Online Appendix

Limited Foresight and Gridlock in Bargaining

A Omitted Proofs for Section 3

Proof. (of Proposition 0) The case of $T = \infty$ is shown in the main text. We thus focus on the case of finite T and proceed by induction on T. The base case is solved in the main text. Suppose the statement of the theorem holds for $T-1$, and consider a decision-maker with foresight horizon T. Given threshold e^*_{T-1} for policy revision with foresight horizon $T-1$, recall the definition

$$\tilde{t}_{T-1} := \min\left\{t : \prod_{i=1}^{t} \tilde{\beta}_i < e^*_{T-1}\right\}$$

from the main text. Note that if the decision-maker revises the policy in the current period, her expected future surplus (and hence utility, since she always institutes her favorite policy) is

$$S_{T-1} = \mathbb{E}[\sum_{t=0}^{\tilde{t}_{T-1}-1} \delta^t \prod i = 1^t \tilde{\beta}_i] + \mathbb{E}[\delta^{\tilde{t}_{T-1}}] S_{T-2}.$$

The decision-maker will thus replace the status quo only if (if)

$$e + \delta S_{T-1} \leq (<) S_{T-1} \qquad \Longrightarrow \qquad e \leq (<)(1-\delta) S_{T-1}.$$

Limiting Case e^*_∞. Finally, I show that $\lim_{T\to\infty} e^*_T := e^*_\infty = 1$. We begin by defining

$$N(x) := \mathbb{E}\left[\sum_{t\in\mathbb{N}:\prod_{i=0}^{t}\tilde{\beta}_i > x} \delta^t \left(\prod_{i=1}^{t} \tilde{\beta}_i\right)\right], \tag{12}$$

$$d(x) := \mathbb{E}\left[\delta^{\min\{t\in\mathbb{N}:\prod_{i=1}^{t}\tilde{\beta}_i \leq x\}}\right]. \tag{13}$$

$N(x)$ is expected efficiency life of a policy revised only once $e \leq x$ and $d(x)$ is the expected δ-power of the time of revision. These will be helpful objects in our analysis. By definition,

$$\begin{aligned} e^*_T &= (1-\delta) S_{T-1} \\ &= (1-\delta)\left\{N(e^{T-1}_*) + d(e^{T-1}_*)N(e^{T-2}_*) + d(e^{T-2}_*)d(e^{T-1}_*)N(e^{T-3}_*) + \dots\right\}, \end{aligned} \tag{14}$$

in terms of the functions $N(\cdot)$ and $d(\cdot)$. Since $\tilde{\beta}$ has a non-atomic distribution, both are continuous in x and therefore also uniformly continuous on the domain $[0,1]$. Thus, for any $\varepsilon > 0$, $\exists \varphi_1, \varphi_2$ such that

$$|x-y| < \varphi_1 \implies |N(x) - N(y)| < \frac{\varepsilon}{2}, \quad |x-y| < \varphi_2 \implies |d(x) - d(y)| < \frac{\varepsilon}{2}.$$

Letting $\varphi := \max\{\varphi_1, \varphi_2\}$ gives us

$$|x-y| < \varphi \implies |N(x) - N(y)| < \varepsilon, \quad |d(x) - d(y)| < \varepsilon. \tag{15}$$

Consider arbitrary $\varepsilon \in (0, 2\delta)$ and the corresponding φ. From Proposition 2(a), the sequence $\{e^*_T\}$ is monotonically increasing and bounded from above (by 1) and so (Cauchy) converges. Hence, there exists T_φ such that $\forall\ m, n > T_\varphi$, $|e^m_* - e^n_*| < \varphi$. Consider the expression Equation 14 for $2T_\varphi$:

$$e^{2T_\varphi}_* = (1-\delta)\left\{N\left(e^{2T_\varphi - 1}_*\right) + d\left(e^{2T_\varphi - 1}_*\right) N\left(e^{2T_\varphi - 2}_*\right) + d\left(e^{2T_\varphi - 1}_*\right) \cdot d\left(e^{2T_\varphi - 2}_*\right) N\left(e^{2T_\varphi - 3}_*\right) + \dots\right\}.$$

Hence,
$$\begin{aligned}\left|e^{2T_\varphi}_* - (1-\delta) N\left(e*^{2T_\varphi}\right) \sum_{i=0}^{2T_\varphi} d\left(e^{2T_\varphi}_*\right)\right| &\le (1-\delta)\left(\sum_{i=1}^{T_\varphi}\left(\frac{\varepsilon}{2}\right)^i + \frac{\delta^{T_\varphi}}{1-\delta}\right), \text{ by using Equation 15} \\ &\le (1-\delta) \cdot \frac{\varepsilon}{2-\varepsilon} + \delta^{T_\varphi} \\ &\le \frac{\varepsilon}{2} + \delta^{T_\varphi}, \text{ since } \frac{\varepsilon}{2} < \delta.\end{aligned}$$

Now, since the statement holds for all T_φ sufficiently large, we can simply choose $T_\varepsilon > T_\varphi$ in the above analysis so that $\delta^{T_\varepsilon} < \frac{\varepsilon}{2}$. Since $\delta < 1$, such a T_ε exists. The above shows that for any $\varepsilon > 0$, there exists $T_\varepsilon > 0$ such that for all $T > T_\varepsilon$,

$$\left|e^{2T_\varepsilon}_* - (1-\delta) N\left(e*^{2T_\varepsilon}\right) \sum_{i=0}^{2T_\varepsilon} d\left(e^{2T_\varepsilon}_*\right)\right| \le \varepsilon.$$

Hence, in the limit as $T \to \infty$,

$$\begin{aligned}\lim_{T\to\infty} e^*_T = e^*_\infty &= (1-\delta) N\left(e^*_\infty\right) \frac{1}{1 - d\left(e^*_\infty\right)} \\ &= (1-\delta)\mathbb{E}\left[\sum_{t\in\mathbb{N}:\prod_{i=1}^t \tilde{\beta}_i > e^*_\infty} \delta^t \left(\prod_{i=1}^t \tilde{\beta}_i\right)\right] \frac{1}{1 - \mathbb{E}\left[\delta^{\min\{t\in\mathbb{N}:\prod_{i=1}^t \tilde{\beta}_i \le e^*_\infty\}}\right]},\end{aligned} \tag{16}$$

substituting in the definitions Equation 12 and Equation 13. Let β^* (possibly equal to 1) be the supremum of the support of the random variable $\tilde{\beta}$. First, note that $e^*_\infty = 1$ solves the fixed point equation Equation 16 since the right-hand side is 1 for any $e^*_\infty \in [\beta^*, 1]$. If $\beta^* < 1$, this also rules out any $e^*_\infty \in [\beta^*, 1)$ as a solution. Furthermore, for any sequence $(x_i)_{i=0}^\infty$, with $x_i \in (e^*_\infty, 1)$,

$$(1-\delta)\frac{1+\delta x_1+\delta^2 x_2+\ldots+\delta^t x_t}{1-\delta^{t+1}} = \frac{1+\delta x_1+\delta^2 x_2+\ldots+\delta^t x_t}{1+\delta+\ldots+\delta^t} > \max_t x_t > e^*_\infty,$$

for any $t \in \mathbb{N}$. Hence, for any realization of $\{\tilde{\beta}_i\}$ such that $\min\{t \in \mathbb{N} : \prod_{i=1}^t \tilde{\beta}_i \leq e^*_\infty\} > 1$, it follows that

$$(1-\delta)\left[\sum_{t\in\mathbb{N}:\prod_{i=1}^t \tilde{\beta}_i > e^*_\infty} \delta^t \left(\prod_{i=1}^t \tilde{\beta}_i\right)\right] \frac{1}{1-\delta^{\min\{t\in\mathbb{N}:\prod_{i=1}^t \tilde{\beta}_i \leq e^*_\infty\}}} > e^*_\infty.$$

Otherwise, it is equal to 1. Noting that for any $e^*_\infty < \beta^*$ there will be delay with positive probability before agreement, it follows that the right-hand side of Equation 16 is greater than e^*_∞ for any such e^*_∞ and thus cannot solve Equation 16. Hence, $e^*_\infty = 1$ is the unique solution to Equation 16, completing the proof. □

Proposition 1. *Consider the general two-party bargaining game of the baseline model. For any $T \in \mathbb{N}$, an equilibrium exists. Moreover, in any equilibrium, agreement occurs at the state $((e, \hat{a}), T)$ only if $e \leq e^*_T$ (and if $e < e^*_T$).*

Proof. We begin with a helpful lemma, which establishes that whenever agreement occurs in equilibrium, it must occur with either bargaining party as the incumbent.

Lemma A.1. *For given equilibrium strategies, define*

$$A^\tau(\hat{p}) := A^\tau_L(\hat{p}) \cap A^\tau_R(\hat{p}).$$

If outcome (e, a) comes into place at $(\hat{p}, \tau)$ for some incumbent on the equilibrium path, then $(e, a) \in A^\tau(\hat{p})$. Moreover, if $A^\tau(\hat{p}) \neq \emptyset$, then some $(1, a) \in A^\tau(\hat{p})$ comes into place in equilibrium for any incumbent.

Proof. If $p^\tau_I(\hat{p}) \notin A^\tau(\hat{p})$, then either

(1) $p^\tau_I(\hat{p}) \notin A^\tau_{-I}(\hat{p})$, in which case, the proposal is rejected by definition of A_{-I}.

