amount, $M^\tau$, while the individual investor's utility depends on its individual choice, $m$. In equilibrium, of course, each individual invests the same amount, so the share of resources invested by any one, $m^{\tau}$ , equals the share of total resources invested overseas, $M^{\tau}$ . Taking the derivative with respect to *m*, we find

$$\frac{\partial EU_i(m)}{\partial m} = \left(r_f - r_d\right)\left[1 - \Omega(M^{\tau})\right]v'\left((1-m)r_d + mr_f\right) - \left(r_d - r_c\right)\Omega(M^{\tau})v'\left((1-m)r_d + mr_c\right) - \Omega'(M^{\tau})\frac{1}{N}\left[v\left((1-m)r_d + mr_f\right) - v\left((1-m)r_d + mr_c\right)\right]. \quad \text{A.3}$$

The first two terms in this expression are familiar and represent the marginal benefit and marginal cost of additional foreign investment, respectively. The third term arises because the investor's decision also influences the probability of a crisis: the more is invested overseas, the lower the probability of a crisis, and the higher expected return. Because the probability of a crisis is decreasing in investment, $\Omega'(M^{\tau}) < 0$, making the sign on that third term positive. This term complicates the maximization problem, but it shrinks to zero as the number of investors gets large since, holding the total pot of investment fixed at one, that no single investor can move the probability of conflict on its own. Setting A.3 to zero and taking the limit as *N* goes to infinity, the first-order condition becomes

$$\left(r_f - r_d\right)\left[1 - \Omega^{\tau}(m^{\tau})\right]v'\left((1-m^{\tau})r_d + m^{\tau}r_f\right) - \left(r_d - r_c\right)\Omega^{\tau}(m^{\tau})v'\left((1-m^{\tau})r_d + m^{\tau}r_c\right) = 0, \quad \text{A.4}$$

where we have now replaced *m* with $m^{\tau}$ in order to solve for the optimal allocation.

This expression does not permit a closed-form solution, and it may have more than one solution. We can show, however, that for any level of investment that is optimal without a treaty, there exists a higher level of investment that is optimal with a treaty. Let $m^{NT}$ denote a solution to A.4 in the absence of a treaty, so the probability of a crisis is $\Omega^{NT}(m^{NT})$. Now assume there is a treaty, so the probability a crisis is given by $\Omega^{T}$, and evaluate the expression at $m^{NT}$. Because, $\Omega^{NT}(m) > \Omega^{T}(m)$ for all *m*, the first term, which captures the marginal benefit of investment, would go up, and the second term, which represents the marginal cost, would go down, making the derivative with respect to *m* positive at that point. Put another way, in the presence of a treaty, the marginal benefit of additional investment exceeds the marginal cost at $m = m^{NT}$ . Hence, there exists an optimal allocation in the presence of the treaty such that $m^{T} > m^{NT}$ , as conjectured in S-2.

Finally, we turn to the treaty signing strategies in S-1. Recall that decisions to sign or not are simultaneous, and a treaty is only concluded if both agree. This means that, at this stage, each state only needs to consider if it is beneficial to agree to a treaty if the other side is a type that would also agree; if the other state is a type that would not agree, then the state's own choice is irrelevant.

For both states, the expected utility of signing a treaty, or not, is a function of the (as yet unrealized) shock. The shock can fall into one of three ranges: if the shock is sufficiently large and positive, so that $s > a + \alpha m^{\tau} \lambda$, then state A will want to initiate a crisis; if the shock is sufficiently large and negative, so that $s < -b$ , then state B will want to initiate a crisis; in between, neither state will have an incentive to start a crisis. Finally, in the event of a crisis, the expected value of the outcome must be conditioned on the fact that the shock was above or below the relevant threshold. For example, the expected territorial division that will result from a crisis initiated by state A is $p_1 + E(s \mid s > a + \alpha m \lambda)$.

With these preliminaries in mind, we first consider state B's incentives. Given that state A is a type that is willing to sign a treaty (i.e., $a > \hat{a}$), B's expected utility as a function of $\tau$ is given by

$$EU_B(\tau) = F(-b)\left[1 - p_1 - E(s \mid s < -b) - b\right] + \left[\int_{\hat{a}}^{\infty} F(u + \alpha m^{\tau}\lambda) g_a(u)du - F(-b)\right](1 - p_1) + \int_{\hat{a}}^{\infty}\left[1 - F(u + \alpha m^{\tau}\lambda)\right]\left[1 - p_1 - E(s \mid s > u + \alpha m^{\tau}\lambda) - b\right] g_a(u)du$$

where the terms correspond to the expected probability and payoff from a low, medium, and high shock, respectively.

