Online Appendix

“Does Informative Opposition Influence Electoral Accountability?”

This online appendix aims to provide formal arguments omitted from the manuscript.

Contents

A.1 Case where $\kappa = 0$

Thus far, we have assumed that $\kappa > 0$; thus, strategic-type party B's equilibrium strategy, ρ^*, is independent of ω (Lemma 1).

Under $\kappa = 0$, when party B is indifferent between $m = 1$ and $m = 0$, the party may change the probability of opposition depending on whether $x_1 = \omega$ simply because of the indifference. This may provide room for improvement in electoral accountability. In the following, we show that this is not the case even when $\kappa = 0$: that is, our result on the irrelevance of the minority party's monitoring still holds even if we allow κ to be zero.

As a preliminary result, we first show the following, which played a key role in proving Proposition 2.

Lemma A.1. *Suppose that $Q(x_1) > 0$. Then,*

(i). There exists no equilibrium where $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) > P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$.

(ii). $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*}) \Leftrightarrow$ Inequality A.2 does not hold.

(iii). If Inequality A.2 holds, $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$.

Proof. (i). Suppose that $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) > P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$. Then, $\rho^{M*}(x_1, \omega) = 0$ for every ω. Therefore, from the proof of Lemma 3, $H(0) > 0$ must hold. However, this never holds, which is a contradiction.

(ii). Suppose that $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$. Then, $\rho^{M*}(x_1, \omega) = 1$ for every ω. Therefore, from the proof of Lemma 3, $H(1) < 0$ must hold, which is equivalent to the property that Inequality A.2 does not hold.

(iii). (i) and (ii) together imply (iii).

□

Applying this lemma, we can establish the result in Proposition 2. Note that (i) in Proposition 2 is straightforward; thus, we only prove (ii). The key is that party B's payoff is independent of ω both directly and indirectly. Due to this structure, party B can change the value of ρ depending on ω if and only if the party is indifferent between $m = 1$ and $m = 0$. For example, when $m = 1$ leads to a higher expected payoff for party B, it must oppose a bill with probability one independently of ω. Furthermore, the party is indifferent between $m = 1$ and $m = 0$ if and only if the winning probability of party B is independent of m. Therefore, party B can change the value of ρ depending on ω if and only if the party's opposition has no influence on the electoral result. Therefore, allowing strategic-type party B's equilibrium strategy to depend on ω is useless in avoiding the irrelevant monitoring equilibrium.

Proposition A.1. *Suppose $p_B < 2(\sqrt{2} - 1)$. For each equilibrium, there exists σ^{N*} such that the equilibrium is an irrelevant monitoring equilibrium corresponding to σ^{N*}.*

Proof. There are five candidates for the equilibrium, other than the irrelevant monitoring equilibrium. We examine the five candidates one by one. The proof is basically the same as in the proof of Proposition 2 (ii).

Case 1. $\sigma^{M*}(0) > 0$ and $\sigma^{M*}(1) < 1$. It suffices to prove that $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$ holds for every x_1 in such an equilibrium. Then, every equilibrium such that $\sigma^{M*}(0) > 0$ and $\sigma^{M*}(1) < 1$ is an irrelevant monitoring equilibrium for some σ^{N*}.

Suppose not. That is, suppose that for some x_1, $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) \neq P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$ holds. Let such x_1 be x'. From Lemma A.1 (i), this implies that $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) \leq P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$ holds for every x_1 and it holds with a strict inequality for x'.

First, we prove that this implies that $\sigma^{M*}(0) \leq \sigma^{M*}(1)$. Since $P_A^M(x', 1 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(x', 0 \mid \sigma^{M*}, \rho^{M*})$ is assumed, $\rho^{M*}(x', \omega) = 1$ for every ω. Hence,

$$\mathbb{E}[P_A^M(x', m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = x'] > \mathbb{E}[P_A^M(x', m \mid \sigma^{M*}, \rho^{M*}) \mid \omega \neq x'],$$

implying that $\sigma^{M*}(0) \leq \sigma^{M*}(1)$.

Here, $\sigma^{M*}(0) \leq \sigma^{M*}(1)$。 As we showed in Proposition 2 (i), this implies Inequality A.2 under $p_B < 2(\sqrt{2} - 1)$. That is, $P_A^M(x', 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(x', 0 \mid \sigma^{M*}, \rho^{M*})$, which is a contradiction.

Therefore, other than the irrelevant monitoring equilibrium, there is no equilibrium where $\sigma^{M*}(0) > 0$ and $\sigma^{M*}(1) < 1$.

Case 2. $\sigma^{M*}(0) = 0$ and $\sigma^{M*}(1) < 1$.

We prove that there is no equilibrium such that $\sigma^{M*}(0) = 0$ and $\sigma^{M*}(1) < 1$. We start by showing that $\rho^{M*}(1, 1) = 0$. Prove by contradiction. Suppose not. Then, $\tilde{p}_A(1, 1) - \tilde{p}_B(1, 1) = \tilde{p}_A^{int}(1)$, while $\tilde{p}_A(1, 0) - \tilde{p}_B(1, 0) = \tilde{p}_A^{int}(1) - \tilde{p}_B(1, 0)$, where $\tilde{p}_B(1, 0) > 0$. Hence, strategic-type party B has no incentive to oppose $x_1 = 1$, which is a contradiction. Therefore, $\rho^{M*}(1, 1) = 0$ holds.

Furthermore, because $\sigma^{M*}(0) \leq \sigma^{M*}(1)$, $P_A^M(0, 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(0, 0 \mid \sigma^{M*}, \rho^{M*})$ holds. Hence, $\tilde{p}_A(0, m) - \tilde{p}_B(0, m) = \tilde{p}_A^{int}(0) - p_B$. This is strictly less than $\tilde{p}_A(1, 0) - \tilde{p}_B(1, 0) = \tilde{p}_A^{int}(1) - p_B$ because $\tilde{p}_A^{int}(1) > \tilde{p}_A^{int}(0)$. That is, strategic-type party B has no incentive to propose policy 0 when $\omega = 1$, which is a contradiction. Therefore, there is no equilibrium where $\sigma^{M*}(0) = 0$ and $\sigma^{M*}(1) < 1$.

Case 3. $\sigma^{M*}(0) = 0$ and $\sigma^{M*}(1) = 1$.