(2) $p^\tau_I(\hat{p}) \notin A^\tau_I(\hat{p})$. Either the proposal is rejected, as in (1), or, if accepted, yields an inferior continuation value for I than maintaining the status-quo and moving to the next period. The latter follows since $A^\tau_I(\hat{p})$ maximizes utility in equilibrium and incumbency is not history-dependent.

On the other hand, if $\exists\ (1,a) \in A^\tau(\hat{p})$, then if proposed, it will be accepted and implemented. By (2), the proposer prefers to implement such an a rather than maintain the status-quo; moreover, any proposal $p_I \notin A^\tau_{-I}(\hat{p})$ will be rejected. Hence, in equilibrium, it must be that some $(1,a) \in A^\tau(\hat{p})$ comes into place. $\square$

The main proof is by induction on the foresight horizon T. Let $V_i^T(e,\hat{a}|\sigma)$ be the continuation value of agent i after new status quo $(e,\hat{a})$ has materialized and prior to the selection of the incumbent, given that agents play profile σ and have foresight horizon T. We first consider $T=1$. With status quo $(e,\hat{a})$, the condition for both agents to agree to a given proposal $(1,a^1)$ in equilibrium is

$$u_{x_i}(1,a^1)\cdot S_0 \geq u_{x_i}(e,\hat{a}) + \delta\mathbb{E}[V_i^1(\tilde{\beta}_1 e,\hat{a}|\sigma)] \tag{17}$$

for each $i \in \{L,R\}$, given that future play is described by profile σ. Noting the form of u_{x_i}, a^1 is in the agreement set of the respective parties if

$$a^1 \geq \frac{e\hat{a}}{S_0} + \frac{\delta\mathbb{E}\left[V_R^1(\tilde{\beta}_1 e,\hat{a}|\sigma)\right]}{S_0} \tag{R}$$

$$a^1 \leq \frac{e\hat{a}}{S_0} + \frac{\delta\mathbb{E}\left[V_R^1(\tilde{\beta}_1 e,\hat{a}|\sigma)\right]}{S_0} + 1 - \frac{\left(\delta S^1(\tilde{\beta}_1 e,\hat{a}|\sigma)+e\right)}{S_0}, \tag{L}$$

where $S^1(e',\hat{a}|\sigma) := \sum_{i\in\{1,2\}} V_i^1(e',\hat{a}|\sigma)$ is the expected total surplus to both bargaining agents when they play profile σ, with status quo e'. By Lemma A.1, if σ is an equilibrium profile, it must dictate (dis)agreement at $(e,\hat{a})$ regardless of the identity of the incumbent. Thus, agreement occurs in equilibrium only if (R) and (L) are satisfied (and if (R) and (L) are satisfied with strict inequality). Thus, from (R) and (L), there exists $\hat{a} \in A^T(e,\hat{a})$ if and only if

$$\frac{e\hat{a}}{S_0} + \frac{\delta\mathbb{E}\left[V_R^1(\tilde{\beta}_1 e,\hat{a}|\sigma)\right]}{S_0} \leq 1, \qquad \text{and} \qquad 1 - \frac{\left(\delta S^1(\tilde{\beta}_1 e,\hat{a}|\sigma)+e\right)}{S_0} \geq 0. \tag{18}$$

Consider the case $e < e_1^* = (1-\delta)S_0$. We know $\mathbb{E}[V_R^1(\tilde{\beta}_1 e,\hat{a}|\sigma)] \leq S^1(\tilde{\beta}_1 e,\hat{a}|\sigma) \leq S_0$, where the final inequality results from previous arguments made in the single decision-maker case.[13] By direct inspection, both conditions in Equation 18 are thus satisfied whenever $e < (1-\delta)S_0$. Hence, agreement occurs in any equilibrium profile σ if $e < e_1^*$.

On the other hand, consider $e > e_1^*$. Under any σ, if agents disagree at $(e,\hat{a})$, there is from above a path

[13] Surplus is maximized when $e < (1-\delta)S_0$ by renewing the policy, which yields expected surplus equal to S_0.

of future play in which agreement occurs after $t \geq 1$ periods. In this case,

$$\begin{aligned} e + \delta \mathbb{E}\left[S^1(\tilde{\beta}_1 e, \hat{a} | \sigma, t)\right] &= e + \mathbb{E}\left[\sum_{j=1}^{t-1} \delta^j \left(\prod_{i=1}^{j} \tilde{\beta}_i\right) e \,\middle|\, \sigma, t\right] + \frac{\delta^t}{1 - \delta\overline{\overline{\beta}}} \\ &= \sum_{j=0}^{t-1} \delta^j \mathbb{E}\left[e \cdot \prod_{i=1}^{j} \tilde{\beta}_i \,\middle|\, \sigma, t\right] + \frac{\delta^t}{1 - \delta\overline{\overline{\beta}}} \\ &> \frac{1-\delta}{1-\delta\overline{\overline{\beta}}}\left(\frac{1-\delta^t}{1-\delta}\right) + \frac{\delta^t}{1-\delta\overline{\overline{\beta}}} = \frac{1}{1-\delta\overline{\overline{\beta}}}. \end{aligned} \tag{19}$$

Here, the final inequality results from noting that each element in the sum of Equation 19 must have $\mathbb{E}\left[e \cdot \prod_{i=1}^{j} \tilde{\beta}_i \middle| \sigma, t\right] > e^* = \frac{1-\delta}{1-\delta\overline{\overline{\beta}}}$, since at any future period j $(j < t)$ for which $e \cdot \prod_{i=1}^{j} \tilde{\beta}_i \leq e^*$, we have from above that there must be agreement in period j, which would contradict the assumption of t periods of disagreement. The final inequality shows that for any $e > e_1^*$, the second condition in Equation 18 is violated. So, agreement occurs under any equilibrium σ only if $e \leq e_1^*$, as desired.

Finally, for existence of equilibrium, note that R makes proposal $p_R^1(e, \hat{a}) = \max\left\{a^1 \in A^1(e, \hat{a})\right\}$, and L proposes $p_L^1(e, \hat{a}) = \min\left\{a^1 \in A^1(e, \hat{a})\right\}$, whenever $e < e_1^*$. The opposition always accepts these. When $e > e_1^*$, any acceptable proposal from $A_I^1(e, \hat{a})$ being made by incumbent I and being rejected by opposition O constitutes an equilibrium profile. At $e = e_1^*$, the incumbent and opposition are indifferent between having the (uniquely) acceptable proposal implemented or rejected, so any profile is admissible in equilibrium.

We now suppose the statement is true for $\tau = T - 1$ and consider the behavior of agents with foresight horizon T. By inductive hypothesis and definition of S_{T-1}, starting at the period in which the next agreement is passed, the remaining expected surplus in the game remains constant at S_{T-1} for any equilibrium strategy profile σ. The proof of the proposition for any $T > 1$ is thus analogous[14] to the base case, substituting S_{T-1} for $S_0 = \frac{1}{1-\delta\overline{\overline{\beta}}}$, $S^T(\cdot)$ for $S^1(\cdot)$, and $V_i^T(\cdot)$ for $V_i^1(\cdot)$. □

Example A.2 (Two Negotiators with Constant Recognition Probabilities). *I now consider a simple example, corresponding to Figure 3, of a two-agent bargaining game consistent with the model of Section 2, where each negotiator has* $T = 1$ *and a constant probability* $\frac{1}{2}$ *of being recognized as the proposer in each period. Without loss of generality, suppose* L *makes a proposal* $a_L^1(e, \hat{a})$ *to* R *from status quo* $(e, \hat{a})$*. In order for the proposal to be accepted, it must be that*

$$a_L^1(e, \hat{a}) \cdot S_0 \geq e \cdot \hat{a} + \delta \mathbb{E}\left[\frac{1}{2} a_L^1(\tilde{\beta}_1 e, \hat{a}) + \frac{1}{2} a_R^1(\tilde{\beta}_1 e, \hat{a})\right] \cdot S_0, \tag{20}$$

[14]The unique diference in the $T > 1$ case is that we, in general, have $p_i^1(e, \hat{a}) \in \arg\max V_i^T(1, \hat{a})$, since the choice of agreement may affect the utility downstream for general $\pi(\cdot)$.

where $a_I^1(\tilde{\beta}_1 e, \hat{a})$ *are the proposals made by* $I \in \{L, R\}$ *in the next period if the stochastic decay shock at the start of the next period is* $\tilde{\beta}_1$*. Similarly, L will only advance such a proposal if it generates more utility than simply waiting until the next period—*

$$(1 - a_L^1(e, \hat{a})) \cdot S_0 \geq e \cdot (1 - \hat{a}) + \delta \left(1 - \mathbb{E}\left[\frac{1}{2} a_L^1(\tilde{\beta}_1 e, \hat{a}) + \frac{1}{2} a_R^1(\tilde{\beta}_1 e, \hat{a})\right]\right) \cdot S_0 \tag{21}$$