Since signing a treaty induces higher investment, it also increase the size of the shock that will cause state A to initiate a crisis. With a treaty, state A will initiate if $s > a + \alpha m^T \lambda$; without a treaty, A will initiate if $s > a + \alpha m^{NT}\lambda$. Since $m^T > m^{NT}$, we can identify three kinds of shocks: (1) for $s < a + \alpha m^{NT}\lambda$, there is no difference to B between having a treaty or not; (2) for $a + \alpha m^T \lambda > s > a + \alpha m^{NT}\lambda$, B would get the status quo with a treaty but would lose territory and pay the costs of conflict without a treaty; and (3) for $s > a + \alpha m^{NT}\lambda$, B will lose territory and pays the costs of conflict with or without a treaty. Since a treaty has no effect in cases (1) and (3), the net expected benefit to B of signing a treaty is determined by the second case:

$$EU_B(T) - EU_B(NT) = \int_{\hat{a}}^{\infty}\left[F(u + \alpha m^T\lambda) - F(u + \alpha m^{NT}\lambda)\right]\left[E(s \mid u + \alpha m^T\lambda > s > u + \alpha m^{NT}\lambda) + b\right] g_a(u)du \,. \qquad \text{A.5}$$

It is easy to see that this expression must be positive for all *b*. Thus, even though investment has no direct effect on its payoffs, state B strictly benefits from a treaty because the additional foreign investment increases state A's costs of challenging the border. Since all types of state B prefer to sign a treaty, $\hat{b} = 0$.

Sate A's expected utility as a function of whether or not there is a treaty is given by

$$EU_A(\tau) = \alpha(1-m^\tau)r_d + \left[1-F(a+\alpha m^\tau\lambda)\right]\left[p_1 + E(s \mid s > a+\alpha m^\tau\lambda) - a + \alpha m^\tau r_c\right] + \left[F(a+\alpha m^\tau\lambda)-\rho\right]\left(p_1+\alpha m^\tau r_f\right) + \int_0^\infty F(-u)\left[p_1 + E(s \mid s < -u) - a + \alpha m^\tau r_c\right] g_b(u)du \qquad \text{A.6}$$

with $\tau \in \{T, NT\}$. The first term of this expression is the payoff from domestic investment, which does not depend on the second period outcome, while the remaining three terms correspond to high, medium, and low (i.e., negative) values of the shock, respectively.

*Lemma 1*. If $\rho < \frac{r_f - r_d}{r_f - r_c}$, A strictly prefer to sign a treaty if $a \geq 1 - p_1$.

*Proof of Lemma 1.* Because the shock, *s*, cannot exceed $1-p_1$, $F(a+\alpha m^T\lambda) = F(a+\alpha m^{NT}\lambda) = 1$ when $a > 1 - p_1 - \alpha m^{NT}\lambda$. Substituting into A.6 yields

$$EU_A(\tau | a > 1-p_1-\alpha m^{NT}\lambda) = \alpha(1-m^\tau)r_d + (1-\rho)\left(q+\alpha m^\tau r_f\right) + \int_0^{p_1} F(-u)\left[p_1 + E(s \mid s < -u) - a + \alpha m^\tau r_c\right] g_b(u)dt$$

The difference in expected utility between signing and not signing a treaty is

$$EU_A^T - EU_A^{NT} \left| (a > 1-p_1-\alpha m^{NT}\lambda) = \alpha(m^T - m^{NT})\left[\rho r_c + (1-\rho)r_f - r_d\right]\right. .$$

Because $m^T > m^{NT}$, this expression is positive if $\rho < \frac{r_f - r_d}{r_f - r_c}$.

*Lemma 2*. $\Delta = EU_A(T) - EU_A(NT)$ is increasing in *a* for all $a \leq 1 - p_1 - \alpha m^{NT}\lambda$.

