Online Appendix

Economic Uncertainty, Aggregate Debt, and the Real Effects of Corporate Finance

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A. Proofs

This appendix provides the proofs of the results in Section 2.

The first proposition solves the firm's capital structure problem before having found the pricing kernel. This is possible because we can deduce enough about aggregate dynamics in advance from the following lemmas.

The first lemma formalizes the dynamics of output deduced in the text.

Lemma 1. *Assume that firm value and optimal debt are linear linear in output – $V^{(i)} = v(\sigma)Y^{(i)}$ and $B^{(i)} = b(\sigma)Y^{(i)}$ – and that $b < v$ for all σ.*

Then aggregate output, including entry and exit effects, obeys the stochastic differential equation:

$$\frac{dY}{Y} = \mu_Y \, dt + \, d\left[\sum_{j=1}^{\mathscr{J}_t} \left((u_t - 1)1_{\{j,+\}} + (d_t - 1)1_{\{j,-\}}\right)\right]. \qquad (17)$$

where $\mu_Y = (\mu + \zeta(I/Y))$ includes the growth in the mass of firms due to aggregate investment, and $1_{\{j,\pm\}}$ are indicators for the sign of the jth jump, and $u_t = u(\sigma_t)$ is

$$\mathrm{E_t}\left[e^{\varphi_j} | \varphi_j > 0\right]$$

and

$$d_t = \mathrm{E_t}\left[e^{\varphi_j} 1_{\{\varphi_j > \varphi^\star\}} | \varphi_j < 0\right] = \int_{\varphi^\star}^{0} e^{\varphi} \, d\mathscr{F}^{\varphi-}(\sigma_t)$$

where $\varphi^\star = \varphi^\star(\sigma)$ is the critical threshold for jumps below which firms *default.*

Proof. Formally, the probabilistic structure of the model assumes a set of $i.i.d.$ firms indexed by $m \in [0, \bar{M}]$ where the interval is technically a dense limit of increasing countable subsets.[35] At time t, the set of firms that have come into existence is indexed by the subinterval $[0, M_t]$, where $M_t < \bar{M}$. For simplicity, we take M_t to be nondecreasing in t, meaning that the "mass" M_t counts firms with zero output (those that have exited). Also notice that the structure implicitly imposes that the distribution of output (or any other characteristic) of entering firms, i.e., those in $[M_t, M_t + dM_t)$, is the same as that of those that have previously entered by time t.

With this set-up, the dynamics of Y before considering entry and exit follow from a law of large numbers applied across firms at each point in time, as described in the text.

We deduce the existence of a single default-inducing jump threshold for all firms from the linearity assumption. (If no such threshold exists at t, take $\varphi^{\star}(t) = -\infty$.) The assumption $b < v$ implies that, absent a jump, there is no default due to changes in the aggregate state σ_t. The effect entry is to increase the mass of firms according to $dM/M = \zeta(I/Y)\, dt$ by assumption. Because the output distribution of the entering firms is the same as that of incumbents, their contribution to dY is just $Y dM$. *QED*

The next lemma characterizes the form of the economy's value function, and deduces the dynamics of consumption and marginal utility.

Lemma 2. *Given an output process described by (17), the representative*

[35] Formally, the "continuum" can be described as the limit of economies with countable, increasing index sets, each of which is endowed with a finitely additive measure of total mass M. Al-Najjar (2004) shows that integration of random variables in the limit economy is well defined and a strong law of large numbers applies.

agent's value function is of the form $J = j(\sigma)\, Y^{1-\gamma}/(1-\gamma)$.

The aggregate consumption process is $C = c(\sigma)Y$. The functions $j(\sigma)$ and $c(\sigma)$ are characterized (respectively) by an ordinary differential equation and an algebraic equation given in the proof.

Let Λ denote the pricing kernel. Its dynamics may be written

$$\frac{d\Lambda}{\Lambda} = \eta_0(\sigma)\, dt + \eta_1(\sigma)\, dW + d\left[\sum_{j=1}^{\mathcal{J}_t}\Big((u(\sigma)^{-\gamma}-1)1_{\{j,+\}} + (d(\sigma)^{-\gamma}-1)1_{\{j,-\}}\Big)\right].$$

Proof. Given the aggregator function $f(C,J)$, the Bellman equation for J tells us that $\max_C\{\mathrm{E}[dJ] + f(C,J)\; dt\} = 0$. Under the conjectured form for $J = J(\sigma,Y)$, and using the known dynamics of σ, and Y, we have $\mathrm{E}[dJ]/J =$

$$\frac{j(\sigma)'}{j(\sigma)}\, m(\sigma) + (1-\gamma)\mu_Y(\sigma) + \tfrac{1}{2}s^2(\sigma)\, \frac{j(\sigma)''}{j(\sigma)} + \tfrac{1}{2}\lambda[(u(\sigma)^{1-\gamma}-1) + (d(\sigma)^{1-\gamma}-1)]$$

using the version of Itô's lemma for jumping processes. Dividing $f(C,J)$ by J and using the conjectured form of C, we get the two terms[36]

$$\beta\theta c(\sigma)^{\rho} j(\sigma)^{-\frac{1}{\theta}} - \beta\theta.$$

Adding these to the $\mathrm{E}[dJ]/J$ terms and multiplying by j gives the ODE:

$$\beta\theta c(\sigma)^{\rho} j(\sigma)^{1-\frac{1}{\theta}} - \beta\theta j(\sigma) + j(\sigma)' m(\sigma) +$$

$$\tfrac{1}{2}s^2(\sigma) j(\sigma)'' + (1-\gamma)\mu_Y j(\sigma) + \tfrac{1}{2}\lambda[(u(\sigma)^{1-\gamma}-1) + (d(\sigma)^{1-\gamma}-1)]j = 0$$