First, as in Case 2, $\rho^{M*}(1, 1) = 0$ holds, implying that $P_A^M(1, 1 \mid \sigma^{M*}, \rho^{M*}) \geq P_A^M(1, 0 \mid \sigma^{M*}, \rho^{M*})$. Second, due to the same logic, $\rho^{M*}(0, 0) = 0$ holds, implying that $P_A^M(0, 1 \mid \sigma^{M*}, \rho^{M*}) \geq P_A^M(0, 0 \mid \sigma^{M*}, \rho^{M*})$.

(i). $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$ for every x_1. In this case, it becomes an irrelevant monitoring equilibrium.

(ii). $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) > P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$ for some x_1. In this case, $\sigma^{M*}(0) = 0$ implies $\sigma^{M*}(1) = 0$, which is a contradiction.

Therefore, other than the irrelevant monitoring equilibrium, there is no equilibrium where $\sigma^{M*}(0) = 0$ and $\sigma^{M*}(1) = 1$.

Case 4. $\sigma^{M*}(0) \in (0, 1)$ and $\sigma^{M*}(1) = 1$.

First, as in Case 2, $\rho^{M*}(0, 0) = 0$ holds, implying that $P_A^M(0, 1) \geq P_A^M(0, 0)$. Second, because $Q(1) \leq \frac{1}{2}$, $p_B < 2(\sqrt{2} - 1)$ implies that $P_A^M(1, 1) = P_A^M(1, 0)$ holds.

(i). $P_A^M(0, 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(0, 0 \mid \sigma^{M*}, \rho^{M*})$. In this case, it becomes an irrelevant monitoring equilibrium.

(ii). $P_A^M(0, 1 \mid \sigma^{M*}, \rho^{M*}) > P_A^M(0, 0 \mid \sigma^{M*}, \rho^{M*})$ for some x_1. In this case, $\sigma^{M*}(0) \in (0, 1)$ implies $\sigma^{M*}(1) = 0$, which is a contradiction.

From (i) and (ii), except for the irrelevant monitoring equilibrium, there is no equilibrium where $\sigma^{M*}(0) \in (0, 1)$ and $\sigma^{M*}(1) = 1$.

Case 5. $\sigma^{M*}(0) = \sigma^{M*}(1) = 1$.

First, as in Case 2, $\rho^{M*}(0, 0) = 0$ holds. Thus, $\tilde{p}_A(0, 0) - \tilde{p}_B(0, 0) = \tilde{p}_A^{int}(0) - p_B$. Furthermore, $p_B < 2(\sqrt{2} - 1)$ implies that $P_A^M(1, 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(1, 0 \mid \sigma^{M*}, \rho^{M*})$ holds; that is, $\tilde{p}_A(1, m) - \tilde{p}_B(1, m) = \tilde{p}_A^{int}(1) - p_B$. Hence, strategic-type party A has an incentive to choose policy 1 when $\omega = 0$ if and only if $\sigma^{N*} = 1$ (see Case 5 in the proof of Proposition 2 (ii)). Therefore, such an equilibrium exists only if $\sigma^{N*} = 1$; that is, other than the irrelevant monitoring equilibrium, there is no equilibrium where $\sigma^{M*}(0) = \sigma^{M*}(1) = 1$.

From cases 1-5, we conclude that the irrelevant monitoring equilibrium is the unique equilibrium if $p_B < 2(\sqrt{2} - 1)$. □

A.2 Party B's Payoff Function

To see that our results do not hinge on specific assumptions on party B's payoff function, suppose that strategic-type party B has the following payoff function: If it is the majority party in period t, its payoff is

$$b + \bar{u}(x_t),$$

whereas if it is the minority party in period t, its payoff is

$$\underline{u}(x_t) - \mathbf{1}_t \kappa,$$

where $\mathbf{1}_t$ is an indicator function that takes one if the party incurs a cost κ to observe ω_t. Note that $\max_{x_t} \bar{u}(x_t) \geq \max_{x_t} \underline{u}(x_t)$ is assumed. When $\bar{u}(x_t) = -|x_t|$ and $\underline{u}_t = 0$, the setting is reduced to the main model.

Suppose that party B chooses m. Then, party B's expected payoff is given by

$$-\underline{u}(x_1) - \mathbf{1}_1\kappa + P_A(x_1, m)\left[\underline{u}(1)\right] + (1 - P_A(x_1, m))\left[b + \max_{x_2} \bar{u}(x_2)\right].$$

Therefore, party B chooses $m = 1$ if $P_A(x_1, 1) < P_A(x_1, 0)$, chooses $m = 0$ if $P_A(x_1, 1) > P_A(x_1, 0)$, and indifferent between them if if $P_A(x_1, 1) = P_A(x_1, 0)$. That is, the analysis in the main text is directly applicable.

As such, the details of party B's utility function does not matter.

A.3 Opposition Probability in the Irrelevant Monitoring Equilibrium

In this section, we examine how the opposition probability of the strategic-type minority party changes depending on the situations. Throughout this subsection, we assume that the distribution about ε, F, is the uniform distribution over $[-0.5, 0.5]$. Furthermore, we focus on the irrelevant monitoring equilibrium corresponding to $\sigma^{N*} = (E^*, E^*)$. These assumptions allow us to explicitly and uniquely derive σ^{N*}, which simplifies the proof.

In our setting, policy 1 is the strategic-type majority party's ideal policy, and thus policy 1 lowers the interim reputation of the majority party. The first issue is whether the implementation of policy 1 increases the probability of opposition by the strategic-type minority party. Interestingly, whether this is the case depends on the initial reputation of the majority party. The result is summarized as follows:

Proposition A.2. *Suppose that F is the uniform distribution over* $[-0.5, 0.5]$. *In the irrelevant monitoring equilibrium corresponding to $\sigma^{N*} = (E^*, E^*)$, the following properties hold:*

(i). If and only if $b > 2/p_B$, $\rho^{M*}(0) > \rho^{M*}(1)$ holds for p_A that is sufficiently close to zero.

(ii). $\rho^{M*}(1) > \rho^{M*}(0)$ holds for p_A that is sufficiently close to one.

Proof. As preliminary results, we first derive σ^{N*} and $\rho^{M*}(x_1)$. By solving $G(\sigma) = 0$,

$$\sigma^{N*}(\omega) = E^* = \min\left\{\frac{-p_A(0.5b + 1) + 1 + \sqrt{0.25p_A^2 b^2 + 1}}{2(1 - p_A)}, 1\right\}.$$

Next, let us consider the value of $\rho^{M*}(x_1)$.