*Proof of Lemma 2.* Proving this lemma requires taking the derivative of A.6 with respect to *a*. It will help to rewrite A.6 dropping terms that are constant in *a* and making explicit terms that vary in *a*, or:

$$* = F(a+\alpha m^\tau\lambda)\left(\alpha m^\tau\lambda\right) - \left[1-F(a+\alpha m^\tau\lambda)\right]a - a\rho + \int_{a+\alpha m^\tau\lambda}^{1-p_1} tf(t)dt$$

Taking the derivative of this expression with respect to a yields[1]

$$\frac{\partial EU_A(\tau)}{\partial a} = -\left[1 - F(a + \alpha m^{\tau}\lambda) + \rho\right].$$

Then

$$\frac{\partial \Delta}{\partial a} = \frac{\partial EU_A(T)}{\partial a} - \frac{\partial EU_A(NT)}{\partial a} = F(a + \alpha m^{T}\lambda) - F(a + \alpha m^{NT}\lambda).$$

The fact that the fact that $m^T > m^{NT}$ implies that $\partial\Delta/\partial a > 0$ for all $a \leq 1 - p_1 - \alpha m^{NT}\lambda$.

Collectively, these lemmas imply that, under the specified conditions, there exists some level of costs, $\hat{a}$, such that A prefers to sign a treaty if and only if $a \geq \hat{a}$, as conjectured in S-1. Note that, without further assumptions, we cannot rule out that $\Delta > 0$ for all *a*, in which case $\hat{a} = 0$. Under these conditions, the absence of a treaty is off the equilibrium path, and we can assign any reasonable belief in the event no treaty is signed. Since the lowest type of A has the lowest net benefit from signing a treaty, a sensible belief in the event no treaty is signed is that $a = 0$.

[1] Yes, it's miraculous that the derivative of A.6 reduces to this. I have a truly marvelous demonstration of this proposition which the margin is too narrow to contain.

PROPOSITION 2. If $d > \tilde{d}$ (defined below), there exists a perfect Bayesian equilibrium to the game with co-ethnics of the following form:

S-4. In period 1, state B always offers to sign a treaty, and state A agrees to sign if and only if $a \geq \hat{a}$, with $\hat{a} > 0$.

S-5. If a treaty is signed, the individuals located at $l_i > \hat{l}^T$ move to state A, and if a treaty is not signed, the individuals located at $l_i > \hat{l}^{NT}$ move to state A, with $\hat{l}^{NT} > \hat{l}^T$. Put in terms of the fraction of co-ethnics who move, $m^T > m^{NT}$.

S-6. After the shock is realized, state A initiates a crisis if and only if $s > \hat{s}^\tau$ with $\tau \in [T, NT]$ and $\hat{s}^T > \hat{s}^{NT}$. State B initiates a crisis if and only if $s < -b$.

On the equilibrium path, beliefs follow Bayes rule.

PROOF. First consider the conditions under which state A will initiate a crisis in the second period. In the text, we showed that A will initiate a crisis if the shock, *s*, satisfies:

$$s + \alpha \left( \frac{\min(\hat{l}^\tau - p_1, s)}{1 - p_1} \right) V > a . \qquad \text{A.7}$$

If $a > \left(\hat{l} - p_1\right)\left(1 + \frac{\alpha V}{1 - p_1}\right)$, then state A will only initiate a crisis if $s > \hat{l} - p_1$, and we can solve for $\hat{s}^\tau$ as follows:

$$\hat{s}^\tau = a - \alpha V \left( \frac{\hat{l}^\tau - p_1}{1 - p_1} \right) = a - \alpha V \left(1 - m^\tau\right). \qquad \text{A.8}$$

Notice that $s > \hat{l}^\tau - p_1$ implies that $p_2 > \hat{l}^\tau$, which means that all remaining co-ethnics will be redeemed in the event of a crisis. When *a* is not in this high range, then there are shocks such that state A will want to initiate a crisis, even though it will not thereby recover all the remaining co-ethnics. In this case, $\hat{s}^{\tau} < \hat{l}^{\tau} - p_1$ and

$$\hat{s}^{\tau} = a\left(1+\frac{\alpha V}{1-p_1}\right)^{-1}. \qquad \text{A.9}$$

Notice that, in both cases, $\hat{s}^{\tau}$ is increasing in *a*. It will be useful to define

$\overline{a}^{\tau} = \left(\hat{l} - p_1\right)\left(1+\frac{\alpha V}{1-p_1}\right)$ as the threshold on costs that defines these different cases.

*Lemma 3.* If $l^T < l^{NT}$, then, $\hat{s}^T \geq \hat{s}^{NT}$, and this inequality holds strictly for *a* sufficiently high.

