Mathematica Code to Replicate the Plots in the Manuscript

Figure 2

Definition of the susceptibility:

In[821]:=

```
μ[β_, a_, B_] = 1 / (1 + Exp[-β (a + B)])
dμdB[β_, a_, B_] = D[μ[β, a, B], B];
χ[β_, a_, B_] = dμdB[β, a, B]
```

Out[821]=

$$\frac{1}{1+e^{-((a+B)\beta)}}$$

Out[823]=

$$\frac{e^{-((a+B)\beta)}\beta}{\left(1+e^{-((a+B)\beta)}\right)^2}$$

Theoretical prediction of the maxima as obtained from

In[833]:=

```
FindRoot[{Tanh[β / 2] == 1 / β}, {β, 1.5}]
```

Out[833]=

```
{β → 1.5434}
```

In[836]:=

```
γ = Exp[-β] / (1 + Exp[-β]) ^2 //. {β → 1.5434046384182085`}
```

Out[836]=

```
0.14505
```

and explained in more detail the SOA in the Proof of Proposition 2.

Define the dotted line: $\chi = \gamma\,\beta$

In[839]:=

```
peak[β_] = γ β
```

Out[839]=

```
0.14505 β
```

Figure 2 a : (a_i = 0)

In[840]:=

```
Block[{a = 0},
 pl1 = Plot[{dμdB[β, a, 0], dμdB[β, a, .1], dμdB[β, a, .2], dμdB[β, a, .3],
    dμdB[β, a, .4], dμdB[β, a, .5], dμdB[β, a, 1]}, {β, 0, 20},
   PlotRange → {-.02, 5}, BaseStyle → {FontSize → 16, FontFamily → Times},
   Frame → True, PlotRangePadding → None, AspectRatio → 1,
   PlotLegends → {"B=0", "B=0.1", "B=0.2", "B=0.3", "B=0.4", "B=0.5", "B=1"}];
 pl2 = Plot[peak[β], {β, 0, 20}, PlotRange → {-.02, 5},
   PlotStyle → {Gray, Dashed}, BaseStyle → {FontSize → 16, FontFamily → Times},
   Frame → True, PlotRangePadding → None, AspectRatio → 1];
 pl = Show[{pl2, pl1}]
]
```

Out[840]=

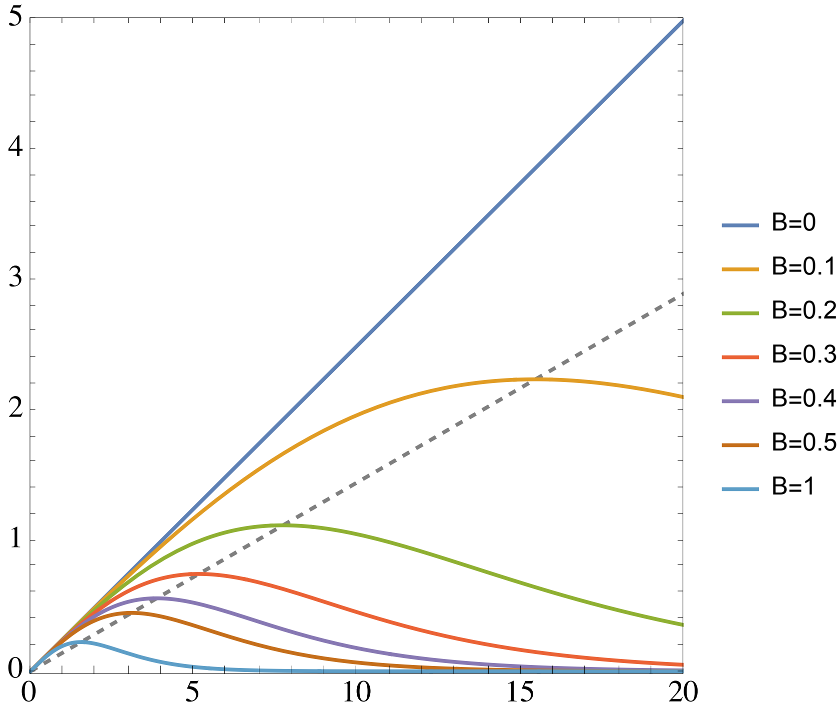


Figure 2 b: (a_i = -0.2)