Case 1. $E^* < 1$. From the proof of Proposition 2, the equilibrium probability of the opposition is given by

$$\rho^{M*}(x_1) = \frac{\left(1 - \tilde{p}_A^{int}(x_1)\right)\left(1 + \tilde{p}_A^{int}(x_1) - p_B\right)}{2(1 - p_B)}.[27] \tag{A.1}$$

[27]This is obtained by rearranging $H(\rho) = 0$.

Case 2. $E^* = 1$**.** On the one hand, $\rho^{M*}(1)$ is given by Equation A.1. On the other hand, $\rho^{M*}(0) = 0$ from the proof of Proposition 2 (see the uniqueness part). Here, when $\sigma^{N*} = 1$, $\tilde{p}_A^{int}(0) = 1$ so that Equation A.1 is equal to zero. Hence, $\rho^{M*}(x_1)$ can be given by Equation A.1.

From cases 1 and 2, we conclude that $\rho^{M*}(x_1)$ is given by Equation A.1.

Based on these arguments, we prove (i) and (ii).

(i) It suffices to prove that $\lim_{p_A \searrow 0} \rho^{M*}(1) < \lim_{p_A \searrow 0} \rho^{M*}(0)$. To this end, we first prove that $\lim_{p_A \searrow 0} \rho^{M*}(1) = 0.5$. It is obvious that $\lim_{p_A \searrow 0} \tilde{p}_A^{int}(1) = 0$. Hence, from Equation A.1, $\lim_{p_A \searrow 0} \rho^{M*}(1) = 0.5$ holds.

We next derive the value of $\lim_{p_A \searrow 0} \rho^{M*}(0)$. Here,

$$\tilde{p}_A^{int}(0) = \frac{p_A}{0.5 p_A b + 1 - \sqrt{0.25 p_A^2 b^2 + 1}} =: \frac{f(p_A)}{g(p_A)}.$$

Since $\lim_{p_A \searrow 0} f(p_A) = 0$ and $\lim_{p_A \searrow 0} g(p_A) = 0$ hold, we apply L'Hopital's rule. That is,

$$\lim_{p_A \searrow 0} \tilde{p}_A^{int}(0) = \frac{\lim_{p_A \searrow 0} f'(p_A)}{\lim_{p_B \searrow 0} g'(p_A)} = \frac{2}{b}.$$

By substituting this into Equation A.1, we have

$$\lim_{p_A \searrow 0} \rho^{M*}(0) = \frac{\left(1 - \frac{2}{b}\right)\left(1 + \frac{2}{b} - p_B\right)}{2(1 - p_B)}.$$

Hence,

$$\lim_{p_A \searrow 0} \rho^{M*}(0) > 0.5 = \lim_{p_A \searrow 0} \rho^{M*}(1) \Leftrightarrow b > \frac{2}{p_B}.$$

(ii) It is easy to observe that there exists $\underline{p} \in (0, 1)$ such that $\sigma^{N*} = 1$ for any $p_A \in [\underline{p}, 1)$. Consider $p_A \in [\underline{p}, 1)$. For such p_A, $\rho^{M*}(0) = 0$ holds because $\sigma^{N*} = 1$. Second, $\rho^{M*}(1)$ is decreasing in $\tilde{p}_A$ when $\tilde{p}_A$ is sufficiently close to one, whereas $\lim_{p_A \nearrow 1} \rho^{M*}(1) = 0$. Hence, when p_A is sufficiently close to one, $\rho^{M*}(1) > 0$ holds. Therefore, we conclude that $\rho^{M*}(1) > \rho^{M*}(0)$ for p_A that is sufficiently close to one. □

Hence, when the initial reputation of the majority party is sufficiently large, the implementation of policy 1 unambiguously increases the probability of the opposition by the strategic-type minority party. On the contrary, when the majority party's reputation is too low, the implementation of policy 1 can rather decrease the probability of the opposition. This result is also confirmed by Figure 3, where the solid line represents $\rho^{M*}(1)$ and the dotted line represents $\rho^{M*}(0)$.

Another issue is the effect of each party's initial reputation. As in Figure 3, the effect of the majority party's initial reputation could be non-monotonic: the moderate reputation of the

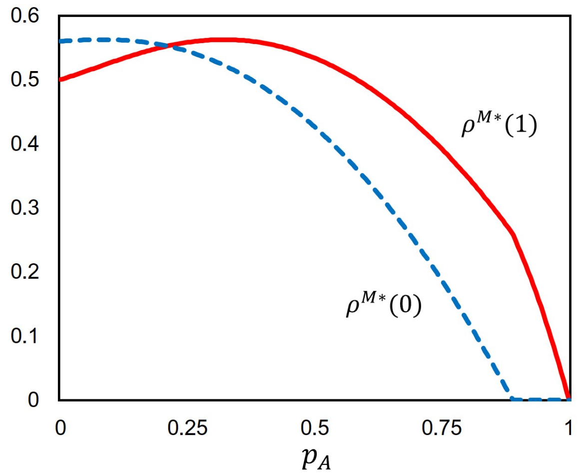

Figure 3: Numerical illustration ($F = U[-0.5, 0.5]$, $b = 10$, $p_B = 0.5$)

majority party increases the opposition probability. While obtaining the analytical result on this effect is hard, we obtain a clear monotonic effect of the minority party's initial reputation, which is summarized in the following proposition:

Proposition A.3. *Suppose that F is the uniform distribution over $[-0.5, 0.5]$. In the irrelevant monitoring equilibrium corresponding to $\sigma^{N*} = (E^*, E^*)$, $\rho^{M*}(x_1)$ is weakly increasing in p_B for any $x_1 \in \{0, 1\}$.*

Proof. From the proof of Proposition A.2, E^* is independent of p_B when F is the uniform distribution. Hence, $\tilde{p}_A^{int}(x_1 \mid \sigma^{N*})$ is also independent of p_B. Therefore, from Equation A.1,

$$\frac{\partial \rho^{M*}(x_1)}{\partial p_B} = \frac{\tilde{p}_A^{int}(x_1)(1 - \tilde{p}_A^{int}(x_1))}{2(1 - p_B)} \geq 0.$$

□

That is, the strategic-type minority party increases the probability of the opposition as its initial reputation becomes higher. When the minority party's reputation is high, the voter believes that the minority party's opposition is likely to indicate not the minority party's harmful activity but the majority party's harmful policymaking. Hence, the strategic-type minority party increases the probability of the opposition. This monotonic effect of the minority party's reputation can be also seen in Figure 4. Note that, in the figure, we do not allow p_B to be larger than 0.8 because the irrelevant monitoring equilibrium may not be the unique equilibrium once p_B exceeds $2(\sqrt{2} - 1)$.