*Proof of Lemma 3.* If $l^T < l^{NT}$, the foregoing logic implies that there are three ranges of *a*. In the lowest range, $a < \overline{a}^T$, and $\hat{s}^T = \hat{s}^{NT} = a\left(1+\frac{\alpha V}{1-p_1}\right)^{-1}$. In this highest range of *a*, $a > \overline{a}^{NT}$, which means that $\hat{s}^T = a - \alpha V\left(1-m^T\right)$ and $\hat{s}^{NT} = a - \alpha V\left(1-m^{NT}\right)$. Since $l^T < l^{NT}$ implies $m^T > m^{NT}$, $\hat{s}^T > \hat{s}^{NT}$. Finally, in the middle range, $\overline{a}^{NT} > a > \overline{a}^T$, which means that $\hat{s}^{NT} = a\left(1+\frac{\alpha V}{1-p_1}\right)^{-1}$ and $\hat{s}^T = a - \alpha V\left(\frac{\hat{l}^T - p_1}{1-p_1}\right)$. It is easy to show that $a > \overline{a}^T$ implies $\hat{s}^T > \hat{s}^{NT}$. Therefore, $\hat{s}^T \geq \hat{s}^{NT}$ for all *a*, and the inequality holds strictly for $a > \overline{a}^T$.

We now turn to co-ethnics' migration decision conditional on whether or not there was a treaty. Because there is a mass of individuals who do not act in a coordinated manner, each individual's decision takes the others as given and has no effect on the probability of being redeemed. In particular, we posit that the co-ethnics plays a cutpoint strategy such that, given treaty decision $\tau$, any individual *i* at location $l_i$ will migrate if and only if $l_i > \hat{l}^{\tau}$. We then show that, given this strategy, no individual has an incentive to deviate.

An individual at location $l_i$ will be recovered if (a) state A wants to initiate a crisis and (b) the share of territory it would get from the crisis is at least $l_i$. Let $\theta^\tau(l_i,a)$ denote the probability that state A with costs *a* will, in the second period, acquire additional territory that includes the location $l_i$. It follows that

$$\begin{aligned}\theta^\tau(l_i,a) &= \Pr(s > \hat{s}^\tau \cap s > l_i - p_1) \\ &= 1 - F\left[\max(\hat{s}^\tau, l_i - p_1)\right]\end{aligned}. \qquad \text{A.10}$$

Clearly, $\theta$ is weakly decreasing in both $l_i$ and *a*: the further you are from the border or the higher state A's costs of conflict are, the less likely you are to be redeemed. Now let $\Theta^\tau(l_i)$ denote the expected probability that State A will take territorial containing $l_i$ conditional on its treaty signing decision. Given the treaty selection strategy in S-4, this means

$$\Theta^T(l_i) = E\left(\theta^T(l_i,a)\middle| a > \hat{a}\right) = 1 - \int_{\hat{a}}^{\infty} F\left(\max(\hat{s}^T(t), l_i - p_1)\right)\frac{g_a(t)}{1-G_a(\hat{a})}dt \text{, and}$$

$$\Theta^{NT}(l_i) = E\left(\theta^{NT}(l_i,a)\middle| a \le \hat{a}\right) = 1 - \int_{0}^{\hat{a}} F\left(\max(\hat{s}^{NT}(t), l_i - p_1)\right)\frac{g_a(t)}{G_a(\hat{a})}dt \text{ .}$$

Although these expressions are complicated, the fact that $\hat{s}^\tau$ is increasing in *a* and $\hat{s}^T \ge \hat{s}^{NT}$ means that $\Theta^{NT}(l_i) > \Theta^T(l_i)$ -- that is, at any given distance, the probability of being redeemed is lower if state A signed a treaty than if it did not. Moreover, both probabilities are weakly decreasing in $l_i$, which means that, individuals farther from the border have a lower probability of being redeemed than those closer (holding treaty status constant).

For each individual, moving gives a certain payoff of $V - d$, while staying put gives payoff of *V* with probability $\Theta^\tau(l_i)$ and zero otherwise. Hence an individual located at $l_i$ will move if

$$\Theta^{\tau}(l_i) < \frac{V-d}{V}. \quad \text{A.11}$$

Since $\Theta^{\tau}(l_i)$ is decreasing in distance, this condition implies that there will be some cutpoint, $\hat{l}^{\tau}$, such that individuals for which $l_i > \hat{l}$ will move, and those closer to the status quo border will remain in place, as conjectured. Moreover, $\Theta^{NT}(l_i) > \Theta^{T}(l_i)$ implies that $l^T < l^{NT}$, completing the proof of S-5).