Adding these two inequalities together allows us to see that there exists a beneficial agreement $a_L^1(e, \hat{a})$ *in the present for both agents if and only if*

$$S_0 \geq e + \delta S_0 \quad \iff \quad e \leq (1 - \delta) S_0,$$

as noted by Proposition 1. On the other hand, for any $e \in [0, 1]$, *Equation 20 with equality defines the least acceptable proposal to R—* $\alpha_R^1(e, \hat{a})$*—- at* $(e, \hat{a})$ *with* $T = 1$*. Of course,* $a_L^1(e, \hat{a}) = \alpha_R^1(e, \hat{a})$ *for any* $e < e_1^* = (1 - \delta) S_0$*. Similarly, taking Equation 21 with equality defines the greatest acceptable proposal—* $\alpha_L^1(e, \hat{a})$*— to L. Furthermore, consider* $\hat{a} = \frac{1}{2}$*, in which case the environment is exactly symmetric for both parties. From any e, therefore, parties' least acceptable proposals will be symmetric about* $\frac{1}{2}$*. In particular, we solve for* $\alpha_R^1(e, \hat{a})$ *as*

$$\alpha_R^1(e, \hat{a}) \cdot S_0 = e \cdot \hat{a} + \frac{1}{2} \cdot S_0$$
$$\implies \alpha_R^1(e, \hat{a}) = \frac{e(1 - \delta\overline{\beta}) + \delta}{2}.$$

Here, I substitute $\frac{1}{2}$ *for* $\mathbb{E}\left[\frac{1}{2} a_L^1(\tilde{\beta}_1 e, \hat{a}) + \frac{1}{2} a_R^1(\tilde{\beta}_1 e, \hat{a})\right]$ *in Equation 20, given the symmetric environment, and simplify. By analogous calculation, from Equation 21,*

$$\alpha_L^1(e, \hat{a}) = 1 - \frac{e(1 - \delta\overline{\beta}) + \delta}{2}.$$

Substituting for $\overline{\beta} = .5$*,* $\delta = .8$*, and for* $e \in \{.9, .7, .3\}$ *now allows us to reproduce Figure 3.*

Proposition 2. *In any equilibrium of the game,*

(a) for $T' < T$ *and any set of other fixed parameters,* $e_{T'}^* < e_T^*$

(b) for any $T \in \mathbb{N}$*,* e_T^* *is strictly decreasing in* δ

Proof. (*of Proposition 2*) (a) From Proposition 1, for every T,

$$e_T^* = (1 - \delta) S_{T-1} = (1 - \delta) \left(\sum_{k=1}^{T} \mathbb{E}\left[\delta^{\sum_{i=1}^{k-1} \tilde{t}_{T-i}} \tilde{\eta}^{T-k}\right]\right) = (1 - \delta) \left(\mathbb{E}\left[\tilde{\eta}^{T-1}\right] + \mathbb{E}\left[\delta^{\tilde{t}_{T-1}} S_{T-2}\right]\right)$$

$$
\begin{aligned}
&> (1-\delta)\mathbb{E}\left[\sum_{i=0}^{\tilde{t}_{T-1}} \delta^i \cdot e^*_{T-1} + \delta^{\tilde{t}_{T-1}} S_{T-2}\right] \\
&= (1-\delta)S_{T-2}\mathbb{E}\left[\sum_{i=0}^{\tilde{t}_{T-1}} \delta^i \cdot (1-\delta) + \delta^{\tilde{t}_{T-1}}\right] \\
&= (1-\delta)S_{T-2} = e^*_{T-1},
\end{aligned}
$$

where the final equality results because $\sum_{i=0}^{\tilde{t}_{T-1}} \delta^i \cdot (1-\delta) + \delta^{\tilde{t}_{T-1}} = 1$ for any integer realization $\tilde{t}_{T-1}$.

(b) For convenience, let $\tilde{\beta}_0 = 1$. We consider first e^*_1. It is clear that

$$
\frac{\partial e^*_1}{\partial \delta} = -\frac{1-\overline{\beta}}{(1-\delta\overline{\beta})^2} < 0.
$$

Next, note that for each $T \geq 2$,

$$
\frac{\partial e^*_T}{\partial \delta} = \frac{\partial \left\{(1-\delta)S_{T-1}\right\}}{\partial \delta} = -S_{T-1} + (1-\delta)\frac{\partial}{\partial \delta}S_{T-1}. \tag{22}
$$

The following auxiliary claim is thus useful in completing the proof—

Lemma A.3. *For each $T \in \mathbb{N}$,*

$$
\frac{\partial}{\partial \delta}S_T = \frac{\prod_{\tau=1}^{T} d(e^*_\tau) \cdot \overline{\beta}}{(1-\delta\overline{\beta})^2}. \tag{23}
$$

Proof of Lemma A.3. We proceed by induction on T. First, note that using definitions Equation 12 and Equation 13, we can write

$$
S_{T-1} = N(e^*_{T-1}) + d(e^*_{T-1})S_{T-2}.
$$

Thus, for the base case $T = 1$,

$$
\begin{aligned}
\frac{\partial}{\partial \delta}S_1 &= \frac{\partial}{\partial \delta}\left(N(e^*_1) + d(e^*_1)\cdot\frac{1}{1-\delta\overline{\beta}}\right) \\
&= N'(e^*_1)\cdot\frac{\partial}{\partial \delta}e^*_1(\delta) + d'(e^*_1)\cdot\frac{\partial}{\partial \delta}e^*_1(\delta)\cdot\frac{1}{1-\delta\overline{\beta}} + d(e^*_1)\cdot\frac{\partial}{\partial \delta}\frac{1}{1-\delta\overline{\beta}} \\
&= -\frac{1}{(1-\delta\overline{\beta})^2}\left[(1-\overline{\beta})\cdot\left(N'(e^*_1) + \frac{d'(e^*_1)}{1-\delta\overline{\beta}}\right) - d(e^*_1)\overline{\beta}\right]
\end{aligned} \tag{24}
$$

We evaluate by rewriting $N(e^*_1)$ and $d(e^*_1)$ in convenient forms for differentiation. For convenience, we let G_i be the cumulative distribution of the stochastic product $\prod_{j=1}^{i}\tilde{\beta}_j$ and g_i its pdf. For any x,

$$
N(x) = 1 + \delta\cdot\mathbb{P}[\tilde{\beta}_1 > x]\cdot\mathbb{E}[\tilde{\beta}_1|\tilde{\beta}_1 > x] + \delta^2\cdot\mathbb{P}[\tilde{\beta}_1 > x]\cdot\mathbb{E}[\tilde{\beta}_1|\tilde{\beta}_1 > x] + \dots
$$

$$= 1 + \delta \cdot \int_x^1 t \cdot g_1(t)\, dt + \delta^2 \cdot \int_x^1 t \cdot g_2(t)\, dt + \dots.$$

Using the Leibniz rule,

$$N'(x) = -x \cdot \sum_{i=1}^{\infty} \delta^i g_i(x). \tag{25}$$

Similarly,

$$d(x) = \delta G_1(x) + \delta^2 (G_2(x) - G_1(x)) + \delta^3 (G_3(x) - G_2(x)) + \dots$$

$$\implies d'(x) = \delta g_1(x) + \delta^2 (g_2(x) - g_1(x)) + \delta^3 (g_3(x) - g_2(x)) + \dots.$$

By directly applying the above expressions, we then obtain

$$N'(e_1^*) + \frac{d'(e_1^*)}{1 - \delta\overline{\beta}} = \sum_{i=1}^{\infty} \left[g_i(e_1^*) \left(-\delta^i e_1^* + \frac{\delta^i}{1 - \delta\overline{\beta}} - \frac{\delta^{i+1}}{1 - \delta\overline{\beta}} \right) \right] = 0,$$

since $e_1^* = \frac{1-\delta}{1-\delta\overline{\beta}}$. Therefore, substituting back into Equation 24,

$$\frac{\partial}{\partial \delta} S_1 = \frac{d(e_1^*)\overline{\beta}}{(1 - \delta\overline{\beta})^2}.$$

This shows the base case. We suppose now the claim holds true for any $\tau = 1, \dots, T-1$. We show it must hold for T.

$$\begin{aligned}
S_T &= N(e_T^*) + d(e_T^*) S_{T-1}. \\
\implies \frac{\partial}{\partial \delta} S_T &= N'(e_T^*) \cdot \frac{\partial e_T^*}{\partial \delta} + d'(e_T^*) \cdot \frac{\partial e_T^*}{\partial \delta} S_{T-1} + d(e_T^*) \cdot \frac{\partial}{\partial \delta} S_{T-1} \\
&= \frac{\partial e_T^*}{\partial \delta} \left(N'(e_T^*) + d'(e_T^*) S_{T-1} \right) + d(e_T^*) \cdot \frac{\partial}{\partial \delta} S_{T-1} \\
&= \frac{\partial e_T^*}{\partial \delta} \cdot \sum_{i=1}^{\infty} \left[g_i(e_T^*) \left(-\delta^i e_T^* + (\delta^i - \delta^{i+1}) S_{T-1} \right) \right] + d(e_T^*) \cdot \frac{\partial}{\partial \delta} S_{T-1},
\end{aligned}$$

where in the final equality we reapply the expressions for $N'(x)$ and $d'(x)$ derived above. Since $e_T^* = (1-\delta) S_{T-1}$, each summand $g_i(e_T^*) \left(-\delta^i e_T^* + (\delta^i - \delta^{i+1}) S_{T-1} \right) = 0$. Hence,

$$\begin{aligned}
\frac{\partial}{\partial \delta} S_T &= d(e_T^*) \cdot \frac{\partial}{\partial \delta} S_{T-1} \\
&= d(e_T^*) \frac{\prod_{\tau=1}^{T-1} d(e_*^\tau)\overline{\beta}}{(1 - \delta\overline{\beta})^2}, \text{ using the inductive hypothesis for } \frac{\partial}{\partial \delta} S_{T-1} \\
&= \frac{\prod_{\tau=1}^{T} d(e_*^\tau)\overline{\beta}}{(1 - \delta\overline{\beta})^2}, \text{ as desired.}
\end{aligned}$$

□

Using Lemma A.3, we can now complete the proof of Proposition 2(b) for any $T \geq 2$.

$$\frac{\partial}{\partial \delta} e_*^T = -S_{T-1} + (1-\delta)\frac{\partial}{\partial \delta} S_{T-1} = -S_{T-1} + \frac{(1-\delta)\prod_{\tau=1}^{T} d(e_*^\tau)\overline{\beta}}{(1-\delta\overline{\beta})^2} < 0,$$

where the concluding inequality results because $\frac{1-\delta}{1-\delta\overline{\beta}} < 1$ and $S_{T-1} > S_0 := \frac{1}{1-\delta\overline{\beta}}$ since from part (a), the S_T are increasing in T. This completes the proof. □

B Omitted Proofs for Section 4

Note: for ease of notation in the proofs in this Appendix, we drop σ from the terms $p^T(\hat{p}, I_T, \sigma)$ and $a^T(\hat{p}, I_t, \sigma)$ for outcomes that materialize in equilibrium given a particular status quo and incumbent.