A.4 Equilibrium under High p_B

We start with providing several lemmas.

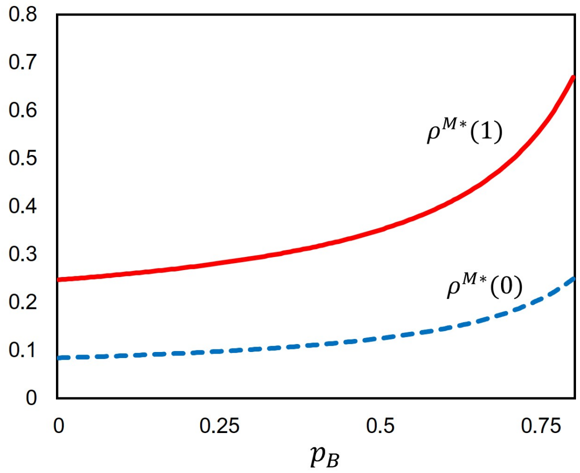

Figure 4: Numerical illustration ($F = U[-0.5, 0.5]$, $b = 10$, $p_A = 0.8$).

Lemma A.2. *Consider an equilibrium that is not an irrelevant monitoring equilibrium corresponding to any σ^{N*}. If $\sigma^{M*}(0) = 0$, $\sigma^{M*}(1) = 1$.*

Proof. Step 1. First, we prove $\rho^{M*}(1) = 0$. Suppose not. Because $\sigma^{M*}(0) = 0$, $x_1 = 1$ only if $\omega_1 = 1$. That is, opposition to policy 1 indicates that the strategic-type party B opposes the bill despite $\omega_1 = 1$. Hence, $\tilde{p}_A(1,1) = \tilde{p}_A^{int}(1), \tilde{p}_B(1,1) = 0$. Similarly, $\tilde{p}_A(1,0) = \tilde{p}_A^{int}(1), \tilde{p}_B(1,0) > p_B$. Therefore, $P_A^M(1,0 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(1,1 \mid \sigma^{M*}, \rho^{M*})$, which contradicts $\rho^{M*}(1) > 0$.

Step 2. Next, we prove $\rho^{M*}(0) > 0$ if $\sigma^{M*}(1) < 1$. Suppose not. Because $\sigma^{M*}(1) < 1$ and $\rho^{M*}(0) = 0$, $\tilde{p}_A(0,1) = 0, \tilde{p}_B(0,1) = 1$. On the other hand, $\tilde{p}_A(0,0) > \tilde{p}_A^{int}(0), \tilde{p}_B(0,0) < p_B$. Therefore, $P_A^M(0,0 \mid \sigma^{M*}, \rho^{M*}) > P_A^M(0,1 \mid \sigma^{M*}, \rho^{M*})$, which contradicts $\rho^{M*}(0) = 0$.

Step 3. Lastly, we prove $\sigma^{M*}(1) = 1$. Suppose not.

From step 1, $\rho^{M*}(1) = 0$. Therefore, $\tilde{p}_A(1,0) = \tilde{p}_A^{int}(1)$, $\tilde{p}_B(1,0) = p_B$, and party B never opposes policy 1 when $\omega_1 = 1$. This implies

$$\mathbb{E}[P_A^M(1, m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1] = P_A^N(1 \mid \sigma^{M*}).$$

From step 2, $P_A(0,0 \mid \sigma^{M*}, \rho^{M*}) \leq P_A(0,1 \mid \sigma^{M*}, \rho^{M*})$ holds. This implies that

$$\mathbb{E}[P_A^M(0, m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1] \leq P_A^N(0 \mid \sigma^{M*}).$$

Furthermore, from the proof of Proposition 1,

$$bP_A^N(0 \mid \sigma^{M*}) - 1 < bP_A^N(1 \mid \sigma^{M*})$$

because $E = 0.5[\sigma^{M*}(1) + \sigma^{M*}(0)] < 0.5$, $G(0.5) > 0$, and $G(E)$ is decreasing in E.

Therefore,

$$\begin{aligned} b\mathbb{E}[P_A^M(0, m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1] - 1 =& bP_A^N(0 \mid \sigma^{M*}) - 1 \\ <& bP_A^N(1 \mid \sigma^{M*}) \\ \leq& b\mathbb{E}[P_A^M(1, m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1]. \end{aligned}$$

This implies that $\sigma^{M*}(1) = 1$, which is a contradiction.

□

Lemma A.3. *Consider an equilibrium that is not an irrelevant monitoring equilibrium corresponding to any σ^{N*}. If $\sigma^{M*}(0) > 0$, $\sigma^{M*}(1) = 1$.*

Proof. Case (i). $P_A^M(x_1, 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(x_1, 0 \mid \sigma^{M*}, \rho^{M*})$ for any x_1. In this case, the equilibrium is an irrelevant monitoring equilibrium, which is a contradiction.

Case (ii). $P_A^M(1, 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(1, 0 \mid \sigma^{M*}, \rho^{M*})$ but $P_A^M(0, 1 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(0, 0 \mid \sigma^{M*}, \rho^{M*})$.

In this case,

$$\begin{aligned} &\mathbb{E}[P_A^M(1, m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0] - \mathbb{E}[P_A^M(0, m) \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0] \\ <&\mathbb{E}[P_A^M(1, m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1] - \mathbb{E}[P_A^M(0, m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1]. \end{aligned}$$

Because

$$\sigma^{M*}(0) > 0 \Leftrightarrow b\left[\mathbb{E}[P_A^M(1, m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0] - \mathbb{E}[P_A^M(0, m) \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0]\right] + 1 \geq 0,$$

this implies that

$$b\left[\mathbb{E}[P_A^M(1, m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1] - \mathbb{E}[P_A^M(0, m) \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1]\right] + 1 > 0.$$

Therefore, $\sigma^{M*}(1) = 1$.