[36]Recall $f(C,J) = \frac{\beta C^{\rho}/\rho}{((1-\gamma)\; J)^{1/\theta-1}} - \beta\theta J$.

or, more compactly,

$$\tfrac{1}{2}s^2 j'' + j'm + \beta\theta c^{\rho} j^{1-\frac{1}{\theta}} + \big((1-\gamma)\mu_Y + \tfrac{1}{2}\lambda[(u^{1-\gamma}-1)+(d^{1-\gamma}-1)] - \beta\theta\big)j = 0 \tag{18}$$

which must hold at the optimal consumption policy. Recall that $\mu_Y = \mu + \zeta(1 - c(\sigma))$. Hence, the FOC for consumption is simply

$$\beta c^{\rho-1} j^{-\frac{1}{\theta}} = \zeta'(1-c). \tag{19}$$

Given any smooth function $c(\sigma)$, the ODE defining j is to be solved on the closed interval $[\underline{\sigma}, \bar{\sigma}]$, and coefficient $s(\sigma)$ on the second order term is zero at the endpoints. This is equivalent to two mixed boundary conditions (i.e., a relation $g(j', j) = 0$), which suffices for existence and uniqueness of a solution. (Baxley and Brown (1981).) Then (19) is just an algebraic equation for $c(\sigma)$ given $j(\sigma)$. Formally, the implicit solution can be inserted in the coefficients of the ODE. In practice, solving the two equations iteratively rapidly yields a convergent solution for the pair of functions. The existence of these solutions verifies the conjectured functional forms.

Given these J and C functions, Duffie and Skiadas (1994) show that the pricing kernel under stochastic differential utility is

$$\Lambda_t = e^{\int_0^t f_J(C_u, J_u)\, du} f_C(C_t, J_t).$$

Here $f_C(C,J) = \beta\ c(\sigma)^{\rho-1} j(\sigma)^{1-\frac{1}{\theta}}\ Y^{-\gamma}$

The drift and diffusion coefficient of Λ can be readily evaluated (as functions of σ) by Itô's lemma and straightforward algebra, and are not of immediate interest. What is important is that $d\Lambda/\Lambda$ inherits the jump structure of $Y^{-\gamma}$, which is equivalent to the conclusion of the lemma. *QED*

We now proceed to the proof of Proposition 1. While the preceding lemma would appear to have characterized aggregate dynamics, in fact, only the *form* of the pricing kernel has been determined. What has not been pinned down are the critical jump threshold $\varphi^\star(\sigma)$ and the size of the downward jump $d(\sigma)$.

Proof of Proposition 1.

To start, assume firm value is linear in output prior to default.

The proposition first asserts that the optimal default policy for equity holders is to abandon if and only if, following a jump to $Y_t^{(i)}$, the value of the firm is below the pre-jump level of optimal debt, $B_{t-}^{(i)}$. If equity holders do not abandon, then their optimal debt policy at t is to adjust to $B_t^{(i)}$. If they do so, they repay the difference $B_{t-}^{(i)} - B_t^{(i)} > 0$ to debt holders, and their claim is now worth $V_t^{(i)} - B_t^{(i)}$. Clearly they will do this if and only if the debt repayment is less than the value they receive:

$$V_t^{(i)} - B_t^{(i)} > B_{t-}^{(i)} - B_t^{(i)} \iff V_t^{(i)} > B_{t-}^{(i)}$$

as asserted.

From this observation, it follows that we can link the optimal leverage ratio prior to a jump with the critical default threshold. Default occurs iff $V_t^{(i)} \leq B_{t-}^{(i)}$. So dividing by $B_{t-}^{(i)}$, we have

$$e^{\varphi^\star} \equiv \frac{Y_t^{(i),\star}}{Y_{t-}^{(i)}} = \frac{V_t^{(i),\star}}{V_{t-}^{(i)}} = \frac{B_{t-}^{(i)}}{V_{t-}^{(i)}}$$

where the second inequality uses the conjectured linearity. Because the left side here is only a function of the aggregate state, the linearity of $V^{(i)}$ thus implies that of $B^{(i)}$. Denote the optimal market leverage ratio ℓ. Then we have characterized the optimal bankruptcy barrier given optimal leverage as

$$e^{\varphi^\star} = b(\sigma)/v(\sigma) = \ell(\sigma). \tag{20}$$

The value of the ith firm is characterized by the condition

$$\mathrm{E}[d\Lambda V^{(i)}]/(\Lambda V^{(i)}) = -((1-\tau)Y^{(i)} + \bar{r}\tau B^{(i)})\,dt/V^{(i)}.$$

The numerator on the right is the firm's after-tax earnings when interest deduction is permitted at the statutory rate $\bar{r}$, and the tax rate is τ.

Let us conjecture that, prior to default, $V^{(i)} = v(\sigma)(1-\tau)Y^{(i)}$. Given the form of the pricing kernel, applying Itô's lemma to the left side of the above condition gives the equation

$$\begin{aligned} \tfrac{1}{2}s^2(\sigma)v'' + [m(\sigma) + \eta_1(\sigma)s(\sigma)]v' + & \\ \big(\eta_0 + \mu + \tfrac{1}{2}\lambda[(u^{1-\gamma}-1) + (d^{1-\gamma}-1) + \bar{r}\tau\ell(\sigma)]\big)v + 1 &= 0. \end{aligned} \tag{21}$$

As with the j equation above, existence and uniqueness of a solution to this equation will verify the linearity conjecture.