In[841]:=

```
Block[{a = -.2},
 pl1 = Plot[{dμdB[β, a, 0], dμdB[β, a, .1], dμdB[β, a, .2], dμdB[β, a, .3],
    dμdB[β, a, .4], dμdB[β, a, .5], dμdB[β, a, 1]}, {β, 0, 20},
   PlotRange → {-.02, 5}, BaseStyle → {FontSize → 16, FontFamily → Times},
   Frame → True, PlotRangePadding → None, AspectRatio → 1,
   PlotLegends → {"B=0", "B=0.1", "B=0.2", "B=0.3", "B=0.4", "B=0.5", "B=1"}];
 pl2 = Plot[peak[β], {β, 0, 20}, PlotRange → {-.02, 5},
   PlotStyle → {Gray, Dashed}, BaseStyle → {FontSize → 16, FontFamily → Times},
   Frame → True, PlotRangePadding → None, AspectRatio → 1];
 pl = Show[{pl2, pl1}]
]
```

Out[841]=

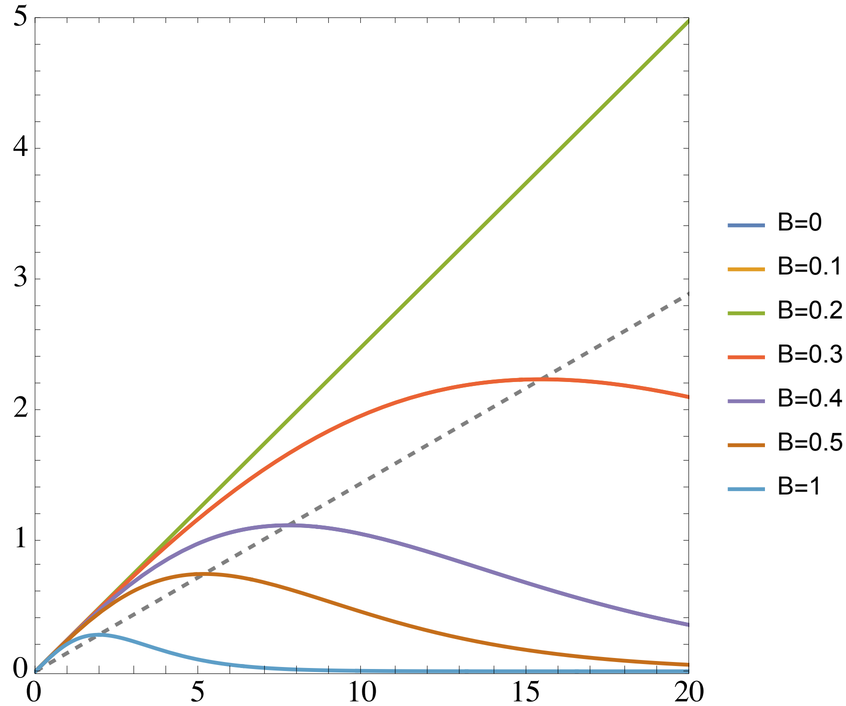


Figure 3

Define mean and variance of the Gaussian as a function of issue attention β, attitude a, media influence B, and population size n:

In[1437]:=

```
μ  = 1 / (1 + Exp[-β (a + B)]);
σ2 = μ (1 - μ) // FullSimplify;
```

In our case a = 0 and B=0 which will simplifies the expression :

In[1439]:=

```
μ  = 1 / (1 + Exp[-β (a + B)]) //. {a → 0, B → 0};
σ2 = μ (1 - μ) //. {a → 0, B → 0};
Print["μ = ", μ, ", σ2 = ", σ2]
```

$\mu = \frac{1}{2}, \sigma 2 = \frac{1}{4}$

The corresponding Gaussian for the ensemble of size n is defined by :

In[1442]:=

```
Clear[P, x]
P[n_] := PDF[NormalDistribution[μ, Sqrt[σ2 / n]], x]
```