Furthermore, $\rho^{M*}(0) = 1$ because $P_A^M(0, 1 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(0, 0 \mid \sigma^{M*}, \rho^{M*})$.

When $\sigma^{M*}(1) = 1$, $x_1 = 0$ only if $\omega_1 = 0$. Hence, $\tilde{p}_A(0, 1) = \tilde{p}_A^{int}(0), \tilde{p}_B(1, 1) = 0$. Similarly, $\tilde{p}_A(0, 0) = \tilde{p}_A^{int}(0), \tilde{p}_B(0, 0) > p_B$. Therefore, $P_A^M(0, 0 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(0, 1 \mid \sigma^{M*}, \rho^{M*})$, which is a contradiction. That is, there is no equilibrium satisfying case (ii).

Case (iii). $P_A^M(1, 1 \mid \sigma^{M*}, \rho^{M*}) = P_A^M(1, 0 \mid \sigma^{M*}, \rho^{M*})$ but $P_A^M(0, 1 \mid \sigma^{M*}, \rho^{M*}) > P_A^M(0, 0 \mid \sigma^{M*}, \rho^{M*})$.

In this case, $\rho^{M*}(0) = 0$. However, when $\sigma^{M*}(1) < 1$, $P_A^M(0, 1 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(0, 0 \mid \sigma^{M*}, \rho^{M*})$. Therefore, $\sigma^{M*}(1) = 1$ must hold.

Case (iv). $P_A^M(1,1 \mid \sigma^{M*}, \rho^{M*}) > P_A^M(1,0 \mid \sigma^{M*}, \rho^{M*})$.

In this case, $\rho^{M*}(1) = 0$. Given this, $\tilde{p}_A(1,1) = 0$ and $\tilde{p}_B(1,1) = 1$. Therefore, $P_A^M(1,1 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(1,0 \mid \sigma^{M*}, \rho^{M*})$, which is a contradiction.

Case (v). $P_A^M(1,1 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(1,0 \mid \sigma^{M*}, \rho^{M*})$.

(a). $P_A^M(0,1 \mid \sigma^{M*}, \rho^{M*}) \leq P_A^M(0,0 \mid \sigma^{M*}, \rho^{M*})$.

In this case,

$$\begin{aligned}&\mathbb{E}[P_A^M(1,m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0] - \mathbb{E}[P_A^M(0,m) \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0]\\ <&\mathbb{E}[P_A^M(1,m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1] - \mathbb{E}[P_A^M(0,m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1].\end{aligned}$$

Therefore, $\sigma^{M*}(0) > 0 \Rightarrow \sigma^{M*}(1) = 1$.

(b). $P_A^M(0,1 \mid \sigma^{M*}, \rho^{M*}) > P_A^M(0,0 \mid \sigma^{M*}, \rho^{M*})$.

In this case, $\rho^{M*}(0) = 0$. Suppose $\sigma^{M*}(1) < 1$. Then $\tilde{p}_A(0,1) = 0$ and $\tilde{p}_B(0,1) = 0$. Therefore, $P_A^M(0,1 \mid \sigma^{M*}, \rho^{M*}) < P_A^M(0,0 \mid \sigma^{M*}, \rho^{M*})$, which is a contradiction. This implies that $\sigma^{M*}(1) = 1$ must hold.

From cases (i)-(v), in the equilibrium, $\sigma^{M*}(1) = 1$ must hold. □

Based on the above lemmas, we obtain the following proposition, which shows that the presence of monitoring (weakly) improves electoral accountability compared to the most disciplined equilibrium in the game without monitoring. Note that $\sigma^{M*}(0) = 2E^* - 1$ may hold when b is low. For example, in an extreme case where $b = 0$, reelection motives do not exist; thus $\sigma^{N*}(0) = \sigma^{M*}(0) = 1$. However, when b is reasonably high, $\sigma^{M*}(0) < 2E^* - 1$ would hold.

Proposition A.4. *Suppose that there is no irrelevant monitoring equilibrium corresponding to any σ^{N*}. There is an equilibrium, and in the equilibrium, $\sigma^{M*}(1) = 1$ and $\sigma^{M*}(0) \leq 2E^* - 1$.*

Proof. The existence of a perfect Bayesian equilibrium is ensured in this class of games.

When $\sigma^{M*}(0) = 0$, $\sigma^{M*}(1) = 1$ holds by Lemma A.2.

When $\sigma^{M*}(0) > 0$, $\sigma^{M*}(1) = 1$ also holds from Lemma A.3. Therefore, it suffices to show that $\sigma^{M*}(0) \leq 2E^* - 1$ when $\sigma^{M*}(0) > 0$.

From case (iii) and case (v) in Lemma A.3,

$$\mathbb{E}[P_A^M(1,m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0] \leq \mathbb{E}[P_A^N(1 \mid \sigma^{M*})];$$

$$\mathbb{E}[P_A^M(0,m \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0] \geq \mathbb{E}[P_A^N(0 \mid \sigma^{M*})].$$

Let

$$L^N(\sigma) := b[P_A^N(1 \mid \sigma) - P_A^N(0 \mid \sigma)] + 1;$$

$$L^M(\sigma) := b[\mathbb{E}[P_A^M(1,m \mid \sigma, \rho^{M*}) \mid \omega = 0] - \mathbb{E}[P_A^M(0,m \mid \sigma, \rho^{M*}) \mid \omega = 0]] + 1.$$

Then,

$$L^M(1, \sigma^{M*}(0)) \leq L^N(1, \sigma^{M*}(0)).$$

Because $L^M(1, \sigma^{M*}(0)) \geq 0$ by $\sigma^{M*}(0) > 0$, this implies that $L^N(1, \sigma^{M*}(0)) \geq 0$.

(a). If $\sigma^{M*}(1) = 1$ and $L^M(1, \sigma^{M*}(0)) > 0$, $L^N(1, 1) > 0$ holds, implying that $\sigma^{N*}(0) = 2E^* - 1 = 1$. Therefore, $\sigma^{M*}(1) = 2E^* - 1$.