B.1 Proofs for Section 4.2

Proposition 3. *For any $T \in \mathbb{N}$, there is an almost-everywhere outcome-unique equilibrium of the game that implements unique proposals $p_i^T(\hat{p})$ with $e_i^T(\hat{p}) = 1$ and $a_i^T(e, \hat{a})$ linear and increasing in $\hat{a}$ for $i \in \{R, L\}$ for any $\hat{p} = (e, \hat{a})$ at which agreement occurs on the equilibrium path. We write for each i,*

$$a_i^T(e, \hat{a}) = C^T(e)\hat{a} + B_i^T(e),$$

where $C^T \in (0,1)$ for $e > 0$ and $C^T(0) = 0$.

We prove the result by induction on the foresight horizon T through two lemmas— a preliminary result followed by the inductive step. The base case of $T = 1$ is in the main text. We will first show that if the main proposition holds for all $\tau \leq T-1$, that linearity in the partisan lean of policy, $\hat{a}$, also holds for several key equilibrium objects.

Let $\mathcal{H}^1 := \frac{1}{1-\delta\overline{\beta}}$ and $\mathcal{J}^1 := 0$. Supposing Proposition 3 holds for $\tau \leq T-1$, define recursively for all such τ—

$$\begin{aligned}
K^\tau(e) &= C^\tau(e) \cdot (1 - \Delta^\tau(e)), \text{ for any } e \leq e_*^\tau \\
M^\tau(e) &= B_L^\tau(e) \cdot (1 - \Delta^\tau(e)) + \left(1 - \frac{\Delta^\tau(e)}{2}\right)\Delta^\tau(e), \text{ for any } e \leq e_*^\tau \\
\mathcal{H}^{\tau+1} &= \mathbb{E}[\tilde{\eta}^\tau] + \mathbb{E}[\delta^{\tilde{t}_\tau} \cdot \mathcal{H}^\tau \cdot K^\tau(\tilde{e}^\tau)] \\
\mathcal{J}^{\tau+1} &= \mathbb{E}\left[\delta^{\tilde{t}_\tau}\left(\mathcal{H}^\tau \cdot M^\tau(\tilde{e}^\tau) + \mathcal{J}^\tau\right)\right]
\end{aligned}$$

Lemma B.1. *Suppose the statement of Proposition 3 holds for every $\tau \leq T-1$ Then, for each $\tau \leq T-1$,* *(i) If I^τ is the incumbent when p^τ is implemented, then for any $e \leq e_*^\tau$,*

$$\mathbb{E}_{I^\tau(e)}\left[a^\tau(e,\hat{a})\right] = K^\tau(e)\cdot\hat{a} + M^\tau(e)$$

(ii) Moreover,

$$W_R^{\tau+1}(a) = \mathcal{H}^{\tau+1}a + \mathcal{J}^{\tau+1}.$$

Proof. We proceed by induction on τ. By hypothesis, since $a_i^\tau(e,\hat{a})$ are linear in $\hat{a}$ with the same coefficient $C^\tau(e)$, it follows that $a_R^\tau(e,\hat{a}) - a_L^\tau(e,\hat{a}) = \Delta^\tau(e)$, which does not depend on $\hat{a}$. For $\tau = 1$, we have

$$\text{(i)} \qquad \begin{aligned}\mathbb{E}_{I_1(e)}\left[a^1((e,\hat{a}),I_1)\right] &= a_L^1(e,\hat{a}) + \left(1 - \frac{a_L^1(e,\hat{a}) + a_R^1(e,\hat{a})}{2}\right)\Delta^1(e)\\ &= a_L^1(e,\hat{a}) + \left(1 - a_L^1(e,\hat{a}) - \frac{\Delta^1(e)}{2}\right)\Delta^1(e)\\ &= (1-\Delta^1(e))\cdot a_L^1(e,\hat{a}) + \left(1 - \frac{\Delta^1(e)}{2}\right)\Delta^1(e)\\ &= \underbrace{(1-\Delta^1(e))C^1(e)}_{:=K^1(e)}\cdot a + \underbrace{(1-\Delta^1(e))B_L^1(e) + \left(1-\frac{\Delta^1(e)}{2}\right)\Delta^1(e)}_{:=M^1(e),\quad \text{by definition.}}\end{aligned}$$

$$\text{(ii)} \qquad \begin{aligned}W_R^2(a) &= a\cdot\mathbb{E}[\tilde{\eta}^1] + \mathbb{E}\left[\delta^{\tilde{t}_1}a^1((\tilde{e}^1,\hat{a}),I_1)\right]\cdot\frac{1}{1-\delta\overline{\beta}}\\ &= a\underbrace{\left(\mathbb{E}[\tilde{\eta}^1] + \mathbb{E}[\delta^{\tilde{t}_1}\mathcal{H}^1K^1(\tilde{e}^1)]\right)}_{:=\mathcal{H}^2} + \underbrace{\mathbb{E}\left[\delta^{\tilde{t}_1}\mathcal{H}^1M^1(\tilde{e}^1)\right]}_{:=\mathcal{J}^2,\quad\text{by definition.}}\end{aligned}$$

Now suppose (i) and (ii) hold for all $\tau < \hat{T} \leq T-1$. First, we show that (i) holds for $\hat{T}$. Note by inductive hypothesis that $W_R^{\hat{T}}$ is increasing. Thus, parties will push their proposals to the extrema of the acceptability set $A^{\hat{T}}(e,\hat{a})$ in equilibrium. We then have

$$\begin{aligned}\mathbb{E}_{I^{\hat{T}}(e)}\left[a^{\hat{T}}((e,\hat{a}),I_{\hat{T}})\right] &= a_L^{\hat{T}}(e,\hat{a}) + \left(1 - \frac{a_L^{\hat{T}}(e,\hat{a}) + a_R^{\hat{T}}(e,\hat{a})}{2}\right)\Delta^{\hat{T}}(e)\\ &= a_L^{\hat{T}}(e,\hat{a}) + \left(1 - a_L^{\hat{T}}(e,\hat{a}) - \frac{\Delta^{\hat{T}}(e)}{2}\right)\Delta^{\hat{T}}(e),\\ &= \left(1-\Delta^{\hat{T}}(e)\right)a_L^{\hat{T}}(e,\hat{a}) + \left(1-\frac{\Delta^{\hat{T}}(e)}{2}\right)\Delta^{\hat{T}}(e)\\ &= \left(1-\Delta^{\hat{T}}(e)\right)C^{\hat{T}}(e)\cdot\hat{a}\\ &\qquad + \left(1-\Delta^{\hat{T}}(e)\right)B_L^{\hat{T}}(e) + \left(1-\frac{\Delta^{\hat{T}}(e)}{2}\right)\Delta^{\hat{T}}(e).\end{aligned}$$

Noting the definition of $K^{\hat{T}}$ and $M^{\hat{T}}$, we see that (i) holds for $\hat{T}$.

$$
\begin{aligned}
\text{And, for (ii),}\quad W_R^{\hat{T}+1}(a) &= a \cdot \mathbb{E}\left[\tilde{\eta}^{\hat{T}}\right] + \mathbb{E}\left[\delta^{\tilde{t}_{\hat{T}}} W_R^{\hat{T}}\left(a^{\hat{T}}((\tilde{e}^{\hat{T}}, a), I_{\hat{T}})\right)\right] \\
&= a \cdot \mathbb{E}\left[\tilde{\eta}^{\hat{T}}\right] + \mathbb{E}\left[\delta^{\tilde{t}_{\hat{T}}} \left\{\mathcal{H}^{\hat{T}}\left(K^{\hat{T}}(\tilde{e}^{\hat{T}}) \cdot a + M^{\hat{T}}(\tilde{e}^{\hat{T}})\right) + \mathcal{J}^{\hat{T}}\right\}\right], \\
&\quad \text{by composing linear functions} \\
&= a \cdot \underbrace{\left(\mathbb{E}\left[\tilde{\eta}^{\hat{T}}\right] + \mathbb{E}\left[\delta^{\tilde{t}_{\hat{T}}} \cdot \mathcal{H}^{\hat{T}} \cdot K^{\hat{T}}(\tilde{e}^{\hat{T}})\right]\right)}_{:= \mathcal{H}^{\hat{T}+1}} + \underbrace{\mathbb{E}\left[\delta^{\tilde{t}_{\hat{T}}}\left(\mathcal{H}^{\hat{T}} \cdot M^{\hat{T}}(\tilde{e}^{\hat{T}}) + \mathcal{J}^{\hat{T}}\right)\right]}_{:= \mathcal{J}^{\hat{T}+1}}, \quad \text{by definition.}
\end{aligned}
$$

This completes the inductive step and thus the proof. □

Lemma B.2 (Inductive Step). *Take any $T \geq 2$. Suppose the statement of Proposition 3 holds for every $\tau \leq T-1$. Then, the statement also holds for T.*

Proof. Given the inductive hypothesis, there is an outcome-unique equilibrium in cut-off strategies for foresight horizon of length $T-1$. Let σ be the profile played therein. As before, we continue to write $\tilde{\eta}^{\tau}$ to be the efficiency life of the agreement which is in place while τ future agreements remain on the foresight horizon, and let $\tilde{t}_{\tau}$ be the number of periods this agreement is in place. It remains to be shown that the proposal functions with a T foresight horizon are linear and unique in any equilibrium.