Now we consider the first order condition that maximizes v with respect to b, or, equivalently, with respect to ℓ.

Differentiating (21), there are contributions from the benefit flow term as well as from the down-jump term $d^{1-\gamma}-1$. The latter term is the expectation of the percentage jump in the product $\Lambda V^{(i)}$. The ratios $V_t^{(i)}/V_{t-}^{(i)}$

and Λ_t/Λ_{t-} are independent given a jump, and the pricing kernel term is $d^{-\gamma}$. The firm takes this component as given and not affected by its default decision. However, the jump in own-firm value is affected. Hence we differentiate

$$\int_{\varphi^\star}^{0} e^{\varphi}\ d\mathscr{F}^{\varphi-}$$

and multiply by $d^{-\gamma}$. From above, we know $\varphi^\star = \log(\ell)$. Differentiating this and using the chain rule gives the FOC as

$$\tfrac{1}{2}\lambda d^{-\gamma} f^{\varphi-}(\varphi^\star) = \bar{r}\tau$$

where $f^{\varphi-}$ is the density function of the negative jumps. And from Lemma 1,

$$d = \int_{\varphi^\star}^{0} e^{\varphi}\ d\mathscr{F}^{\varphi-}.$$

The preceding two equations form a system whose solutions are d and $\varphi^\star$. This closes the problem. It is easy to see that the first equation describe a locus of points d that is monotonically increasing from zero in $|\varphi^\star|$. The second describes a locus that monotonically decreases to zero as long as the density function does so, which has been assumed. Hence the system has a unique interior solution. (The fact that $\varphi^\star = 0$ is not a solution verifies the assertion that $b < v$ for all σ. That is, jumps alone can trigger default.)

So far, the derivation has assumed that leverage would be chosen to maximize the value of the firm. The proposition also asserts that resulting policy would also followed by managers who could not commit to maximizing firm value, and instead maximized the value of equity. Intuitively, this is a consequence of the stipulation that the price, p, of the debt contract per unit face value is always one, which implies that no policy can expropriate value from existing debt holders.

Formally, if the firm is at the firm-value maximizing value, policy pair V', B' then equity holders can costlessly move to any V'', B'' by paying (or receiving if negative) the difference in debt amounts $B' - B''$. Including this payment, equity holders will have achieved net value $V'' - B'$. But, by assumption, this is strictly less than the original value they had, $V' - B'$.

QED

Proof of Corollary 2.1

Let P denote the value of an arbitrary debt contract and $p = P/B$ be its price per unit face value. Let τ denote the sooner of the firm's default time and the repayment time of the contract. (The debt contract considered in the paper has no formal maturity. However, the firm has the right to alter the amount outstanding costlessly at any time. We can consider a repayment of amount ΔB as applying *pro rata* randomly across bonds. So any individual bond can be considered to have a stochastic retirement time.) Then on $[0, \tau)$, p solves the valuation equation

$$\begin{aligned} & \tfrac{1}{2}s^2(\sigma)p'' + [m(\sigma) + \eta_1(\sigma)s(\sigma)]p' + \\ \left(\eta_0 + \tfrac{1}{2}\lambda[(u^{-\gamma}\mathrm{E_t}\left[\frac{p^+}{p}\right] - 1) + (d^{-\gamma}\mathrm{E_t}\left[\frac{p^-}{p}\right] - 1)]\right)p + \Gamma & = 0. \quad (22) \end{aligned}$$

where Γ is the coupon rate and $\frac{p^+}{p}$ and $\frac{p^-}{p}$ denote the fractional changes in p conditional on an up and down jump, respectively.

We require that Γ be set such that $p = 1$ solves this equation. And we are assuming $p = 0$ on default. In that case, the equation reads

$$\eta_0 + \tfrac{1}{2}\lambda[(u^{-\gamma} - 1) + (d^{-\gamma}\mathscr{F}^{\varphi-}(\varphi^\star) - 1)] + \Gamma = 0.$$

or

$$\eta_0 + \tfrac{1}{2}\lambda[(u^{-\gamma} - 1) + (d^{-\gamma} - 1)] - \tfrac{1}{2}\lambda d^{-\gamma}(1 - \mathscr{F}^{\varphi-}(\varphi^\star))] + \Gamma = 0.$$

We then recognize that the first two term are the drift rate of the pricing kernel, Λ, which is equal to minus the instantaneous riskless rate, r. Hence,

$$\Gamma = r + \tfrac{1}{2}\lambda d^{-\gamma}(1 - \mathscr{F}^{\varphi-}(\varphi^\star)).$$

QED

Proof of Corollary 2.2

The assumption now is that, creditors of a firm that has defaulted receive a payment ΘB_{t-} where B_{t-} is the face value of debt prior to default. The government does not have the ability to create the value lost due to default, however. Those losses create the same decline in aggregate consumption as in the base case. (So implicitly a tax on all households must fund the creditors' insurance payout.)