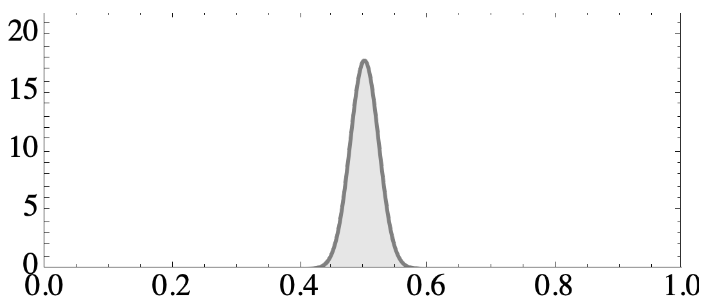
For a population on n= 500 agents we can draw the stationary distribution as follows:

Stationary Distribution:

In[1444]:=

```
Plot[P[500], {x, 0, 1}, PlotPoints → 100, PlotRange → {0, 22},
 Filling → Axis, Frame → True, PlotStyle → {Thick, Gray}, AspectRatio → 2 / 5,
 PlotRangePadding → None, BaseStyle → {FontSize → 16, FontFamily → Times}]
```

Out[1444]=



Evolution of the Aggregate Opinion

For the example of the stochastic evolution of the aggregate opinion first we consider a single agent. It’s opinion switches stochastically between 0 and 1 according to the update rates for up and down transitions:

In[952]:=

```
Clear[ru, rd]
ru[β_, a_] = 1 / (1 + Exp[-β (a + B)]) //. B → 0;
rd[β_, a_] = 1 / (1 + Exp[β (a + B)]) //. B → 0;
```

where here we assume B = 0.

The evolution is described by a Markov Process that is specified in the following Module (below). Here m refers to the transition matrix of the Markov Process. The parameter Δt specifies the duration of individual time bins, which when multiplied by a rate will return a probability. We will set it to Δt = 0.01. Furthermore we will assume $\beta = 2$.

First, pecify the Random Seed:

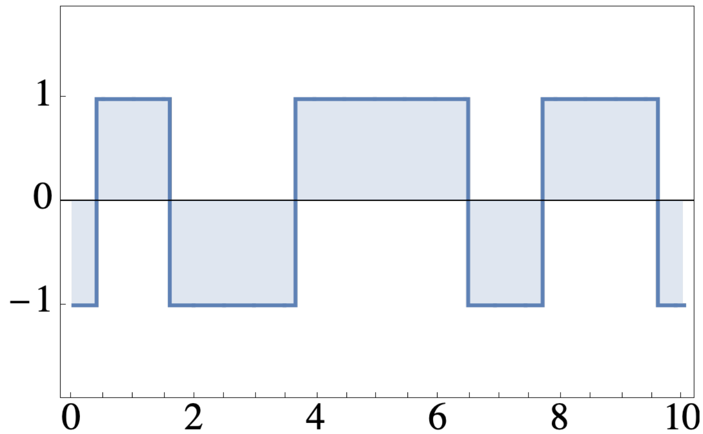
In[1348]:=
```
SeedRandom[1234]; RandomReal[]
```
Out[1348]=
```
0.876608
```
In[1340]:=
```
Clear[setinit]
setinit := Module[{}, If[RandomReal[{-1, 1}] > 0, init = {0, 1}, init = {1, 0}]]
```
In[1342]:=
```
TelegraphSteadyState[β_, a_, Δt_] := Module[{ββ = β, bb = a, Δtt = Δt},
   pu = ru[β, a] Δt;
   pd = rd[β, a] Δt;
   m = {{1 - pu, pu}, {pd, 1 - pd}};
   𝒫 = DiscreteMarkovProcess[setinit, m];
   x = Range[0, 20, Δt];
   data = 2 (RandomFunction[𝒫, {0, Length[x] - 1}] - 1.5);
   y = data["Values"];

   Nsp = (Length[x] + 1) / 2;
   (*Cutting out the first elements of x and last elements of y*)
   xs = x[[1 ;; Nsp]];
   ys = y[[-Nsp ;;]];
   (*Replacing data by an array based on shortened entries*)

   rf = Transpose[{xs, ys}]];
 (* This funtion removes the first half of the trajectory to "forget" the
 initial condition and to make sure the system has relaxed into a steady state *)
```
An individual trajectory for an agent with a = 0 will look as follows:

In[1318]:=
```
rf = TelegraphSteadyState[2, 0, .01];
```

In[1319]:=

```
ListStepPlot[rf, Filling → Axis, Ticks → {Automatic, {-1, 1}},
 PlotRange → {-1.9, 1.9}, BaseStyle → {FontSize → 20, FontFamily → Times},
 Frame → True, FrameTicks → {{{-1, 0, 1}, None}, {Automatic, None}}]
```

Out[1319]=



We use that module to simulate n = 500 agents, from which we then obtain the aggregate opinion x by pooling. Here the parameter Nens also denotes the population size n.