The argument is analogous to the proof of the base case in the main text. Consider $e \leq (1-\delta)S_{T-1}$. As in Equation 17, a new agreement a is acceptable to both players at $\hat{p} = (e, \hat{a})$ if and only if

$$W_i^T(a) \geq u_{x_i}(\hat{p}) + \delta \mathbb{E}[V_i^T(\tilde{\beta}_1 e, \hat{a}|\sigma)].$$

Let $y_R^T(e, \hat{a})$ and $y_L^T(e, \hat{a})$ be the minimal (maximal) policies accepted by R and L, respectively whenever $e \leq (1-\delta)S_{T-1}$. We have then

$$W_R^T(y_R^T(e, \hat{a})) = e\hat{a} + \delta \mathbb{E}[V_R^T(\tilde{\beta}_1 e, \hat{a}|\sigma)] \qquad W_L(y_L^T(e, \hat{a})) = e(1-\hat{a}) + \delta \mathbb{E}[V_L^T(\tilde{\beta}_1 e, \hat{a}|\sigma)] \tag{26}$$

From Proposition 1, agreement occurs in any equilibrium in the ensuing period should it not occur in the current one. Hence, $\sum_{i \in \{R,L\}} V_i^T(\tilde{\beta}_1 e, \hat{a}|\sigma) = S_{T-1}$ for any realization of $\tilde{\beta}_1$. We can thus express the equation for L in Equation 26 as

$$
\begin{aligned}
S_{T-1} - W_R^T(y_L^T(e, \hat{a})) &= e - e\hat{a} + \delta\left(S_{T-1} - \mathbb{E}\left[\hat{V}_R^T(\tilde{\beta}_1 e, \hat{a})\right]\right) \\
W_R^T(y_L^T(e, \hat{a})) &= (1-\delta)S_{T-1} - e + W_R^T(y_R^T(e, \hat{a}), \ \text{by substitution from Equation 26} \\
\implies W_R^T(y_L^T(e, \hat{a})) - W_R^T(y_R^T(e, \hat{a})) &= (1-\delta)S_{T-1} - e
\end{aligned} \tag{27}
$$

Finally, by Lemma B.1, linearity of $W_R^T(\cdot)$ allows us to rewrite the left-hand side of Equation 27 as

$$\mathcal{H}^T \cdot (y_L^T(e,\hat{a}) - y_R^T(e,\hat{a})) = (1-\delta)S_{T-1} - e.$$

By Lemma B.1 and indutive hypothesis, W_R^T is increasing so that R (L) will prefer greater (lesser) agreements. That is, $a_R^T(e,\hat{a}) = y_L^T(e,\hat{a})$ and $a_L^T(e,\hat{a}) = y_R^T(e,\hat{a})$. This immediately yields

$$\Delta^T(e) := a_R^T(e,\hat{a}) - a_L^T(e,\hat{a}) = \frac{(1-\delta)S_{T-1} - e}{\mathcal{H}^T}, \tag{28}$$

a function only of e. The previously proven structure of cut-off equilibrium with foresight horizon T (Proposition 1), then allows us to derive $a_L^T(e,\hat{a})$ as the solution to $W_R^T(a_L^T(e,\hat{a}) = e\hat{a} + \delta\mathbb{E}\left[V_R^T(\beta_1 e,\hat{a})\right]$. Using again Lemma B.1,

$$\begin{aligned}
W_R^T(a_L^T(e,\hat{a})) &= e\hat{a} + \delta\mathbb{E}\left[W_R^T(a^T((\tilde{\beta}_1 e,\hat{a}),I_1))\right], \text{ since we have a cut-off equilibrium. Thus,}\\
\mathcal{H}^T a_L^T(e,\hat{a}) + \mathcal{J}^T &= e\hat{a} + \delta\mathbb{E}\left[W_R^T(a_L^T(\tilde{\beta}_1 e,\hat{a})) + \left(1 - \frac{a_L^T(\tilde{\beta}_1 e,\hat{a}) + a_R^T(\tilde{\beta}_1 e,\hat{a})}{2}\right)\mathcal{H}^T \cdot \Delta^T(\tilde{\beta}_1 e)\right]\\
&= e\hat{a} + \delta\mathbb{E}\Bigg[\mathcal{H}^T \cdot a_L^T(\tilde{\beta}_1 e,\hat{a})\left(1 - \Delta^T(\tilde{\beta}_1 e)\right)\\
&\qquad + \left(1 - \frac{\Delta^T(\tilde{\beta}_1 e)}{2}\right)\mathcal{H}^T\Delta^T(\tilde{\beta}_1 e)\Bigg] + \delta\mathcal{J}^T.
\end{aligned}$$

Rearranging gives us

$$a_L^T(e,\hat{a}) = \frac{e\hat{a}}{\mathcal{H}^T} + \delta\mathbb{E}\left[\left(1 - \frac{\Delta^T(\tilde{\beta}_1 e)}{2}\right)\Delta^T(\tilde{\beta}_1 e)\right] - (1-\delta)\frac{\mathcal{J}^T}{\mathcal{H}^T} + \delta\mathbb{E}\left[\left(1 - \Delta^T(\tilde{\beta}_1 e)\right)a_L^T(\tilde{\beta}_1 e,\hat{a})\right]$$

Note that the above equation has an $a_L^T(\tilde{\beta}_1 e,\hat{a})$ term on the right-hand side. This gives us a recursive formulation of $a_L^T(\tilde{\beta}_1 e,\hat{a})$. Reapplying this recursive formula allows us to obtain

$$\begin{aligned}
a_L^T(e,\hat{a}) = &\underbrace{\left(\sum_{i=0}^{\infty}\delta^i \cdot \mathbb{E}\left[\prod_{j=1}^{i}\left(1 - \Delta^T\left(\prod_{k=1}^{j}\tilde{\beta}_k \cdot e\right)\right)\tilde{\beta}_j\right]\right)\frac{e}{\mathcal{H}^T}}_{C^T(e)} \cdot \hat{a} + \\
&\sum_{i=1}^{\infty}\delta^i \cdot \mathbb{E}\Bigg[\prod_{j=1}^{i-1}\left(1 - \Delta^T\left(\prod_{k=1}^{j}\tilde{\beta}_k \cdot e\right)\right) \cdot \\
&\qquad \left\{\left(1 - \frac{\Delta^T\left(\prod_{k=1}^{i}\tilde{\beta}_k \cdot e\right)}{2}\right)\Delta^T\left(\prod_{k=1}^{i}\tilde{\beta}_k \cdot e\right) - \frac{(1-\delta)\mathcal{J}^T}{\delta\mathcal{H}^T}\right\}\Bigg],
\end{aligned} \tag{29}$$

where the final term is $B_L^T(e)$. As before, $a_R^T(e,\hat{a}) = a_L^T(e,\hat{a}) + \Delta^T(e)$. Thus, both parties' equilibrium proposals are uniquely determined by Equation 29, linear, and strictly increasing in $\hat{a}$ for $e > 0$, which follows because $C^T(e) > 0$ for all $e > 0$. Moreoever, the sum in the coefficient of $C^T(e)$ is less than $\mathcal{H}^T$, by noting the recursive definitions in the prelude to Lemma B.1. Hence, $C^T(e) < 1$, completing the proof. □

Proposition 4 (Short-Run Dynamics). *When $e \leq e_T^*$, $\mathbb{P}[R\ wins]$ is decreasing in $\hat{a}$ and $\mathbb{E}[a^T(e,\hat{a})]$ is increasing in $\hat{a}$.*

Proof.
$$\mathbb{P}_t[\text{R wins}] = \mathbb{P}\left[m_t > \frac{a_L^T(e,\hat{a}) + a_R^T(e,\hat{a})}{2}\right] = 1 - \frac{a_L^T(e,\hat{a}) + a_R^T(e,\hat{a})}{2},$$

where m_t is the position of the t-period median voter. The first equality is a result of the median voter selecting the policy she myopically favors, and the final equality results from the assumption that m_t is distributed uniformly on $[0,1]$. By Proposition 3, $a_I^T(e,\hat{a})$ is increasing in $\hat{a}$ for $I \in \{L, R\}$, so that $\mathbb{P}_t$[R wins] is decreasing in $\hat{a}$. The result for $\mathbb{E}[a^T(e,\hat{a})]$ is direct from Lemma B.1, noting that the definition of $K^T(e)$ means that $K^T \geq 0$ since $C^T \geq 0$ from Proposition 3. In both cases, monotonicity is strict when $e > 0$, again from direct inspection of $K^T(e)$ and $C^T(e)$. □