To derive the effect on optimal capital structure, we revisit the equation (21) for firm value. Previously, the contribution from the expected change in $\Lambda\, V^{(i)}$ from down jumps was $\frac{1}{2}\lambda$ times

$$d^{-\gamma}\int_{\varphi^\star}^{0} e^{\varphi}\, d\mathscr{F}^{\varphi-} - 1.$$

Now there is an additional contribution to the left-hand term from the default insurance that creditors collect:

$$d^{-\gamma}\left[\int_{\varphi^\star}^{0} e^{\varphi}\, d\mathscr{F}^{\varphi-} + \Theta\frac{B^{(i)}}{V^{(i)}}\int_{-\infty}^{\varphi^\star} d\mathscr{F}^{\varphi-}\right].$$

Differentiating the new term with respect to $\ell = B^{(i)}/V^{(i)}$ adds the two terms

$$\Theta\int_{-\infty}^{\varphi^\star} d\mathscr{F}^{\varphi-} + \Theta\, f^{\varphi-}(\varphi^\star).$$

So the full FOC becomes

$$\tfrac{1}{2}\lambda\, d^{-\gamma}\,[(1-\Theta)\, f^{\varphi-}(\varphi^\star) - \Theta\, \mathscr{F}^{\varphi-}(\varphi^\star)] = \bar{r}\tau.$$

As in Proposition 1, this FOC can be solved jointly with the equation $d = \int_{\varphi^\star}^{0} e^{\varphi}\, d\mathscr{F}^{\varphi-}$.

Besides altering the optimal leverage, the firm value equation must be solved with the extra term given above. In addition, the solution for the credit spread picks up a factor of $(1-\Theta)$. *QED*

Proposition 2 now simply finishes the characterizations of the quantities in Lemma 2 above. Now that d and $\varphi^\star$ have been determined explicitly, the coefficients in the differential equation for $j(\sigma)$ and the algebraic equation for $c(\sigma)$ are fully specified. The proof just finishes the description of the pricing kernel.

Proof of Proposition 2

The Lemma determined that

$$\Lambda_t = e^{\int_0^t f_J(C_u,J_u)\,du}\, f_C(C_t,J_t),$$

and

$$f_C(C,J) = \beta\; c(\sigma)^{\rho-1} j(\sigma)^{1-\frac{1}{\theta}}\; Y^{-\gamma}.$$

Denote the product of c and j terms in this expression as $a(\sigma)$. Also, after some cancellations,

$$f_J(C,J) = \beta\theta\left[(1-\frac{1}{\theta})\; c(\sigma)^{\rho} j(\sigma)^{-\frac{1}{\theta}} - 1\right].$$

The task is to evaluate $d\Lambda/\Lambda$. The integral term just contributes an f_J term to the drift. To this we add df_C/f_C, which is

$$\left[\tfrac{1}{2}\frac{a''}{a}s^2 + \frac{a'}{a}m + \mu_Y\right] dt + \frac{a'}{a}s\, dW + d\left[\sum_{j=1}^{J_t}\Big((u^{-\gamma}-1)1_{\{j,+\}} + (d^{-\gamma}-1)1_{\{j,-\}}\Big)\right].$$

The diffusion coefficient here is $sa'/a = s[(\rho-1)c'/c + (1-1/\theta)j'/j]$, which is called η_1 in the Proposition. Likewise η_0 is the drift term plus f_J. The full expression for a''/a is omitted for brevity. The expression in the proposition for riskless rate is just minus the drift of $d\Lambda/\Lambda$. *QED*

Likewise, there is nothing formally to prove for Proposition 3, because the proof of Proposition 1 already deduced the ODE solved by $v(\sigma) = V^{(i)}/(1-\tau)Y^{(i)}$. There it was only necessary to observe its form in order to take the first order condition for optimal debt. Now that the kernel and the debt policy have been explicitly obtained, the ODE is fully specified and (as observed above) a unique solution exists. We can redefine v to be that solution times $(1-\tau)$ to obtain the solution in terms of pre-tax output $V^{(i)} = v(\sigma)Y^{(i)}$.

The following corollary computes the risk premia for the firm's claims.

Corollary A.1. *The expected excess return to the firm's assets is*

$$\pi_V = -\frac{v'}{v}s\eta_1 + \tfrac{1}{2}\lambda\left((u-1)+(d-1)+(u^{-\gamma}-1)+(d^{-\gamma}-1)-(u^{1-\gamma}-1)-(d^{1-\gamma}-1)\right).$$

The expected excess return to the firm's debt is

$$\pi_F = \tfrac{1}{2}\lambda\ (d(\sigma)^{-\gamma}-1)\ (1-\mathscr{F}^{-\varphi}(\varphi^{\star})).$$

The expected excess return to the firm's equity, π_E is given by the solution to

$$\pi_V = \frac{1}{1-\ell}\pi_E + \frac{\ell}{1-\ell}\pi_F.$$

Proof of Corollary

The valuation ODE for V equates

$$\tfrac{1}{2}\frac{v''}{v}s^2 + \frac{v'}{v}m + \mu + \frac{1-\tau}{v} + \bar{r}\tau\frac{b}{v} + \eta_0$$

to

$$-\left(\eta_1 s\frac{v'}{v} + \tfrac{1}{2}\lambda((u^{1-\gamma}-1)+(d^{1-\gamma}-1)\right).$$