Finally, we turn to the treaty signing decisions. As in Proposition 1, it is easy to show that state B always prefers to sign a treaty if doing so increases migration. Although state B's payoffs are not directly affected by the migration decision, higher flows reduce the incentives for state A to contest the border, decreasing the chances of losing territory and paying the costs of conflict. Thus, $m^T > m^{NT}$ is sufficient to ensure that state B is always willing to sign.

What remains is to show that State A's treaty signing decision in S-4 is optimal given the other strategies. As in Proposition 1, let $\Delta = EU_A(T) - EU_A(NT)$ denote the difference in expected utility between signing a treaty and not as a function of *a*. To prove the existence of a cutpoint, $\hat{a}$, such that state A prefers a treaty if and only if $a > \hat{a}$, we need to show three things: $\Delta > 0$ for the highest cost types of state A, $\Delta < 0$ for the lowest costs types of state A, and $\Delta$ is (weakly) increasing in *a* for all types in between. The first point is established in the following lemma:

*Lemma 4.* Types of state A for which $a > 1 - p_1 + \alpha V \equiv \tilde{a}$ always prefer to sign a treaty.

*Proof of Lemma 4.* Even if all co-ethnics remain in state B, there is no chance that state A will initiate a crisis if $a > 1 - p_1 + \alpha V$. For these highest cost types,

$$EU_A(\tau \,|\, a > \tilde{a}) = \alpha m^\tau (V-d) + (1-\rho) p_1 + \int_0^{p_1} F(-u)\left[p_1 + E(s \mid s < -u) - a\right] g_b(u)dt \,,$$

where $\rho$ is defined, as above, as the ex ante probability that state B will initiate a crisis. It follows immediately that, because $m^T > m^{NT}$

$$EU_A(T \,|\, a > \tilde{a}) - EU_A(NT \,|\, a > \tilde{a}) = \alpha(V-d)(m^T - m^{NT}) > 0 \,.$$

Intuitively, if state A's costs of conflict are so high that it will never contest the status quo border, then the government strictly benefits from signing a treaty and thereby signaling to its co-ethnics that they should move.

For the remainder of the proof, it is useful to start by considering the expected value for starting a crisis, conditional on the shock exceeding the relevant threshold. For each shock, *s*, state A would acquire $p_2 = p_1 + s$ in territory and recovers a share of co-ethnics equal to $\frac{\min(\hat{l}^\tau - p_1, s)}{1-p_1}$. As we saw earlier, high costs types (i.e., those for which $a > \overline{a}^\tau$) will only initiate a crisis if $p_2 > \hat{l}^\tau$ or $s > \hat{l}^\tau - p_1$, in which case it always recovers all remaining co-ethnics (i.e., $1-m^\tau$) conditional on a crisis. Lower cost types will initiate a crisis even if, in doing so, they may not recover all remaining co-ethnics. Letting $c_A^\tau$ denote state A's expected utility from a crisis given that $s > \hat{s}^\tau$ as a function of *a*,

$$c_A{}^\tau = p_1 + E(s \mid s > \hat{s}^\tau) - a + \alpha V(1-m^\tau) \text{ if } a > \overline{a}^\tau \text{, and}$$

$$\begin{aligned} c_A{}^\tau &= p_1 + E(s \mid s > \hat{s}^\tau) - a + \alpha V \left\{ \int_{\hat{s}^\tau}^{\hat{l}^\tau - p_1} \frac{t}{1-p_1} \frac{f(t)}{1-F(\hat{s}^\tau)} dt + \int_{\hat{l}^\tau - p_1}^{1-p_1} \left(1-m^\tau\right) \frac{f(t)}{1-F(\hat{s}^\tau)} dt \right\} \\ &= p_1 + E(s \mid s > \hat{s}^\tau) - a + \frac{\alpha V}{1-F(\hat{s}^\tau)} \left\{ \int_{\hat{s}^\tau}^{\hat{l}^\tau - p_1} \frac{t}{1-p_1} f(t)dt + \left[1 - F(\hat{l}^\tau - p_1)\right]\left(1-m^\tau\right) \right\} \end{aligned} \text{ otherwise.}$$

It follows that

$$EU_A(\tau) = \alpha m^\tau (V-d) + \left[1 - F(\hat{s}^\tau)\right] c_A^\tau + \left[F(\hat{s}^\tau) - \rho\right] p_1 + \int_0^{p_1} F(-u)\left[p_1 + E(s \mid s < -u) - a\right] g_b(u) dt .$$