Proposition 5. *Consider $T' < T$. Whenever $e \leq e_{T'}^* < e_T^*$, $\Delta^T(e) > \Delta^{T'}(e)$. Moreover, $\Delta^T(e)$ is strictly decreasing in δ.*

Proof. We prove the first part of the proposition first. To show this, note that it suffices to show $\Delta^{T+1}(e) > \Delta^T(e)$ for $e \leq e_*^T < e_{T+1}^*$. By Equation 28 (See, proof of Lemma B.2),

$$\Delta^\tau(e) = \frac{(1-\delta)S_{\tau-1} - e}{\mathcal{H}^\tau}.$$

Define

$$\varphi(e) := \Delta^{T+1}(e) - \Delta^T(e) = (1-\delta)\left(\frac{S_T}{\mathcal{H}^{T+1}} - \frac{S_{T-1}}{\mathcal{H}^T}\right) + \left(\frac{1}{\mathcal{H}^T} - \frac{1}{\mathcal{H}^{T+1}}\right)e$$

on the domain $e \in [0, (1-\delta)S_{T-1}]$. Importantly, φ is a monotonic function in e. At $e = (1-\delta)S_{T-1}$, $\Delta^T(e) = 0$ by definition, while $\Delta^{T+1}(e) = (1-\delta)\frac{(S_T - S_{T-1})}{\mathcal{H}^{T+1}} > 0$. Hence, $\varphi((1-\delta)S_{T-1}) > 0$. To show that $\varphi(0) > 0$, which therefore completes the proof, it suffices to show that $\mathcal{H}^T > \mathcal{H}^{T+1}$.

Note that $\mathcal{H}^\tau$ expresses the time-averaged effect of the current partisan lean of policy on the stream of agreements on a foresight of horizon τ. Hence, it is indeed intuitive that the more re-negotiations of policy are considered, the more dilute the current partisan-lean of policy becomes on the full stream of agreements. To see this concretely, we can compare $\mathcal{H}^T$ to $\mathcal{H}^{T+1}$ for a particular realization of efficiency shocks $\beta = \{\tilde{\beta}\}$—

$$\begin{aligned}
\mathcal{H}^T(\beta) &= 1 + \delta\tilde{\beta}_1 + \ldots + \delta^{\tilde{t}_T} K^{T-1}\left(\prod_{i=1}^{\tilde{t}_T}\tilde{\beta}_i\right)\cdot\mathcal{H}^{T-1} \\
&= 1 + \delta\tilde{\beta}_1 + \ldots + \left(1 - \Delta^{T-1}\left(\prod_{i=1}^{\tilde{t}_T}\tilde{\beta}_i\right)\right) \\
&\qquad \cdot \sum_{i=\tilde{t}_T}^{\infty}\left(\delta^i\left[\prod_{j=\tilde{t}_T+1}^{i}\left(1 - \Delta^{T-1}\left(\prod_{k=\tilde{t}_T+1}^{j}\tilde{\beta}_k\left(\prod_{i=1}^{\tilde{t}_T}\tilde{\beta}_i\right)\right)\right)\tilde{\beta}_j\right]\prod_{i=1}^{\tilde{t}_T-1}\tilde{\beta}_i\right)
\end{aligned}$$

by substituting $K^{T-1}(e) = \left(1 - \Delta^{T-1}(e)\right)C^{T-1}(e)$ from the recursive definitions preceding Lemma B.1 and further using the derivation of $C^{T-1}(e)$ from Equation 29. It then follows that

$$\begin{aligned}
\mathcal{H}^T(\beta) &> 1 + \delta\tilde{\beta}_1 + \ldots + \left(1 - \Delta^T\left(\prod_{i=1}^{\tilde{t}_T}\tilde{\beta}_i\right)\right)\sum_{i=\tilde{t}_T}^{\infty}\delta^i\left[\prod_{j=\tilde{t}_T+1}^{i}\left(1 - \Delta^T\left(\prod_{k=\tilde{t}_T+1}^{j}\tilde{\beta}_k\cdot\left(\prod_{i=1}^{\tilde{t}_T}\tilde{\beta}_i\right)\right)\right)\tilde{\beta}_j\right]\left(\prod_{i=1}^{\tilde{t}_T}\tilde{\beta}_i\right) \\
&> 1 + \delta\tilde{\beta}_1 + \ldots \\
&\quad + \left(1 - \Delta^T\left(\prod_{i=1}^{\tilde{t}_{T+1}}\tilde{\beta}_i\right)\right)\sum_{i=\tilde{t}_{T+1}}^{\infty}\delta^i\left[\prod_{j=\tilde{t}_{T+1}+1}^{i}\left(1 - \Delta^T\left(\prod_{k=\tilde{t}_{T+1}+1}^{j}\tilde{\beta}_k\cdot\left(\prod_{i=1}^{\tilde{t}_{T+1}}\tilde{\beta}_i\right)\right)\right)\tilde{\beta}_j\right]\left(\prod_{i=1}^{\tilde{t}_{T+1}}\tilde{\beta}_i\right) \\
&= \mathcal{H}^{T+1}(\beta).
\end{aligned}$$

Because $e^*_{T+1} > e^*_T$, it follows that $\tilde{t}_{T+1} \leq \tilde{t}_T$. This accounts for the second inequality. That is, with a foresight horizon of $T+1$, the first agreement arrived at in the game will be in force for a weakly shorter duration than with a foresight horizon T, given the same realization of shocks $\beta := \left\{\tilde{\beta}\right\}$. With larger initial agreement duration, by inspection, each element of the sequence $\left(\prod_{i=1}^{j}\tilde{\beta}_j\right)_{j=0}^{\infty}$ is being multiplied by a larger number in the sum above. Taking the expectation over the sequences β gives us $\mathcal{H}^T > \mathcal{H}^{T+1}$. As argued above, this directly implies $\Delta^{T+1}(e) > \Delta^T(e)$, completing the proof of the first part of the proposition.

We now show that $\Delta^T(e)$ is decreasing in δ for arbitrary T. Note that

$$\frac{\partial}{\partial\delta}\Delta^T(e) = \frac{\partial}{\partial\delta}\left[\frac{(1-\delta)S_{T-1} - e}{\mathcal{H}^T}\right] = \frac{\frac{\partial}{\partial\delta}e^*_T\cdot\mathcal{H}^T - \frac{\partial}{\partial\delta}\mathcal{H}^T\cdot(e^*_T - e)}{\left(\mathcal{H}^T\right)^2}. \tag{30}$$

From Proposition 2, we have shown that $\frac{\partial}{\partial\delta}e^*_T < 0$, so that in order for the expression in Equation 30 to be negative, we need only show $\frac{\partial}{\partial\delta}\mathcal{H}^T > 0$. To complete the proof, we demonstrate this auxiliary claim inductively on T. For the base case $T = 1$,

$$\frac{\partial}{\partial \delta}\mathcal{H}^1 = \frac{\partial}{\partial \delta}S_0 = \frac{\overline{\beta}}{(1-\delta\overline{\beta})^2} > 0.$$

Suppose $\frac{\partial}{\partial \delta}\mathcal{H}^T > 0$. We show that this holds for $T+1$ as well. Let $t^* = \min\left\{t : \prod_{j=1}^{t} \tilde{\beta}_j \leq z\right\}$ and define for convenience

$$\hat{K}^T(z) := \mathbb{E}\left[\delta^{t^*} \cdot K^T\left(\prod_{j=1}^{t^*} \tilde{\beta}_j\right)\right]. \tag{31}$$

By definition,

$$\mathcal{H}^{T+1} = N(e_*^T) + \hat{K}^T(e_T^*)\mathcal{H}^T.$$

Hence,

$$\begin{aligned}\frac{\partial}{\partial \delta}\mathcal{H}^{T+1} &= \frac{\partial e_*^T}{\partial \delta}\left(N'(e_T^*) + \frac{\partial}{\partial e}\hat{K}^T(e_T^*)\mathcal{H}^T\right) + \frac{\partial}{\partial \delta}\mathcal{H}^T\hat{K}^T(e_T^*) \\ &> \frac{\partial e_*^T}{\partial \delta}\left(N'(e_T^*) + \frac{\partial}{\partial e}\hat{K}^T(e_T^*)\mathcal{H}^T\right),\end{aligned} \tag{32}$$

where the inequality follows from the inductive hypothesis. We use again the notation that g_k represents the distribution over any k-prodcut of i.i.d. random variables drawn from the underlying distribution F that governs $\tilde{\beta}$. By definition Equation 31,

$$\begin{aligned}\hat{K}^T(z) = \delta\int_0^z K^T(y)g_1(y)\,dy + \delta^2\int_{x=z}^1\int_{y=0}^{\frac{z}{x}} K^T(xy)\cdot g_1(y)g_1(x)\,dy\,dx \\ + \delta^3\int_{x=z}^1\int_{y=0}^{\frac{z}{x}} K^T(xy)\cdot g_1(y)g_2(x)\,dy\,dx + \ldots\end{aligned}$$