Again, specify a random seed:

In[1349]:=

```
SeedRandom[12]; RandomReal[]
```

Out[1349]=

```
0.140802
```

In[1350]:=

```
Block[{Δt = 0.01, β = 2, a = 0, Nens  = 500},
  pm = Table[{rf = TelegraphSteadyState[β, a , Δt];
     tmp = rf⟦All, 2⟧;
     tmp  =  (tmp + 1) / 2;
     rf⟦All, 2⟧ = tmp;
     p[i] = rf}, {i, 1, Nens}]];
```

Now do the pooling :

In[1351]:=

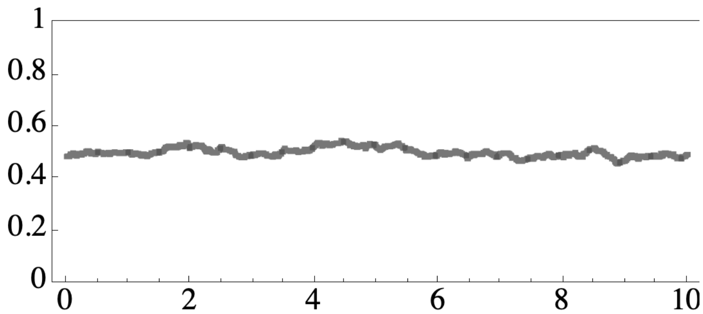
```
Nm = Dimensions[pm]⟦1⟧;
pmav = Sum[pm⟦i⟧, {i, 1, Nm}] / Nm;
```

And plot it:

In[1353]:=

```
pl1 = ListStepPlot[pmav, Ticks → {Automatic, {-1, 1}}, PlotRange → {0, 1},
  BaseStyle → {FontSize → 16, FontFamily → Times}, Frame → True, FrameTicks →
   {{{0, 0.2, .4, .6, .8, 1}, None}, {Automatic, None}}, AspectRatio → 2 / 5,
  PlotStyle → Directive[Darker[Gray], Thickness[0.008], Opacity[.8]]]
```

Out[1353]=

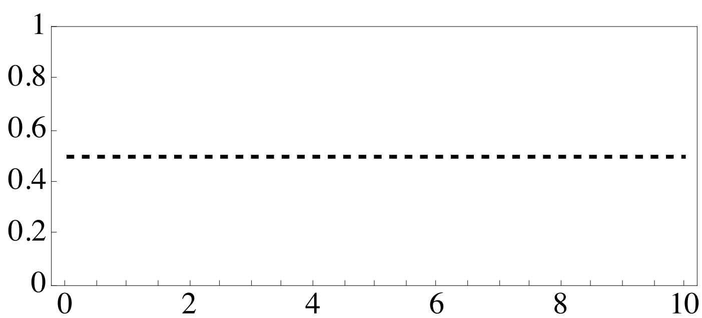


Plot the baseline (dashed)

In[1354]:=

```
pl0j = Plot[.5 , {x, 0, 10}, Ticks → {Automatic, {-1, 1}}, PlotRange → {0, 1},
  BaseStyle → {FontSize → 16, FontFamily → Times}, Frame → True,
  FrameTicks → {{{0, 0.2, .4, .6, .8, 1}, None}, {Automatic, None}},
  AspectRatio → 2 / 5, PlotStyle → Directive[Black, Dashed]]
```

Out[1354]=



And combine both plots:

In[1278]:=

```
Show[pl0j, pl1]
```

Out[1278]=

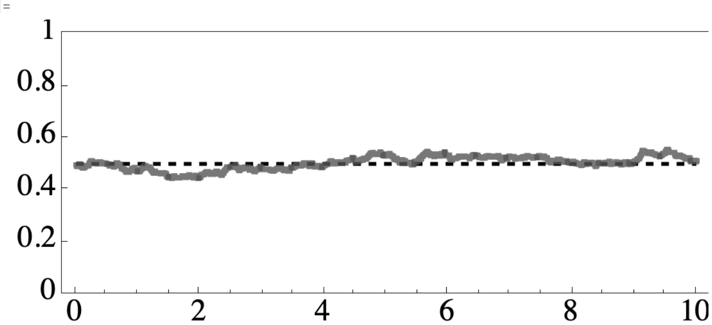


Figure 4

Next we proceed along the same line for a heterogenous population comprised of 60% (fp = 0.6) agents with positive attitude, ap = 0.424, and 40% (fm = 0.4) agents with negative attitude, am = – 0.693.

Define mean and Variance of the Gaussian as a function of issue attention β, attitude a, media influence B, and population size n. Again we assume $\beta = 2$.

The parameter of the attitudes are chosen such that the means of the positive and negative group (approximately) satisfy: μp = 0.7 and μm = 0.2. Let's check:

In[989]:=

```
μp  = 1 / (1 + Exp[-β (a + B)]) //. {B → 0, β → 2, a → 0.424}
```

Out[989]=

```
0.700147
```

In[990]:=

```
μm  = 1 / (1 + Exp[-β (a + B)]) //. {B → 0, β → 2, a → -0.693}
```

Out[990]=

```
0.200047
```

Fair enough! Parameters are chosen such that the mean opinion across **all** agents will still be $\mu = 0.5$:

In[992]:=

```
μ = .6 μp + .4 μm
```

Out[992]=

```
0.500107
```

For specificity, let' s summarize the parameters:

In[1413]:=

```
μ = 0.5;
μp = 0.7;
μm = 0.2;
fp = 0.6;
fm = 0.4;
n = 500;
np = n fp;
nm = n fm;
ap =  0.424;
am =  -0.693;
```

From this we obtain for the variances at the individual level:

In[1423]:=

```
σ2p = μp (1 - μp)
σ2m = μm (1 - μm)
σ2  = fp σ2p + fm σ2m
```

Out[1423]=

```
0.21
```

Out[1424]=

```
0.16
```

Out[1425]=

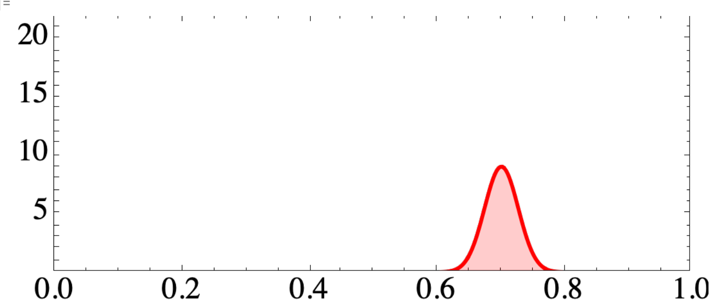
```
0.19
```

Next we plot the steady state distribution for the population with **positive** attitude. Note that it is weighted by fp.

In[1426]:=

```
Clear[P]
P[np_] := PDF[NormalDistribution[μp, Sqrt[σ2p / np]], x];
plp = Plot[fp P[300], {x, 0, 1}, PlotPoints → 100, PlotRange → {0.01, 22},
  Filling → Axis, Frame → True, PlotStyle → {Thick, Red}, AspectRatio → 2 / 5,
  PlotRangePadding → None, BaseStyle → {FontSize → 16, FontFamily → Times}]
```

Out[1427]=

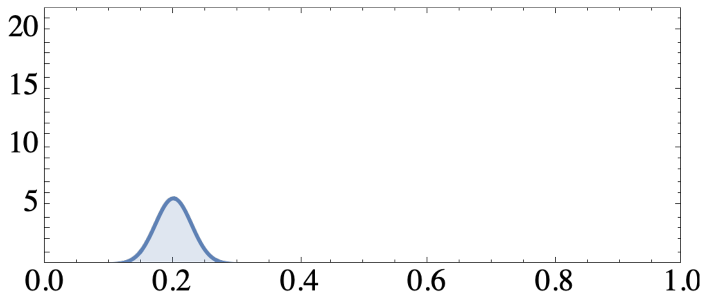


Next we plot the steady state distribution for the population with **negative** attitude. Note that it is weighted by fm.