(b). Suppose that $L^M(1, \sigma^{M*}(0)) = 0$. Here, it is easily observed that $L^N(1, \sigma(0))$ is decreasing in $\sigma(0)$ (note that L^N corresponds to G in the proof of Proposition 1). Therefore, $L^M(1, \sigma^{M*}(0)) = 0$ implies that $L^N(1, \sigma(0)) > 0$ for all $\sigma(0) < \sigma^{M*}(0)$. Therefore, $2E^* - 1 = \sigma^{N*}(0) \geq \sigma^{M*}(0)$.

□

Note that it could be also the case where there is an irrelevant monitoring equilibrium corresponding to some σ^{N*}, but there also exist other equilibria. In such a case, by using the same procedure, it is shown that the equilibrium other than irrelevant monitoring equilibria should be $\sigma^{M*}(1) = 1$ and $\sigma^{M*}(0) \leq 2E^* - 1$.

A.5 Strategic Incentive of Sincere Type

Thus far, we have assumed that the sincere-type minority party has two properties: its policy preference is aligned with voters, and it non-strategically tells the truth due to psychological lying cost. However, one might be interested in what happens if the sincere-type's preference is aligned with voters but has no lying cost. Indeed, the literature of cheap-talk games has shown that a sender may tell a lie even if her or his preference is aligned with a receiver's interest.

To examine this case, suppose that the objective of the sincere-type minority party is to maximize

$$\sum_{t=1}^{2} -|x_t - \omega_t|.$$

The minority party does not know the type of majority party. Thus, the minority party updates the belief about the type of the majority party given x_1 and ω_1. Let this updated belief be $\tilde{q}_A^{int}(x_1, \omega)$. Note that this is not equal to voters' interim belief $\tilde{p}_A^{int}(x_1)$ because voters observe only x_1, whereas the minority party observes both x_1 and ω_1.

Given this updated belief, the sincere-type minority party's payoff when choosing $m = 1$ is given by

$$P_A(x_1, 1) \times [-0.5(1 - \tilde{q}_A^{int}(x_1, \omega))] + (1 - P_A(x_1, 1)) \times 0 = -0.5 P_A(x_1, 1)(1 - \tilde{q}_A^{int}(x_1, \omega)). \quad (A.2)$$

On the other hand, the sincere-type minority party's payoff when choosing $m = 0$ is given by

$$-0.5 P_A(x_1, 0)(1 - \tilde{q}_A^{int}(x_1, \omega)). \quad (A.3)$$

Hence, the sincere-type minority party has an incentive to oppose a bill if and only if

$$\text{Equation } A.2 \geq \text{Equation } A.3 \iff (1 - \tilde{q}_A^{int}(x_1, \omega))(P_A(x_1, 0) - P_A(x_1, 1)) \geq 0.$$

As long as $\tilde{q}_A^{int}(x_1, \omega) < 1$, this is further rewritten as

$$P_A(x_1, 0) - P_A(x_1, 1) \geq 0.$$

This is the same as the incentive condition of the strategic-type minority party, indicating that the sincere-type minority party has an incentive to tell a lie as the strategic type does.

The mechanism is understood as follows. The sincere-type minority party knows that itself is of the sincere type, whereas it has a concern that the majority party may be the strategic type. Thus, in terms of implementing the voter-optimal policy in period 2, the sincere-type minority party should maximize its probability of winning independently of whether the proposed bill is good or bad. Consequently, without lying cost, even the sincere-type minority party tells a lie.

In sum, if we allow the sincere-type minority party to strategically oppose a bill, information transmission never arises; and thus monitoring by the minority party does not work at all.

While this is an important result, assuming no lying cost may be too pessimistic given the massive experimental evidence of cheap talk games. Hence, in our basic model, we consider a case where the sincere type is assumed to tell the truth due to lying costs. Our results in the main text show that even in this case, monitoring does not influence electoral accountability despite its informativeness.

A.6 Repeated Policymaking

Thus far, we have assumed that the number of bills discussed on the floor is one. However, in reality, a legislature sequentially takes a vote on various bills. This subsection is devoted to the analysis of this repeated structure.

For this purpose, let us add period 0 to our original model. That is, the timing of the game is given as follows:

Period 0

1. Nature determines the values of ω_0, ω_1, and ω_2. Parties A and B observe the value of ω_0.
2. Party A (the majority party in period 1) chooses x_0.
3. Party B (the minority party in period 1) chooses m_0.
4. The voter observes x_0 and m_0.

Period 1

1. Parties A and B observe the value of ω_1.

2. Party A (the majority party in period 1) chooses x_1.
3. Party B (the minority party in period 1) chooses m_1.
4. The voter observes x_1 and m_1, and ε is realized.

Period 2

1. The voter decides for which party to vote.
2. The parties observe the value of ω_2.
3. The majority party chooses x_2.
4. The payoffs are realized.

Let the voter's belief about party B's type at the beginning of period 1 be $\tilde{p}_B(x_0, m_0)$.

In the legislative process of period 0, both parties take into account its effect on period 1. For example, the minority party may want to build its good reputation to make the monitoring in period 1 work effectively. As shown in the main analysis, opposition in period 1 never influences the electoral result when $\tilde{p}_B(x_0, m_0) < 2(\sqrt{2} - 1)$. Hence, in period 0, party B may try to build a reputation higher than $2(\sqrt{2} - 1)$. The majority party might have a similar incentive.

For the characterization of the equilibrium strategies in period 0, we need to take these incentives into account. Hence, the characterization is quite hard. Indeed, even in the model without monitoring by the minority party, studies deriving the equilibrium in such a repeated policymaking setting are limited.

Instead, as the bottom line, it is shown that with a positive probability, period 1 opposition by the minority party does not influence the electoral result. The formal result is summarized as follows.

Proposition A.5. *Suppose that $p_B < 2(\sqrt{2} - 1)$. With a positive probability, the stage game equilibrium of period 1 is characterized as the irrelevant monitoring equilibrium.*

Proof. From the Bayes rule, there exists on-path (x_0, m_0) such that $\tilde{p}_B(x_0, m_0) \leq p_B$. Therefore, for such (x_0, m_0), $\tilde{p}_B(x_0, m_0) < 2(\sqrt{2} - 1)$. From Proposition 2, this implies that the stage game equilibrium in period 1 is the irrelevant monitoring equilibrium given such (x_0, m_0). Note that such (x_0, m_0) □

Period 1 represents the period where the election is approaching. In the end of the term, depending on the legislative process of the previous period, opposition by the minority party does not influence the electoral result. In this sense, our results are at least partially preserved in the repeated policymaking setting.