In this expression, the first term is the payoff from the co-ethnics who moved, the second term represents the payoff from a crisis started by state A, the third term represents the status quo payoff, and the final term captures the expected payoff from a crisis started by B. The net expected value of a treaty is then

$$\Delta = \alpha(V-d)(m^T - m^{NT}) + \left[F(\hat{s}^T) - F(\hat{s}^{NT})\right] p_1 + \left[1 - F(\hat{s}^T)\right] c_A^T - \left[1 - F(\hat{s}^{NT})\right] c_A^{NT} .$$

Note that, unlike in the game with investors, this difference does not hinge on the likelihood and expected outcome of a crisis started by B, as these are the same with or without a treaty.

Since this value depends on how a compares to the thresholds $\bar{a}^\tau$ , we need to consider three cases, corresponding to high, medium, and low values of *a*.

Case 1: $a > \bar{a}^{NT} > \bar{a}^T$

In this case,

$$\begin{aligned}
\Delta = {} & \alpha(V-d)(m^T - m^{NT}) + \left[F(\hat{s}^T) - F(\hat{s}^{NT})\right] p_1 + \\
& \left[1 - F(\hat{s}^T)\right]\left[p_1 + E(s \mid s > \hat{s}^T) - a + \alpha V\left(1 - m^T\right)\right] - \\
& \left[1 - F(\hat{s}^{NT})\right]\left[p_1 + E(s \mid s > \hat{s}^{NT}) - a + \alpha V\left(1 - m^{NT}\right)\right] \\
= {} & \alpha(V-d)(m^T - m^{NT}) + \left[1 - F(\hat{s}^T)\right]\left[\alpha V\left(1 - m^T\right) - a\right] - \left[1 - F(\hat{s}^{NT})\right]\left[\alpha V\left(1 - m^{NT}\right) - a\right] - \int_{\hat{s}^{NT}}^{\hat{s}^T} tf(t) dt
\end{aligned}$$

Taking the derivative with respect to *a*,

$$
\begin{aligned}
\frac{\partial \Delta}{\partial a} &= -\left[1-F(\hat{s}^T)\right] - f(\hat{s}^T)\frac{\partial \hat{s}^T}{\partial a}\left[\alpha V\left(1-m^T\right)-a\right]+ \\
&\quad \left[1-F(\hat{s}^{NT})\right] + f(\hat{s}^{NT})\frac{\partial \hat{s}^{NT}}{\partial a}\left[\alpha V\left(1-m^{NT}\right)-a\right]+ \\
&\quad -\hat{s}^T f(\hat{s}^T)\frac{\partial \hat{s}^T}{\partial a} + \hat{s}^{NT} f(\hat{s}^{NT})\frac{\partial \hat{s}^{NT}}{\partial a} \\
&= F(\hat{s}^T) - F(\hat{s}^{NT})
\end{aligned}
$$

where the second step relies on the fact that $\hat{s}^\tau = a - \alpha V\left(1-m^\tau\right)$ (see expression A.8). Since $\hat{s}^T > \hat{s}^{NT}$, $\partial\Delta/\partial a > 0$ for all $a$ in this range.

Case 2: $\overline{a}^{NT} > a > \overline{a}^T$

In this range,

$$
\begin{aligned}
\Delta &= \alpha(V-d)(m^T - m^{NT}) + \left[F(\hat{s}^T) - F(\hat{s}^{NT})\right]p_1 + \\
&\quad \left[1-F(\hat{s}^T)\right]\left[p_1 + E(s \mid s > \hat{s}^T) - a + \alpha V\left(1-m^T\right)\right] - \\
&\quad \left[1-F(\hat{s}^{NT})\right]\left\{p_1 + E(s \mid s > \hat{s}^{NT}) - a + \frac{\alpha V}{1-F(\hat{s}^{NT})}\left[\int_{\hat{s}^{NT}}^{\hat{l}^{NT}-p_1} \frac{t}{1-p_1} f(t)dt + \left[1-F(\hat{l}^{NT}-p_1)\right]\left(1-m^{NT}\right)\right]\right\} \\
&= \alpha(V-d)(m^T - m^{NT}) + \left[1-F(\hat{s}^T)\right]\left[\alpha V\left(1-m^T\right)-a\right] + \left[1-F(\hat{s}^{NT})\right]a - \int_{\hat{s}^{NT}}^{\hat{s}^T} tf(t)dt - \\
&\quad \alpha V\left[\int_{\hat{s}^{NT}}^{\hat{l}^{NT}-p_1} \frac{t}{1-p_1} f(t)dt + \left[1-F(\hat{l}^{NT}-p_1)\right]\left(1-m^{NT}\right)\right]
\end{aligned}
$$