In[1428]:=

```
Clear[P];
P[nm_] := PDF[NormalDistribution[μm, Sqrt[σ2m / nm]], x];
plm = Plot[fm P[200], {x, 0, 1}, PlotPoints → 100, PlotRange → {0.01, 22},
  Filling → Axis, Frame → True, PlotStyle → {Thick}, AspectRatio → 2 / 5,
  PlotRangePadding → None, BaseStyle → {FontSize → 16, FontFamily → Times}]
```

Out[1428]=



Next we plot the steady state distribution for the aggregate opinion in the population, which is normalized to 1.

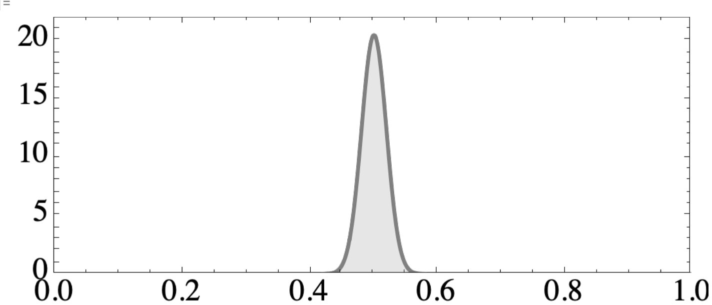
In[1108]:=

```
Clear[P]
```

In[1429]:=

```
P[n_] := PDF[NormalDistribution[μ, Sqrt[(fm σ2m + fp σ2p) / n]], x];
pl = Plot[P[500], {x, 0, 1}, PlotPoints → 100, PlotRange → {0, 22},
  Filling → Axis, Frame → True, PlotStyle → {Thick, Gray}, AspectRatio → 2 / 5,
  PlotRangePadding → None, BaseStyle → {FontSize → 16, FontFamily → Times}]
```

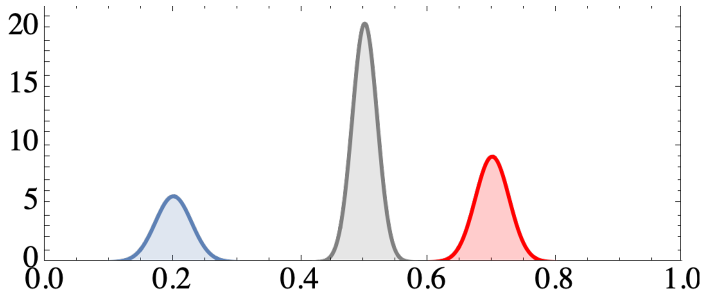
Out[1429]=



In[1430]:=

```
Show[pl, plm, plp]
```

Out[1430]=



Evolution of the Aggregate Opinion

For the example of the stochastic evolution of the aggregate opinion of the heterogenous population we proceed as in the homogenous case. We use the Modules we defined above (so make sure to run the code above to load these functions).

First the agents with negative attitude:

In[1355]:=

```
SeedRandom[17]; RandomReal[]
Clear[pm]
Block[{Δt = 0.01, β = 2, a = am, Nens = 200},
  pm = Table[{rf = TelegraphSteadyState[β, a , Δt];
      tmp = rf⟦All, 2⟧;
      tmp = (tmp + 1) / 2;
      rf⟦All, 2⟧ = tmp; p[i] = rf}, {i, 1, Nens}]];
(* pool it *)
Nm = Dimensions[pm]⟦1⟧;
pmav = Sum[pm⟦i⟧, {i, 1, Nm}] / Nm;
```

Out[1355]=

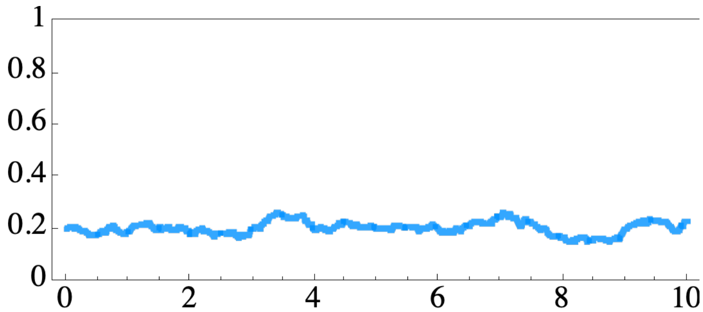
```
0.949674
```

And plot it:

In[1360]:=

```
pl1 = ListStepPlot[pmav, Ticks → {Automatic, {-1, 1}}, PlotRange → {0, 1},
  BaseStyle → {FontSize → 16, FontFamily → Times}, Frame → True,
  FrameTicks → {{{0, 0.2, .4, .6, .8, 1}, None}, {Automatic, None}},
  AspectRatio → 2 / 5, PlotStyle →
   Directive[RGBColor[0, 145 / 255, 255 / 255], Thickness[0.008], Opacity[.8]]]
```

Out[1360]=



Agents with positive attitude:

In[1361]:=

```
SeedRandom[192]; RandomReal[]
Clear[pp]
Block[{Δt = 0.01, β = 2, a = ap, Nens = 300},
  pp = Table[{rf = TelegraphSteadyState[β, a , Δt];
     tmp = rf⟦All, 2⟧;
     tmp = (tmp + 1) / 2;
     rf⟦All, 2⟧ = tmp; p[i] = rf}, {i, 1, Nens}]];
(* pool it *)
Np = Dimensions[pp]⟦1⟧;
ppav = Sum[pp⟦i⟧, {i, 1, Np}] / Np;
```

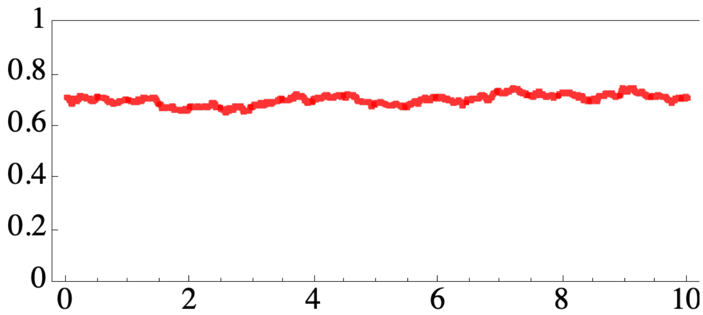
Out[1361]=

```
0.508857
```

In[1366]:=

```
pl2 = ListStepPlot[ppav, Ticks → {Automatic, {-1, 1}}, PlotRange → {0, 1},
  BaseStyle → {FontSize → 16, FontFamily → Times}, Frame → True, FrameTicks →
   {{{0, 0.2, .4, .6, .8, 1}, None}, {Automatic, None}}, AspectRatio → 2 / 5,
  PlotStyle → Directive[RGBColor[1, 0, 0], Thickness[0.008], Opacity[.8]]]
```

Out[1366]=



Pooled over all agents :

In[1367]:=

```
pj = Join[pm[[1 ;; Nm]], pp[[1 ;; Np]]];
Nj = Dimensions[pj][[1]]
pjav = Sum[pj[[i]], {i, 1, Nj}] / Nj;
```

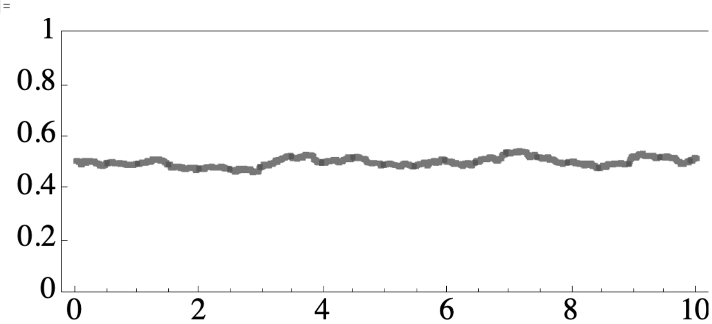
Out[1368]=

```
500
```

In[1370]:=

```
pl0 = ListStepPlot[pjav, Ticks → {Automatic, {-1, 1}}, PlotRange → {0, 1},
  BaseStyle → {FontSize → 16, FontFamily → Times}, Frame → True, FrameTicks →
   {{{0, 0.2, .4, .6, .8, 1}, None}, {Automatic, None}}, AspectRatio → 2 / 5,
  PlotStyle → Directive[Darker[Gray], Thickness[0.008], Opacity[.8]]]
```