A.7 Effect of Free Media

Thus far, we have assumed that the voter does not receive any signal about the desirability of policy implementation other than the cheap-talk message of party B. We believe that this is a useful approach to highlight whether the minority party plays a role as a monitor without other monitoring devices. However, in democratic countries, the independent mass media exists as another monitor of the majority party's activities. To analyze how it changes our conclusion, we model the news reported by the media as an exogenous signal received by the voter. We do not model an endogenous decision by the media because the media's decision is outside the scope of this study.

In particular, we assume that the voter receives a signal $n \in \{G, B, \emptyset\}$,$n \in \{G, B, \emptyset\}$, where $n = G$ (resp. $n = B$) represents the signal indicating that $x = \omega$ (resp. $x \neq \omega$), and $n = \emptyset$ represents the null signal. $\Pr(n = \emptyset \mid x, \omega) = \varepsilon \in (0, 1)$ and $\Pr(n = G \mid x = \omega) = \Pr(n = B \mid x \neq \omega) = \theta(1 - \varepsilon)$, where $\theta \in (0.5, 1]$. This signal is interpreted as news reported by the media, and we assume that this signal is observable to all players. What matters in the analysis is the timing of the signal realization, i.e., whether it is revealed before or after the minority party's decision. We consider each of the two cases in the following.

News-then-minority party's message: First, we consider the case where the voter receives a signal; then party B decides whether to oppose a bill or not. This timing is employed by Stone (2013). In reality, news about controversial issues is frequently reported during the legislative process; thus, the final vote in the legislature (the timing of the minority party's decision) is after news reporting.

The irrelevant monitoring equilibrium is the equilibrium where the equilibrium outcome is the same as in the case without monitoring, which is formally defined as follows in the current context. Let the equilibrium probability of party A's winning given (x_1, n) in the model without monitoring be $P_A^N(x_1, n)$. Similarly, let the equilibrium probability of party A's winning given (x_1, n, m) in the model with the monitoring of the minority party be $P_A^M(x_1, n, m)$.

Definition A.1. *An equilibrium is called the irrelevant monitoring equilibrium if*

(i). $P_A^M(x_1, n, m \mid \sigma^{M*}, \rho^{M*}) = P_A^N(x_1, n \mid \sigma^{N*})$ *holds for any* $(x_1, m, n) \in \{0, 1\} \times \{0, 1\} \times \{G, B, \emptyset\}$ *if* (x_1, n, m) *is on the equilibrium path; and*

(ii). $\sigma^{M*}(\omega_1) = \sigma^{N*}(\omega_1)$ *holds for any* $\omega_1 \in \{0, 1\}$.

Let $\tilde{p}_A^{int}(x_1, n)$ be the equilibrium probability that party A is of the sincere type, given x_1 and n. Given this notation, we obtain the following result on the existence of an irrelevant monitoring equilibrium. To simplify the analysis, we focus on a reasonable case where $\sigma^{N*}(\omega) \in (0, 1)$ for all ω.

Proposition A.6. *There exists an irrelevant monitoring equilibrium corresponding to* σ^{N*} *if*

$$p_B \leq \min\left\{\frac{1 - Q(1 \mid \sigma^{N*})(1 - \tilde{p}_A^{int}(1, n \mid \sigma^{N*})^2)}{1 - Q(1 \mid \sigma^{N*})(1 - \tilde{p}_A^{int}(1, n \mid \sigma^{N*}))}, \frac{1 - Q(0 \mid \sigma^{N*})(1 - \tilde{p}_A^{int}(0, n \mid \sigma^{N*})^2)}{1 - Q(0 \mid \sigma^{N*})(1 - \tilde{p}_A^{int}(0, n \mid \sigma^{N*}))}\right\} \tag{A.4}$$

holds for any n.

Proof. Suppose the irrelevant monitoring equilibrium. If there exists ρ such that $P_A^M(x_1, n, 1 \mid \sigma^{N*}, \rho) = P_A^M(x_1, n, 0 \mid \sigma^{N*}, \rho)$ holds for any (x_1, n), the equilibrium is the irrelevant monitoring equilibrium as in Lemma 2. Thus, it suffices to derive the condition under which $P_A^M(x_1, n, 1 \mid \sigma^{N*}, \rho) = P_A^M(x_1, n, 0 \mid \sigma^{N*}, \rho)$ holds.

The procedure to obtain the condition is the same as that in the proof of Lemma 3. By replacing $\tilde{p}_A^{int}(x_1)$ in the proof with $\tilde{p}_A^{int}(x_1, n)$, we obtain the above condition as that corresponding to Inequality A.2. □

Therefore, an irrelevant monitoring equilibrium exists under a certain condition. Intuition is understood as follows. Given that the majority party proposes x_1 and the news received by the voter is n, the minority party faces a decision on whether to oppose the bill or not. In the mixed-strategy equilibrium, $P_A^M(x_1, n, 1)$ and $P_A^M(x_1, n, 0)$ are equalized as in the main analysis. As such, the existence of free media does not necessarily resolve the irrelevance of the minority party's monitoring.

In sum, the irrelevance of the minority party's monitoring is not necessarily resolved when the minority party's decision comes after news reporting.

Minority party's message-then-news: Next, consider another scenario where the minority party decides whether to oppose a bill or not; then the voter receives a signal. This describes a situation where the legislative activities of the minority party trigger news reports, which is employed by Kishishita (2019). From now on, we assume $\theta = 1$ for simplicity. That is, the voter receives either a perfect signal or a null signal.

We define the irrelevant monitoring equilibrium in the current context as follows.

Definition A.2. *An equilibrium is called the irrelevant monitoring equilibrium if*

(i). $\mathbb{E}[P_A^M(x_1, m, n \mid \sigma^{M*}, \rho^{M*}) \mid x_1, m, \omega_1] = \mathbb{E}[P_A^N(x_1, n \mid \sigma^{N*}) \mid x_1, \omega_1]$ *holds for any* $(x_1, m, n) \in \{0, 1\} \times \{0, 1\} \times \{G, B, \emptyset\}$ *if* (x_1, m, n) *is on the equilibrium path; and*

(ii). $\sigma^{M*}(\omega_1) = \sigma^{N*}(\omega_1)$ *holds for any* $\omega_1 \in \{0, 1\}$.