If we add to each side $\frac{1}{2}\lambda((u^{-\gamma}-1)+(d^{-\gamma}-1))$ we can then substitute out the sum of these terms and η_0 for $-r$ in the top expression. Then add to each side $\frac{1}{2}\lambda((u-1)+(d-1))$ and the top expression becomes the (true) expected excess returns to dV/V. We conclude that π_V is

$$-\frac{v'}{v}s\eta_1+\tfrac{1}{2}\lambda\big((u-1)+(d-1)+(u^{-\gamma}-1)+(d^{-\gamma}-1)-(u^{1-\gamma}-1)-(d^{1-\gamma}-1)\big).$$

which we can also write as

$$\pi_V = -\frac{v'}{v}s\eta_1 + \tfrac{1}{2}\lambda\big[(u-1)(u^{-\gamma}-1)+(d-1)(d^{-\gamma}-1)\big].$$

The debt contract has no expected change per unit time, outside of default. So its expected excess return is the coupon rate minus the riskless rate plus the instantaneous default intensity. But this is just the difference between the credit spread, determined above, (which is also the risk neutral default intensity) and the true default intensity, giving the expression in the corollary. Finally, by no arbitrage, the risk premium on the firm is the value weighted combination of debt and equity claims. This is the last assertion in the corollary.

QED

B. Data and Estimation

This appendix describes the data and estimation procedures used in Section 3.

Aggregate Moments

The model has several simplifying assumptions about firms and the economy that make choice of empirical counterparts somewhat subjective. The list below discusses the proxies chosen and some possible alternatives.

Leverage:

The quantity b in the model is firm's debt face value divided by (pre-tax) cash-flow, or output. In the data, I thus need to choose pairs of (debt,output) measures that correspond to the same set of firms. The firm is supposed to be representative of the entire economy. The broadest measure, and the main proxy used, is from the Federal Reserves Z1 reports (The flow-of-funds accounts) for the U.S. nonfinancial corporate sector. Specifically debt is bank loans and bonds (long and short term), and output is net operating surplus plus consumption of fixed capital.[37] In one test, debt is scaled instead by firm assets (historical cost).[38]

The main tests in the paper use net debt, subtracting cash and cash equivalents.[39] To check robustness to the inclusion of off-balance sheet liabilities in total debt, another version adds retirement entitlements (pensions and healthcare liabilities, FL103152025.Q) to the numerator.

For further robustness, I also consider a broader measure of the private sector. Debt, also from the flow of funds accounts, includes the noncorporate sector, meaning primarily private firms, partner-

[37] The respective Z1 data items are FL104122005.Q, FL104123005.Q and FU106402101.Q, FU106300005.Q.

[38] Series FL102000115.Q.

[39] Specifically, the definition follows the construction of Table L.103 in the Flow of Funds reports. Cash is checkable, time, and foreign deposits, holding of money market mutual funds, and Treasury securities and other bonds. These are Z1 series FL103020005.Q, FL103030003.Q, FL103034003.Q, FL103091003.Q, FL102051003.Q and FL103061103.Q.

ships, and proprietorships. The debt measure is constructed from the same variables as for the corporate series (FL104104005.Q plus FL114123005.Q). The corresponding output measure is now taken to be the non-farm business GDP number from NIPA Table 1.3.5. Fixed assets of nonfinancial corporate and noncorporate sectors are from NIPA Table 6.3.

To address concerns that aggregate data is dominated by large firms, a final alternative leverage measure is constructed from median firm values in quarterly Compustat data. Specifically, for all nonfinancial firms with a reporting quarter ending within each calendar quarter, I compute net debt as long term debt total plus debt in current liabilities minus cash and short term investment. Cash-flow is operating profits before interest, depreciation, and taxes. I then take the median value across firms of the ratio of net debt to cash-flow. Firms for which the denominator is non-positive are excluded, as are firms missing any of the numerator items. The quarterly Compustat series is available from 1976:Q1.

Credit spread:

In choosing a credit spread series, the main consideration is, again, that the model speaks to a firm representative of the entire corporate sector. The question then is how to define the average creditworthiness of firms overall.

Based on the average default rate of the entire private sector (see below) and long-term default frequencies from S&P by rating, the representative firm in the economy appears to be approximately of credit grade BB or BBB. The natural candidate to measure credit in this range is the time-series of seasoned Baa-rated yields-to-maturity from Moody's that goes back to 1916. (A Moody's Baa rating corresponds to an S&P BBB rating.) According to Moody's, this series is for debt with maturity of at least 20 years. So I subtract the constant maturity 20-year Treasury yield from the Federal Reserve Bank of St. Louis (FRED), interpolating between 10 and 30 year yields when the 20 year series in unavailable.[40] I measure both yields at the end of calendar quarters. The Treasury yields are available from 1953:Q1.

For comparison with other works in the asset pricing literature, I also consider the credit spread defined as the difference between Baa yields and Aaa yields. Some authors have viewed Treasury bonds as an inappropriate benchmark because of potential liquidity premia or tax effects embedded in their prices.

To address the concern that the firm's borrowing cost in the model is for floating-rate debt and should therefor correspond to a short-maturity interest rate, I also construct a credit spread based on commercial paper yields. The main drawback with this series is that commercial paper is only issued by large high-quality borrowers, making its spread unrepresentative. Commercial paper rates are obtained from FRED. I concatenate separate pre- and post-1998 se-

[40] Choosing a series that fixes the rating level over time does impose a measurement bias because it misses fluctuations in the population credit quality. Intuitively, the effect of this bias should be straightforward: it should mean that fluctuations in credit spreads are understated.

ries for 90-day maturity. (The post-1998 series is for A2/P2 rated nonfinancial issuers. The earlier series does not specify the issuer type.) The CP-TB spread is defined as the difference between this rate and the current 3-month Treasury bill rate, also from FRED.