Taking the derivative with respect to *a*, we find

$$
\begin{aligned}
\frac{\partial \Delta}{\partial a} &= \left[F(\hat{s}^T) - F(\hat{s}^{NT})\right] + \left[\hat{s}^{NT}\left(1+\frac{\alpha V}{1-p_1}\right) - a\right] f(\hat{s}^{NT})\frac{\partial \hat{s}^{NT}}{\partial a}. \\
&= F(\hat{s}^T) - F(\hat{s}^{NT})
\end{aligned}
$$

where the cancellation in the second step follows from expression A.9, which means that the expression in large brackets reduces to zero. Once again, this derivative must be positive for all $a$ in this range.

Case 3: $a < \bar{a}^T$

Recall from A.9 that, for these types, $\hat{s}^T = \hat{s}^{NT} = \hat{s}$, which means that the size of shock needed to initiate a crisis does not depend on whether or not a treaty had been signed. All that a treaty does is change the number of individuals who move. It follows that

$$\Delta = \alpha(V-d)(m^T - m^{NT}) + \left[1 - F(\hat{s})\right]\left(c_A^T - c_A^{NT}\right), \qquad \text{A.12}$$

where

$$c_A^T - c_A^{NT} = \frac{\alpha V}{1-F(\hat{s})}\left\{\int_{\hat{s}}^{\hat{l}^T - p_1} \frac{t}{1-p_1} f(t)dt + \left[1 - F(\hat{l}^T - p_1)\right]\left(1 - m^T\right) - \int_{\hat{s}}^{\hat{l}^{NT} - p_1} \frac{t}{1-p_1} f(t)dt - \left[1 - F(\hat{l}^{NT} - p_1)\right]\left(1 - m^{NT}\right)\right\}.$$

Noting that

$$\int_{\hat{s}}^{\hat{l}^{NT} - p_1} \frac{t}{1-p_1} f(t)dt = \int_{\hat{l}^T - p_1}^{\hat{l}^{NT} - p_1} \frac{t}{1-p_1} f(t)dt + \int_{\hat{s}}^{\hat{l}^T - p_1} \frac{t}{1-p_1} f(t)dt, \text{ and}$$

$$1 - F(\hat{l}^T - p_1) = \left[1 - F(\hat{l}^{NT} - p_1)\right] + \left[F(\hat{l}^{NT} - p_1) - F(\hat{l}^T - p_1)\right],$$

this becomes

$$c_A^T - c_A^{NT} = \frac{\alpha V}{1-F(\hat{s})}\left\{-\int_{\hat{l}^T - p_1}^{\hat{l}^{NT} - p_1} \frac{t}{1-p_1} f(t)dt - \left[1 - F(\hat{l}^{NT} - p_1)\right]\left(m^T - m^{NT}\right) + \left[F(\hat{l}^{NT} - p_1) - F(\hat{l}^T - p_1)\right]\left(1 - m^{NT}\right)\right\}.$$

Though this expression is complicated, we know that the second term in the curved brackets is negative. Therefore, the entire expression in curved brackets is negative if

$$\int_{\hat{l}^T - p_1}^{\hat{l}^{NT} - p_1} \frac{t}{1-p_1} f(t)dt \geq \left[F(\hat{l}^{NT} - p_1) - F(\hat{l}^T - p_1)\right]\left(1 - m^T\right).$$

Rewriting the right-hand side, recognizing that $1-m^T = \dfrac{\hat{l}^T - p_1}{1-p_1}$, we find

$$\int_{\hat{l}^T - p_1}^{\hat{l}^{NT} - p_1} \frac{t}{1-p_1} f(t)dt \geq \int_{\hat{l}^T - p_1}^{\hat{l}^{NT} - p_1} (1-m^T) f(t)dt$$
$$\geq \int_{\hat{l}^T - p_1}^{\hat{l}^{NT} - p_1} \frac{\hat{l}^T - p_1}{1-p_1} f(t)dt$$

which must be true given the lower bound of the integral. All this is to show that $c_A^T - c_A^{NT} < 0$. This makes sense: all other things equal, conflict is less attractive if a treaty has been signed because there are fewer co-ethnics to redeem through conflict.