By convention, let $g_0 := 0$. Applying the Leibniz rule gives us

$$\begin{aligned}\frac{\partial}{\partial e}\hat{K}^T(z) &= \delta K^T(z)g_1(z) + \sum_{t\geq 1}\delta^{t+1}\left[-g_t(z)\int_0^1 K^T(z\cdot y)g_1(y)\,dy + K^T(z)\int_{x=z}^1 \frac{1}{x}g_1\left(\frac{z}{x}\right)g_t(x)\,dx\right] \\ &= \delta K^T(z)g_1(z) + \sum_{t\geq 1}\delta^{t+1}\left[-g_t(z)\int_0^1 K^T(z\cdot y)g_1(y)\,dy + K^T(z)g_{t+1}(z)\right] \\ &= \sum_{t\geq 0}\left\{\delta^{t+1}\left(-g_t(z)\mathbb{E}\left[K^T\left(\tilde{\beta}_{t+1}\cdot z\right)\right] + K^T(z)g_{t+1}(z)\right)\right\},\end{aligned} \tag{33}$$

The second equality results from using the definition of the partial distribution function of a product of independent random variables. Utilizing Equation 33 and substituting Equation 25 for $N'(e_T^*)$ allows us to write

$$N'(e_T^*) + \frac{\partial}{\partial e}\hat{K}^T(e_T^*)\mathcal{H}^T = -e_T^* \cdot \sum_{t\geq 1}\delta^t g_t(e_T^*) + \mathcal{H}^T \cdot \left(K^T(e_T^*) - \delta\mathbb{E}\left[K^T(\tilde{\beta}\cdot e_T^*)\right]\right) \cdot \sum_{t\geq 1}\delta^t g_t(e_T^*), \tag{34}$$

where $\tilde{\beta}$ is any shock drawn from the distribution F. By directly substituting in the form of $K^T(e)$,

$$\mathbb{E}\left[K^T(e_T^*) - K^T(\tilde{\beta}\cdot e_T^*)\right] = \sum_{i=0}^{\infty}\delta^i \mathbb{E}\left[\prod_{j=0}^{i}\left(1 - \Delta^T\left(\prod_{k=0}^{j}\tilde{\beta}_k \cdot e_T^*\right)\right)\tilde{\beta}_j\right]\frac{e_T^*}{\mathcal{H}^T} - \\ \sum_{i=0}^{\infty}\delta^{i+1}\mathbb{E}\left[\prod_{j=0}^{i}\left(1 - \Delta^T\left(\prod_{k=0}^{j}\tilde{\beta}_k \cdot \tilde{\beta}e_T^*\right)\right)\tilde{\beta}_j \cdot \tilde{\beta}\right]\frac{e_T^*}{\mathcal{H}^T} \tag{35}$$

Note that for each i, the $(i+1)^{th}$ summand in the former sum is equal to the i^{th} summand in the latter sum of Equation 35. Hence, all but the first summand in the former sum remains when we evaluate this difference. We thus obtain

$$\mathbb{E}\left[K^T(e_T^*) - K^T(\tilde{\beta}\cdot e_T^*)\right] = \frac{e_*^T}{\mathcal{H}^T}$$

Substituting back into Equation 34 gives

$$N'(e_T^*) + \frac{\partial}{\partial e}\hat{K}^T(e_T^*)\mathcal{H}^T = 0.$$

Therefore, Equation 32 straightforwardly gives $\frac{\partial}{\partial \delta}\mathcal{H}^{T+1} > 0$, as desired. ☐

B.2 Proofs for Section 4.3

Proposition 6 (**Long-Run Policy Dynamics**)**.** *Let $(L_T)^N(F)$ be the distribution of policies after N agreements have been reached between agents with foresight horizon T, when the initial agreement is drawn from distribution F. Then, there exists distribution G^*, symmetric about $\frac{1}{2}$, such that for any F, as $N \to \infty$, $(L_T)^N(F)$ converges to G^*.*

Proof. Step One. First, I show that the policy operator acts as a contraction, in expectation. Consider two different policies $x, y \in [0,1]$. Without loss of generality, assume $x < y$, and let $\pi_i^T(e,z)$ be the probability that i wins the election at state $((e,z),T)$, for $e < e_T^*$. Denote $\tilde{e}^T$ the (stochastic) efficiency at which revision occurs, and H its cdf. We then have

$$\mathbb{E}\left[\left|a^T(\tilde{e}^T, x) - a^T(\tilde{e}^T, y)\right|\right] = \int_{[0,e_T^*]}\Big\{\left|a_L^T(e^T,x) - a_L^T(e^T,y)\right| \cdot \pi_L^T(x,e^T) + \left|a_R^T(e^T,x) - a_R^T(e,y)\right| \cdot \pi_R^T(y,e^T) \\ + \left|a_R^T(e^T,x) - a_L^T(e^T,y)\right| \cdot (1 - \pi_L^T(x,e^T) - \pi_R^T(y,e^T))\Big\}\, dH(e^T)$$

$$
\begin{aligned}
&= \int_{[0,e_T^*]} \Big\{ C^T(e^T) \cdot |x-y| \cdot \left(\pi_L^T(x,e^T) + \pi_R^T(y,e^T)\right) \\
&\qquad + \left|a_R^T(e^T,x) - a_L^T(e^T,y)\right| \cdot \left(1 - \pi_L^T(x,e^T) - \pi_R^T(y,e^T)\right) \Big\} \, dH(e^T) \\
&\leq \mathbb{E}\left[C^T(\tilde{e}^T)(1+\Delta^T(\tilde{e}^T))\right] \cdot |x-y|, \text{ by using the expressions for } \pi_i^T(z,e) \\
&\leq \left(1 - \mathbb{E}[B_R^T(\tilde{e}^T)^2]\right) \cdot |x-y|.
\end{aligned}
$$

Thus, the policy operator acts as a contraction in expectation; over time, the expected distance between policy outcomes that stem from x and y grows closer by factor less than $\alpha := 1 - \mathbb{E}[B_R^T(\tilde{e}^T)^2] < 1$.

Step Two. Next, consider any two distributions F and G on [0,1] from which the initial policy in the sequence of agreements will be drawn.[15] I show that the operator L_T is a contraction mapping. Let $\xi \in [0, e_T^*] \times \{L, R\} := X$ be a particular realization of $(\tilde{e}_T, I_T(\tilde{e}_T))$.

$$
\begin{aligned}
\int_0^1 |L_T(F)(z) - L_T(G)(z)| \, dz &= \int_0^1 \left| \int_X L_T(F)(z|\xi) - L_T(G)(z|\xi) \, dH(\xi) \right| dz \\
&\leq \int_0^1 \int_X |L_T(F)(z|\xi) - L_T(G|\xi)| \, dH(\xi) \, dz, \text{ by the Cauchy-Schwarz inequality} \\
&= \int_X \int_0^1 |L_T(F)(z|\xi) - L_T(G|\xi)| \, dz \, dH(\xi) \\
&= \int_X \int_0^1 \left| L_T(F|\xi)^{-1}(t) - L_T(G|\xi)^{-1}(t) \right| \, dt \, dH(\xi), \qquad (36)
\end{aligned}
$$

where the final equality in the above series transforms the Riemann-Stieljes integral into the corresponding Lebesgue integral. I integrate over the differences between "percentiles" of the distributions $L_T(F)$ and $L_T(G)$, conditional on a particular realization, $\xi \in X$. Note that conditional on the realization ξ, each percentile of the transformed policy distributions has the same relative ordering as the percentiles of the initial distribution F and G. This is because $a_I^T(e^T, \cdot)$ is monotone for fixed $\xi = (e^T, I)$. Hence,

$$
\begin{aligned}
\left| L_T(F|\xi)^{-1}(t) - L_T(G|\xi)^{-1}(t) \right| &= \left| a_I^T(e^T, F^{-1}(t)) - a_I^T(e^T, G^{-1}(t)) \right|, \text{ where } \xi = (e^T, I) \\
&\leq \alpha \cdot |F^{-1}(t) - G^{-1}(t)|, \text{ from work done in Step One.}
\end{aligned}
$$

Hence, substituting this expression into Equation 36, we now have

$$
\begin{aligned}
\int_0^1 |L_T(F)(z) - L_T(G)(z)| &\leq \int_X \int_0^1 \left| L_T(F|\xi)^{-1}(t) - L_T(G|\xi)^{-1}(t) \right| \, dt \, dH(\xi) \\
&\leq \int_X \int_0^1 \alpha \cdot |F^{-1}(t) - G^{-1}(t)| \, dt \, dH(\xi)
\end{aligned}
$$

[15] With apologies, I am abusing notation here since F and G have been previously been used for distributions corresponding to the decay shocks to the efficiency of policy. Here, they assume no such contextual meaning.

$$= \alpha \int_0^1 |F^{-1}(t) - G^{-1}(t)| \, dt$$
$$= \alpha \int_0^1 |F(z) - G(z)| \, dz,$$

transforming the Lebesgue back into the Riemann-Stieljes integral. We now have that since $\alpha < 1$, L_T is a contraction mapping for any $T \in \mathbb{N}$ over the complete metric space of probability distributions on X, itself a complete metric space. We can therefore apply the Contraction Mapping Theorem to conclude that there exists a unique distribution G^* such that for any initial distribution F, $(L_T)^N(F)$ converges to G^* as $N \to \infty$. If we start with degenerate distribution F that places probability 1 on $\frac{1}{2}$, then at each step N of the process $(L_T)^N(F)$, the resulting distribution will be symmetric about $\frac{1}{2}$. Hence, G^* is symmetric about $\frac{1}{2}$. □

C A Micro-Foundation for Limited Foresight

As discussed in Section 2, negotiators exhibit *limited foresight* when they give import to only a finite number of downstream agreements that result from the current implementation of policy. A natural way this behavior arises is through the inability of agents to fully account for the impact current policy will have on optimal choices at future decision nodes. Agents respond by truncating the dynamic linkage of downstream agreements to a chain of length T.