Default rate:

To assess representative borrower quality in the U.S. corporate sector, I obtain a time-series of annual total bankruptcy filings by U.S. firms for 1981-2015 from the American Bankruptcy Institute. [41] I divide total bankruptcies by the total number of firms in the U.S. from the Statistics of U.S. Businesses [42] compiled from the U.S. Census Bureau's Survey of Business Ownership. The latter series are available from 1988-2012. I average the annual ratio of the two numbers to obtain the unconditional default frequency 0.0087 used in the estimation.

For comparison to rated bond issuers, average global default rates by rating and issuer type are available for 1981-2014 are obtained from S&P's Global Corporate Default Study (Vazza and Kraemer, 2015). For such issuers, the average one-year default rate for nonfinancial firms worldwide over this period is 0.0181 (Table 16).

Investment rate:

In the model, investment is made directly by households through their savings decisions. Therefore I measure average investment as the

[41] http://www.abi.org/newsroom/bankruptcy-statistics
[42] https://www.sba.gov/advocacy/firm-size-data#susb

personal savings rate (savings as a fraction of disposable household income) from NIPA Table 2.1. The value used in the estimation is the average of annual rates from 1980-2015.

Equity valuation:

The model's equity valuation as a fraction of output is again supposed to be representative of the entire economy. The flow of funds tables include market valuation of equity (less intercompany holdings) for the nonfinancial corporate public sector. This is value is divided by the cashflow series constructed as described above. The value used in the estimation is the average of quarterly ratios from 1980:Q1-2015:Q1.

Estimation

The two models in Section 2 are estimated by minimum-square-error criterion applied to the moments (or statistics) listed in the text's Table 1, with one exception. Instead of targeting the default rate itself, the estimation targets the ratio of the credit spread to the default rate in order to better identify the credit risk premium. In addition, because the trade-off model assumes zero recovery on debt, the estimation for that specification deflates the model's value by an average loss factor (one minus recovery rate) of 0.6 for comparison with the empirical counterpart.

Model moments at each point in the parameter space are computed by sampling from the stationary distribution of the σ process. Moment errors are scaled by the estimation error in each target statistic, and squared. Because the statistics are computed from distinct samples over different dates and frequencies, I do not attempt to estimate cross-moment errors.

Hence the scheme corresponds to a diagonal weighting matrix.[43]

The estimation fixes six of the parameters to be the same in both models. The jump intensity is held constant at $\lambda = 1$ for ease of interpretation, e.g., so that jump magnitudes can be viewed as expected annual rates. The jump shape parameter L is fixed at 4.0 and the scale of the production function ζ_1 is fixed at 0.975. These parameters are poorly identified by the data moments. For comparability across specifications, the upper and lower bounds of the state variable are held fixed at $\sigma_l = 0.05, \sigma_u = 0.60$. Finally, the tax-rate is fixed at 0.30, as it is not really a free parameter.

The resulting estimates for the remaining parameters are given in Table A1 for each specification.

Table A1: Parameter Estimates

Parameter		Model:T-O	Model:D-I
Risk aversion	γ	7.7074	8.4004
E.I.S.	ψ	0.3927	3.8295
Subjective discount rate	β	0.0595	0.1292
Interest deduction rate	$\bar{r}$	0.1246	0.0130
Debt recovery/insurance rate	θ	0.4000	0.4013
Production function curvature	ζ_0	0.1465	0.0808
Output growth constant	μ	0.0577	0.0814
Uncertainty mean	$\bar{\sigma}$	0.0925	0.0988
Uncertainty mean-reversion	κ	0.2271	0.1808
Uncertainty diffusion	s_0	0.2599	0.1805

Description: The table gives the point estimates of the parameters for the two versions of the model fitted by the method of simulated moments.

[43] The delta method is used to approximate the sampling error of the credit spread-default rate ratio.

Panel Regressions

The panel regressions shown in Table 3 follow closely Covas and den Haan (2011) in the sample construction, definition of the variables, and controls used. Details on these may be found in the data appendix for that paper, available from wouterdenhaan.com. For reasons described there, the sample starts in 1980 and excludes financial firms, utilities, firms involved in major mergers, as well as Ford, Chrysler, GM, and GE. The sample runs through 2011, which is the extent of the uncertainty series from Jurado *et al.* (2015). For parsimony, Covas and den Haan (2011) include lagged cash-flow and Tobin's Q as the sole controls. We do likewise.

Firm-quarter observations are required to have non-missing, strictly positive assets, sales, and shareholder equity. The debt numerator is long-term debt plus debt in current liabilities minus cash and short-term investments. This definition follows Strebulaev and Yang (2013). The three leverage measures – but not their changes – are winsorized at the 1% and 99% levels.

The definition of net external equity in Panel B of the table again follows Covas and den Haan (2011) and Fama and French (2005) in using the change in shareholder equity net of retained earnings. The Compustat data item SEQQ already nets out of balance sheet equity the cumulative total of extraordinary items as well as treasury stock repurchased. However it is still inclusive of retained earnings. So the the item REQ is subtracted out. Changes in the resulting quantity, known as paid-in capital, arise from stock issuance and repurchases. Dividend payments are not captured.

C. Robustness

This appendix provides additional evidence for the empirical relations documented in Section 3.