We can then conclude two things from these expressions. First, once $\left(c_A^T - c_A^{NT}\right)$ is multiplied by $\left[1-F(\hat{s})\right]$, the latter cancels, and none of the remaining terms in A.12 are a function of *a*, so $\dfrac{\partial \Delta}{\partial a} = 0$. Hence, for these lowest cost types, the difference having a treaty and not having a treaty is constant for all *a*. Whether or not these types prefer a treaty to no treaty cannot be determined without further assumptions. However, because the $c_A^T - c_A^{NT} < 0$, we know that as *d* approaches *V*, the net benefit of a treaty must become negative, since the positive first term shrinks. Let $\tilde{d}$ denote the value of *d* such that $\Delta = 0$ for *a* in this range.[2] It follows that $\Delta < 0$ if $d > \tilde{d}$. Note that it is possible that $\tilde{d} = 0$, in which case this restriction is true by assumption.

Thus, the highest cost types (for which $a > \tilde{a}$) strictly prefer a treaty (by Lemma 4), the lowest costs types (for which $a < \overline{a}^T$) strictly prefer not to sign a treaty as long as $d > \tilde{d}$, and in

[2] Formally, $\tilde{d} = V - \dfrac{\left[1-F(\hat{s})\right]\left(c_A^{NT} - c_A^T\right)}{\alpha(m^T - m^{NT})}$.

between the net expected benefit of a treaty is strictly increasing in *a*. This implies that there exists some $\hat{a} > 0$ such that state A prefers a treaty if and only if $a > \hat{a}$, completing the proof of S-4.

PROPOSITION 3. In any equilibrium in which A offers to sign a treaty if and only if $a \le \hat{a}$, B never offers to sign a treaty.

PROOF. Consider first an equilibrium in which B offers to sign if $b \le \hat{b}$. In this case, the conclusion of a treaty signals that both states have low costs of conflict. As a result, the probability of a crisis is higher in the presence of a treaty than without. In the game with investors, the logic of Proposition 1 means that investment is lower in the presence of a treaty; in the game with co-ethnics, beliefs about B's costs are irrelevant to the migration decision, but flipping the signal associated with A's willingness to sign means that more people will move without a treaty than with. In both cases, then, $m^T < m^{NT}$. However, both propositions showed that state B is better off when flows are higher, because higher flows reduce the likelihood that state A will find it profitable to initiate a crisis. For example, in the model with investors, for shocks such that $a + \alpha m^{NT}\lambda > s > a + \alpha m^T \lambda$, the status quo will be preserved in the absence of a treaty, while state B will lose territory and pay the costs of conflict with a treaty. Hence, the net expected benefit to B of signing a treaty (analogous to A.5) becomes

$$EU_B(T) - EU_B(NT) = \int_0^{\hat{a}} \left[ F(u + \alpha m^{NT}\lambda) - F(u + \alpha m^T \lambda) \right] \left[ -E(s \mid u + \alpha m^{NT}\lambda > s > u + \alpha m^T \lambda) - b \right] g_a(u)du ,$$

which is negative for all *b*. Therefore, no type of state B will agree to sign a treaty.

Now consider an equilibrium in which B offers to sign if $b \ge \hat{b}$. In this case, a treaty signals that A has low costs of conflict, but B has high costs. In the game with co-ethnics, B's type is irrelevant to the migration decision, so the fact that a treaty signals A's low costs ensures that $m^T < m^{NT}$. By the foregoing logic, no type of B wants to conclude a treaty under those circumstances, because B benefits from higher migration. In the game with investors, there are two possibilities. One possibility is that, even with B's revelation, the risks of a second period conflict are high enough with a treaty that $m^T < m^{NT}$. In that case, the foregoing logic implies once again that no type of B will sign. The second possibility is that the net effect of the treaty is to encourage more investment, in which case $m^T > m^{NT}$. But we saw in Proposition 1 that $m^T > m^{NT}$ is sufficient to ensure that high costs types of A have greater expected benefit from a treaty than do low cost types. That is, $m^T > m^{NT}$ is a sufficient condition for the claims in Lemmas 1 and 2, which ensures a cutpoint strategy in which high costs types sign a treaty. In this case, then, the strategy for A conjectured in the statement of the proposition cannot hold in equilibrium. Therefore, in any equilibrium in which state A plays the conjectured strategy, it must be the case that $m^T < m^{NT}$, and state B has no incentive to agree to a treaty.