C.1 Endogenizing Foresight in Equilibrium

To illustrate how this may happen, suppose the status quo is $(e, \hat{a})$ with incumbent I. For a foresight horizon T, the proposal functions of the parties $\{a_i^\tau\}_{\tau=1}^T$ are well-defined at those states at which agreement is reached, within any equilibrium σ.

(1) The agreement reached in the current state has only one possibility— $a_I^T(e, \hat{a})$— given the information set of the agents.

(2) The subsequent agreement that is arrived at on the projected path of play of the agents, on the other hand, is uncertain. This is given by $a_{I_{T-1}}^{T-1}(\tilde{e}^{T-1}, a_I^T(e, \hat{a}))$, where the uncertainty is over both the eventual incumbent I_{T-1} when this agreement is hatched, as well as the efficiency level, $\tilde{e}^{T-1}$ at which this occurs.

(3) Continuing along, the next agreement arrived at is $a_{I_{T-2}}^{T-2}\left(\tilde{e}^{T-2}, a_{I_{T-1}}^{T-1}(\tilde{e}^{T-1}, a_I^T(e, \hat{a}))\right)$. Here, there is uncertainty on *four dimensions*: the incumbent I_{T-2} and the efficiency level $\tilde{e}^{T-2}$ when this agreement is made, along with the incentives induced by the *previous agreement*, $a_{I_{T-1}}^{T-1}$, which itself encapsulated uncertainty along two dimensions.

Clearly, there are more possibilities as one projects agreements further downstream from the current negotiation. In particular, uncertainty inherent in one negotiation continues to compound when considering all agreements whose outcomes are dependent on its resolution.

Formally, given a foresight horizon T, and initial state $((e,\hat{a}),T)$ and incumbent I, we let $X^{(T+1)-\tau}((e,\hat{a}),I)$ be the set of possible agreements reached τ re-negotiations from the present. That is, $a \in X^{(T+1)-\tau}((e,\hat{a}),I)$ if and only if there exists some sequence of $\{\tilde{e}^{\tau'}\}_{\tau'=T}^{T-\tau}$ and $\{I_{\tau'}\}_{\tau'=T}^{T-\tau}$ such that

$$a = a_{I_{T-\tau}}^{T-\tau}\left(\tilde{e}^{T-\tau}, a_{I_{T-\tau+1}}^{T-\tau+1}\left(\tilde{e}^{T-\tau+1},\ldots\right)\right)$$

in any given equilibrium σ. We can now define the following notion of strategic complexity in the problem faced by a decision-maker by foresight horizon T:

Definition C.1. *For given equilibrium σ, the total statistical entropy over future agreements within a foresight horizon T is denoted by*

$$\mathcal{E}^T((e,\hat{a}),I) := -\sum_{\tau=1}^{T}\left(\sum_{x\in X^{(T+1)-\tau}((e,\hat{a}),I)} \mathbb{P}(x)\cdot\ln\mathbb{P}(x)\right). \tag{37}$$

This definition, of course, is analogous to the standard definition of entropy in an information theory setting from Shannon (1948). Here, Shannon entropy is summed over each of the agreements up to T links from the present decision node. Since the sets $X^{(T+1)-\tau}$ themselves depend on the equilibrium σ, $\mathcal{E}^T((e,\hat{a},I)$ is itself an equilibrium object.

I introduce also a *cognitive constraint* $B \in (0,1)$ for agents that represents the degree of uncertainty they are able to accurately incorporate within their maximization problem. Requiring that in equilibrium, $\mathcal{E}^T((e,\hat{a},I)$ not exceed B thus captures the generic inability of negotiatosr to consider the infinite chain of agreements that flow from the current one. This requirement is delineated in the following refinement of our earlier notion of equilibrium—

Definition C.2 (Endogenous Foresight Equilibrium). *Given a cognitive constraint B, exponential discount factor δ, and stochastic decay shock $\tilde{\beta}$, an endogenous foresight equilibrium of the game is given by strategies σ_i^T (as given in Definition 2.3) for $i \in \{R,L\}$ and a foresight horizon T such that*

(1) The strategy profile $\sigma := (\sigma_L^T, \sigma_R^T)$ is a Markov perfect equilibrium

(2) The foresight horizon T is the maximal T' such that for each $(e,\hat{a})$ at which agreement occurs under σ and every incumbent I,

$$\mathcal{E}^{T'}((e,\hat{a}),I) \leq B$$

We can think of Definition C.2 as follows: the analysis within the main text provides a roadmap for computing equilibrium strategies for each foresight horizon T; the cognitive restraint B thus selects the unique one of these equilibria that is consistent with parameter B, using (2) above. Finally, note that the uniqueness of equilibrium we derived in Proposition 4 for the legislative bargaining setting continues to hold under this alternative definition. This is because, since there is a unique equilibrium for each foresight horizon T, the unique endogenous foresight equilibrium equilibrium is given from the mapping from $B \to T$ that is induced by (2) in Definition C.2.

C.2 Discussion

Under this endogenization of the foresight horizon, the strategic setting of negotiations can have a large impact on the foresight horizon of agents, and thus on the outcome.

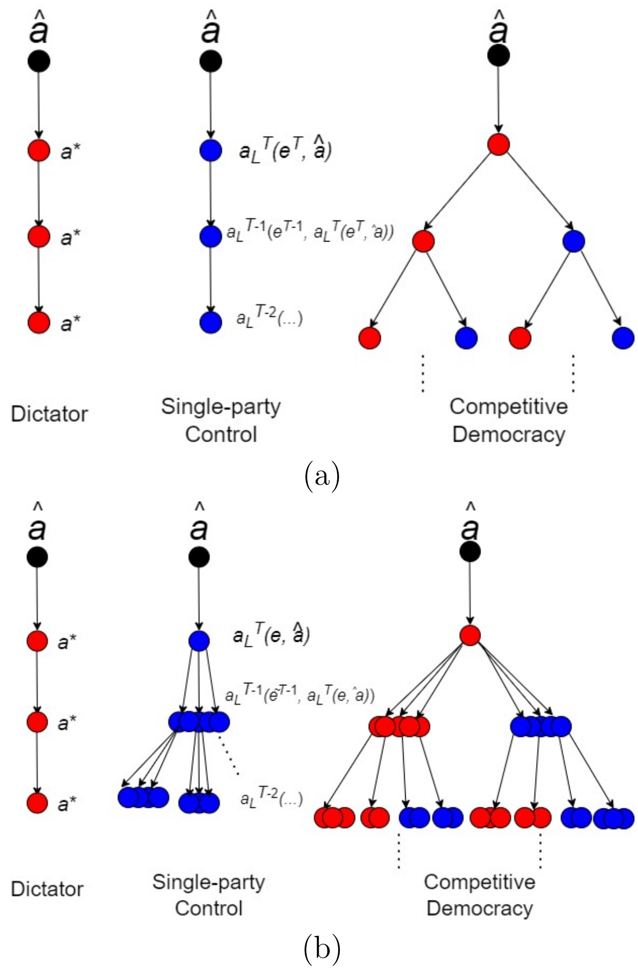

Figure 7: Total statistical entropy increases due to uncertainty over both the incumbent at the time of a break-through in negotiations along with that over the efficiency of policy at any future of time.

For illustration, consider legislative bargaining. Various situations are shown in Figure 7. The bargaining game can be trivial, in the sense that decisions are made by a dictator D in each period; here, there is never any strategic complexity, since in each period, the dictator simply implements her ideal point, a^*. Given this behavior, it easy to see that for any $T \in \mathbb{N}$, and from any status quo policy $\hat{p}$, $\mathcal{E}^T((e, \hat{a}), D) = 0$.

We might also consider a democracy in which there is single-party control over legislative institutions; here, the opposition does not have an effective means to oppose legislation. This situation mirrors the United States Congress in the 1960's and early 1970's when Democrats controlled a *filibuster-proof* majority in the Senate along with a majority in the House of Representatives. If the shocks $\tilde{\beta}$ were deterministic, as in Figure 7(a), then there would be no uncertainty within the entire chain of agreements passed since agents are certain about the incentives— the status quo policy and the identity of the incumbent— they will face in each of the relevant decision nodes. Hence, uncertainty only exists through the randomness of $\tilde{\beta}$.

Finally, in a competitive democracy, such as the one that has existed in the United States from 1994 until the present,[16] the entropy is largest. Here, uncertainty accumulates not only through uncertainty over the identity of the incumbent, but also through uncertainty over the incentives that any incumbent would face when $\tilde{\beta}$ is non-deterministic.

Under Definition C.2, *ceteris paribus*, the foresight exhibited by politicians is more restricted when the environment they face is more electorally competitive. As the incidence of gridlock is increasing with more limited foresight (Corollary 1), we then have a straightforward predicted relationship: *as uncertainty over control of legislative institutions increases, gridlock also increases.* This is among the most important causes of modern-day U.S. gridlock posited by political scientists— seminal work in this literature includes Lee (2016), Binder (1999), among others.

[16] In 1994, Republicans gained control of the House of Representatives for the first time in five decades