First, the empirical rejection of the trade-off model was primarily attributable to the positive correlation between credit spreads and leverage. That correlation was illustrated visually in Figure 4 using the main measures of each quantity as described in Appendix B. Table A2 shows the correlations for pairwise combinations of the additional measures described there.

Next, Table A3 presents extended results on the empirical relation between aggregate debt and uncertainty that employ alternative uncertainty proxies. The table shows results for 12 regressions: using three proxies, two specifications, and two aggregate debt and output series. All 12 support the conclusion of a statistically and economically significant positive association.

The regressions in Section 3 utilize the JLN uncertainty series. Here the table also uses the well-known CBOE VIX index[44], the dispersion in economists' forecast of GDP, as tabulated from the Survey of Professional Forecasters (SPF)[45], and a third series (RED) that measures firm-level, rather than aggregate uncertainty, from Johnson and Lee (2014). It is constructed from the cross-firm dispersion of residual operating earnings,

[44]The VIX series is extend backward from 1990 to 1984 using implied volatility data on individual SPX options.

[45]The SPF series averages the dispersion in current-quarter forecasts of real and nominal GDP. Using other forecast horizons and extracting a principal component from the dispersion series produce similar results.

after orthogonalizing with respect to aggregate output changes. Note that each of the series is constructed from entirely distinct underlying data.

The table shows regressions of year-on-year changes in log debt on year-on-year log changes in the uncertainty series. In the top panel, debt is measured net of cash items using the Flow of Funds (Z.1) accounts for the nonfinancial corporate sector. The specification in the left panel scales debt by corporate cashflow. In the right panel debt changes themselves are the dependent variable, with contemporaneous cashflow changes included as a control variable. The tests are repeated in the bottom panel using a broader measure of debt that includes the noncorporate private sector, and scaling by Gross Domestic Product of nonfarm U.S. businesses, from NIPA Table 1.3.5. Standard errors correct for the serial correlation in overlapping residuals.

Table A2: Leverage Credit Spread Correlations

	Baa-20yr	Baa-Aaa	CP-TB
Nonfinancial corporate debt/cashflow	0.5229	0.2595	0.0278
minus cash	0.5641	0.3191	0.0603
plus retirement liabilities	0.4997	0.2568	0.0587
	1969:Q1-2015:Q1	1969:Q1-2015:Q1	1980:Q1-2015:Q1
Nonfinancial corporate+noncorporate net debt/			
non-farm business GDP	0.3237	0.1025	0.2120
	1969:Q1-2015:Q1	1969:Q1-2015:Q1	1980:Q1-2015:Q1
Median Compustat nonfinancial net debt/cashflow	0.1187	0.1383	0.1248
	1976:Q1-2015:Q2	1976:Q1-2015:Q2	1980:Q1-2015:Q1

Description: The table shows the time-series correlation between credit spreads and leverage using three credit spread series and six leverage definitions.

Interpretation: The positive correlation is robust to the choice of measurement series.

Table A3: Aggregate Debt Dynamics

Panel A: Nonfinancial Corporate Debt ; Cash-flow						
Dependent var	$(b_t - y_t) - (b_{t-4} - y_{t-4})$			$(b_t - b_{t-4})$		
Sample	1984Q1-2018:Q1	1969Q4-2014:Q4	1964Q2-2017:Q4	1984Q1-2018:Q1	1969Q4-2014:Q4	1964Q2-2017:Q4
Δ VIX	0.0827			0.0601		
	(3.28)			(3.01)		
Δ SPF		0.0762			0.0358	
		(3.34)			(2.13)	
Δ RED			0.2070			0.1297
			(3.79)			(2.48)
$b_{t-4} - y_{t-4}$	-0.2341	-0.1785	-0.1286			
	(3.27)	(2.70)	(3.31)			
$y_t - y_{t-4}$				0.0161	-0.0351	0.0132
				(0.12)	(0.24)	(0.10)
N	133	181	215	133	181	215

Panel B: Nonfin. Corp. & Noncorp. Debt; Bus. GDP						
Dependent var	$(b_t - y_t) - (b_{t-4} - y_{t-4})$			$(b_t - b_{t-4})$		
Sample	1984Q1-2018:Q1	1969Q4-2014:Q4	1964Q2-2017:Q4	1984Q1-2018:Q1	1969Q4-2014:Q4	1964Q2-2017:Q4
Δ VIX	0.0591			0.0625		
	(2.83)			(3.37)		
Δ SPF		0.0454			0.0429	
		(3.01)			(2.86)	
Δ RED			0.1174			0.1225
			(2.92)			(2.92)
$b_{t-4} - y_{t-4}$	-0.1521	-0.1364	-0.0970			
	(1.85)	(2.69)	(3.99)			
$y_t - y_{t-4}$				0.8120	0.7247	0.7446
				(4.33)	(4.10)	(4.69)
N	133	181	215	133	181	215

Caption to Table A3

Description: The table reports time-series regressions of quarterly aggregate debt on measures of economic uncertainty, in changes. If the first panel, b is total debt securities and loans, minus cash items (including checkable, time and savings deposits, money-market funds, and foreign deposit) for the U.S. nonfinancial corporate sector and y is the total cashflow of this sector measured as net operating surplus plus consumption of fixed capital. All data are from the Federal Reserve's Flow of Funds accounts. In the second panel, b is the combined total debt (net of cash items) for corporate and noncorporate nonfinancial businesses and y is gross domestic product of nonfarm U.S. businesses from NIPA Table 1.3.5. Debt and output variables are in logarithms. VIX, SPF, and RED are respectively the CBOE VIX index (extend backward from 1990 by the author), the dispersion in current-quarter forecasts of real and nominal GDP as tabulated by the Survey of Professional Forecasters (SPF), and the residual earnings dispersion measure of Johnson and Lee (2014). The uncertainty series are year-on-year log differences contemporaneous with the dependent variable. Numbers in parentheses are Newey and West (1987) T-statistics (in absolute value) using 8 lags.