In the timing of the minority party's cheap-talk message, the signal has not been realized yet. Thus, we take the expectation with respect to n in condition (i). Furthermore, the distribution of n depends on the true state of the world and the proposed bill; therefore, we consider the expectation conditional on (x_1, ω_1). If condition (i) holds, the message from the minority party does not influence the expected electoral result from the majority party's perspective; thus (ii) holds.

Based on this definition, we obtain the following result, which indicates that no irrelevant monitoring equilibrium exists.

Proposition A.7. *When the voter receives an external signal after the minority party's cheap-talk message, there is no irrelevant monitoring equilibrium.*

Proof. Without loss of generality, we consider the case where $x_1 = 1$. Let $I(x_1, \omega)$ be the net effect of opposing the bill given ω. That is,

$$\begin{aligned} I(1,1) =& \mathbb{E}[P_A^M(1,0,n \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1] - \mathbb{E}[P_A^M(1,1,n \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 1] \\ =& \varepsilon P_A^M(1,0,\emptyset \mid \sigma^{M*}, \rho^{M*}) + (1-\varepsilon) P_A^M(1,0,G \mid \sigma^{M*}, \rho^{M*}) \\ & - \left[\varepsilon P_A^M(1,1,\emptyset \mid \sigma^{M*}, \rho^{M*}) + (1-\varepsilon) P_A^M(1,1,G \mid \sigma^{M*}, \rho^{M*})\right]; \end{aligned} \tag{A.5}$$

$$\begin{aligned} I(1,0) =& \mathbb{E}[P_A^M(1,0,n \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0] - \mathbb{E}[P_A^M(1,1,n \mid \sigma^{M*}, \rho^{M*}) \mid \omega = 0] \\ =& \varepsilon P_A^M(1,0,\emptyset \mid \sigma^{M*}, \rho^{M*}) + (1-\varepsilon) P_A^M(1,0,B \mid \sigma^{M*}, \rho^{M*}) \\ & - \left[\varepsilon P_A^M(1,1,\emptyset \mid \sigma^{M*}, \rho^{M*}) + (1-\varepsilon) P_A^M(1,1,B \mid \sigma^{M*}, \rho^{M*})\right]. \end{aligned}$$

Then,

$$\begin{aligned} & I(1,1) - I(1,0) \\ =& (1-\varepsilon)\times \\ & \left[P_A^M(1,0,G \mid \sigma^{M*}, \rho^{M*}) - P_A^M(1,1,G \mid \sigma^{M*}, \rho^{M*}) - P_A^M(1,0,B \mid \sigma^{M*}, \rho^{M*}) + P_A^M(1,1,B \mid \sigma^{M*}, \rho^{M*})\right]. \end{aligned}$$

Here, signal G perfectly reveals that party A proposed a good bill. Thus, $\tilde{p}_A(1, m, G)$ is independent of m, while $\tilde{p}_B(1,1,G) < \tilde{p}_B(1,0,G)$ obviously holds. Therefore, $P_A^M(1,0,G \mid \sigma^{M*}, \rho^{M*}) < P_A^M(1,1,G \mid \sigma^{M*}, \rho^{M*})$ is obtained. Similarly, $P_A^M(1,0,B \mid \sigma^{M*}, \rho^{M*}) > P_A^M(1,1,B \mid \sigma^{M*}, \rho^{M*})$ holds. Therefore, it is shown that $I(1,1) - I(1,0) < 0$ holds, implying that the net benefit of opposing a bill for the minority party is higher when $(x_1, \omega_1) = (1,1)$ than when $(x_1, \omega_1) = (1,0)$.

Lastly, since $I(1,1) - I(1,0) < 0$, either $I(1,1)$ or $I(1,0)$ is non-zero. Furthermore, by construction,

$$I(x_1, \omega_1) = \mathbb{E}[P_A^M(x_1, 0, n \mid \sigma^{M*}, \rho^{M*}) \mid x_1, m, \omega_1] - \mathbb{E}[P_A^N(x_1, 1, n \mid \sigma^{M*}, \rho^{M*}) \mid x_1, \omega_1].$$

Therefore, for some ω_1,

$$\mathbb{E}[P_A^M(x_1, 0, n \mid \sigma^{M*}, \rho^{M*}) \mid x_1, m, \omega_1] \neq \mathbb{E}[P_A^N(x_1, 1, n \mid \sigma^{M*}, \rho^{M*}) \mid x_1, \omega_1],$$

which violates the condition for the irrelevant monitoring equilibrium. □

Intuitively, the irrelevance of the minority party's monitoring is resolved because the ex-post signal creates a lying cost for the minority party. When the minority party opposes the bill but $x = \omega$ is revealed, the reputation of party B is severely undermined, which works as a lying cost. Therefore, the game becomes a costly signaling game; that is, the game is no longer a cheap-talk game. Consequently, the monitoring by party B changes the electoral result; thus, the irrelevant monitoring equilibrium no longer exists in this scenario.

However, it should be noted that as ε goes to one, the electoral effect of the minority party's message also goes to zero. To see this, remember, for example, that the effect of the minority party's opposition when $x_1 = \omega_1 = 1$,

$$I(1,1) = (1-\varepsilon)\left[\mathbb{E}[P_A^M(1,0,n \mid \sigma^{M*}, \rho^{M*}) \mid \omega_1 = 1] - \mathbb{E}[P_A^N(1,1,n \mid \sigma^{M*}, \rho^{M*}) \mid \omega_1 = 1]\right],$$

is given by Equation A.5. Apparently, this value vanishes to zero as ε goes to one. In this sense, there is no discontinuity; an arbitrary small probability of the true state being revealed is not sufficient for effective monitoring by the minority party. In certain situations, the probability that news reports the truth might not be so high because of factors on the supply side or demand side of the media bias. Our results indicate that the minority party's cheap-talk message in such a situation can be approximated by our main analysis.

Summary: We have analyzed the two cases, each of which reasonably captures a certain situation where news is reported. The results indicate that the presence of free media may or may not resolve the irrelevance of the minority party's monitoring, depending on whether news comes before the minority party's message or not. Furthermore, even if it resolves the irrelevance result, it does not imply that the minority party can serve as a monitor alone. The presence of free media, an additional monitoring device, is at least necessary (not sufficient). These results indicate that the minority party's ability in monitoring the majority party's activities is limited, as in our main analysis.