Out[1370]=

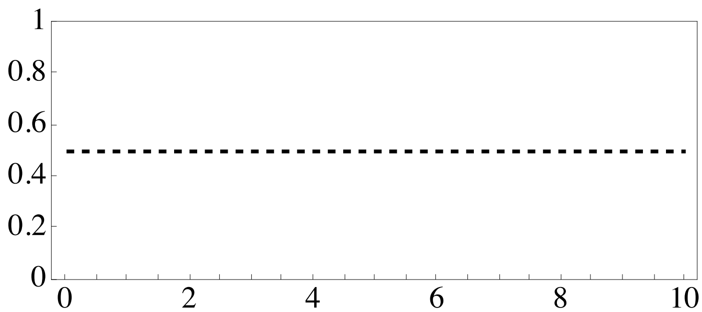


Plot the baselines

In[1371]:=

```
pl0j = Plot[.5 , {x, 0, 10}, Ticks → {Automatic, {-1, 1}}, PlotRange → {0, 1},
  BaseStyle → {FontSize → 16, FontFamily → Times}, Frame → True,
  FrameTicks → {{{0, 0.2, .4, .6, .8, 1}, None}, {Automatic, None}},
  AspectRatio → 2 / 5, PlotStyle → Directive[Black, Dashed]]
```

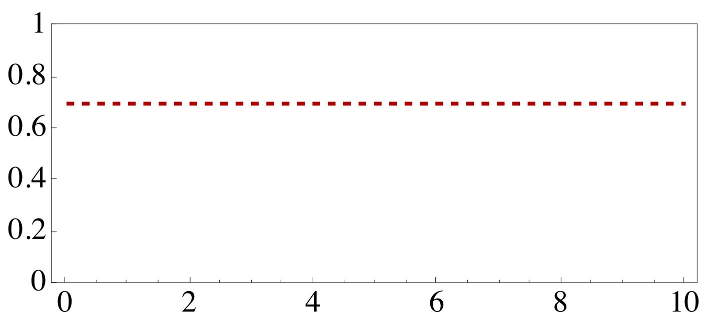
Out[1371]=



In[1372]:=

```
pl0p = Plot[.7 , {x, 0, 10}, Ticks → {Automatic, {-1, 1}}, PlotRange → {0, 1},
  BaseStyle → {FontSize → 16, FontFamily → Times}, Frame → True,
  FrameTicks → {{{0, 0.2, .4, .6, .8, 1}, None}, {Automatic, None}},
  AspectRatio → 2 / 5, PlotStyle → Directive[Darker[RGBColor[1, 0, 0]], Dashed]]
```

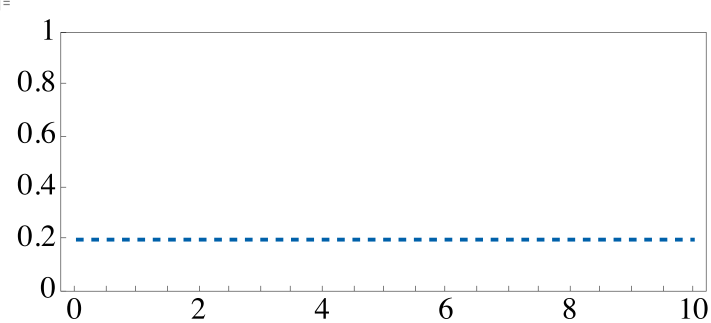
Out[1372]=



In[1373]:=

```
pl0m = Plot[.2 , {x, 0, 10}, Ticks → {Automatic, {-1, 1}}, PlotRange → {0, 1},
  BaseStyle → {FontSize → 16, FontFamily → Times}, Frame → True, FrameTicks →
   {{{0, 0.2, .4, .6, .8, 1}, None}, {Automatic, None}}, AspectRatio → 2 / 5,
  PlotStyle → Directive[Darker[RGBColor[0, 145 / 255, 255 / 255]], Dashed]]
```

Out[1373]=



And combine all plots to the final one:

In[1374]:=

```
Show[pl0j, pl0p, pl0m, pl1, pl2, pl0]
```

Out[1374]=