Interpretation: The positive leverage-uncertainty relation is robust to the choice of uncertainty proxy and the choice of aggregate debt. The same result holds using debt changes as the dependent variable.

Next, Table A4 reverts to the JLN uncertainty measure and shows additional regressions in which changes in (log) debt are the dependent variable.[46] Panel A shows the positive relation persists in these specifications, and is robust to controlling for additional predictors.

Because uncertainty changes are negatively autocorrelated, a natural question is whether the positive relationship in changes is actually proxying for a *negative* relationship between debt *changes* and uncertainty *levels*.[47] Panel B allows the data to consider both possibilities. The results unambiguously support a positive relation in changes, with no statistical support

[46] The table utilizes the nonfinancial corporate series. Results using the broader private sector series are similar and are omitted for brevity. The variables and sample are defined in the caption to Table 2.

[47] The two relations are not econometrically inconsistent. In the context of the models studied here, however, a levels-on-changes regression is a misspecification. See also the discussion in footnote 26.

for a negative relation between debt changes and uncertainty levels.

The specifications so far do not distinguish between expected and unexpected changes in uncertainty. This is appropriate in the sense that the models in Section 2 imply that debt levels respond to levels of uncertainty, whether or not they were expected. Moreover, in the presence of real-world planning delays, it seems likely that debt issuance would be driven mostly by expected changes. Panel C of the Table quantifies both responses by splitting uncertainty changes into two components via an auxiliary regression of these changes on a set of lagged predictors (listed in the column labeled Projection). When both the fitted and the residual components of this projection are included in the debt regression, each is seen to have a statistically and economically significant positive impact. Summing the two coefficients, the total impact is substantially larger than in the baseline specifications in Panel A.

Table A4: Uncertainty and Debt Dynamics

PANEL A: Baseline			
$\Delta v_{(t+4:t)}$			Controls:
0.3028 (3.69)			$\Delta y_{(t+4:t)}$
0.1989 (3.16)			$\Delta y_{(t+4:t)}, \Delta y_{(t:t-4)}, \Delta b_{(t:t-4)}$
0.1420 (2.56)			*all*
PANEL B: Uncertainty changes vs levels			
$\Delta v_{(t+4:t)}$	v_t		Controls:
0.2777 (3.79)	-0.0486 (0.57)		$\Delta y_{(t+4:t)}$
0.1457 (2.34)	0.0155 (0.23)		*all*
PANEL C: Uncertainty innovations vs expected changes			
$U\ \Delta v_{(t+4:t)}$	$E\ \Delta v_{(t+4:t)}$	Controls:	Projection:
0.2450 (3.71)	0.4691 (2.28)	$\Delta y_{(t+4:t)}$	$v_t, \Delta v_{(t:t-4)}, R_t^{mkt}, \lvert R_t^{mkt} \rvert$
0.1247 (2.15)	0.2197 (2.81)	*all*	$v_t, \Delta v_{(t:t-4)}, R_t^{mkt}, \lvert R_t^{mkt} \rvert$

Description: The table shows regressions of aggregate changes in log debt, $\Delta b_{(t+4:t)}$, on uncertainty changes. The variables are described in the caption to Table 2. In the column labeled Controls, *all* refers to the variables $\Delta y_{t+4:t}, \Delta y_{t:t-4}, \Delta b_{t:t-4}, R_t^{mkt}, SP\_PE_t, Y_t^{10yr}, CR\_SP_t, \Delta CPI_{t:t-4}$. In Panel C, $U\ \Delta v_{(t+4:t)}$ and $E\ \Delta v_{(t+4:t)}$ denote the residual and fitted values, respectively, from a first-stage regression of uncertainty changes on the variables listed in the Projection column. Numbers in parentheses are Newey and West (1987) T-statistics (absolute value) using 8 quarterly lags.

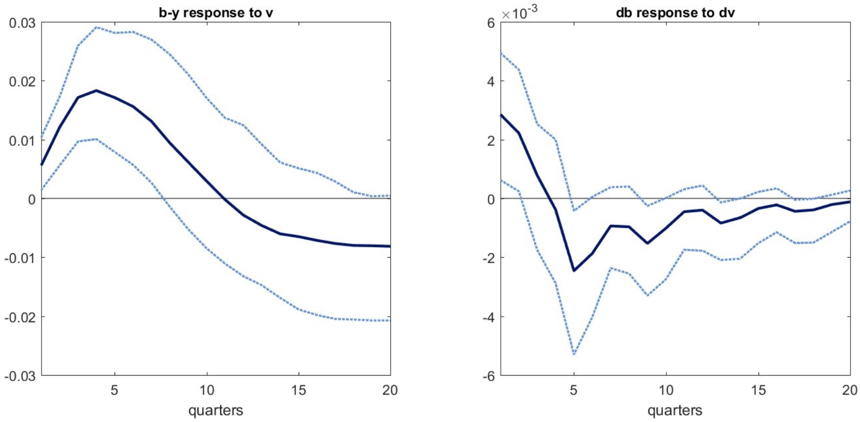
Interpretation: The positive debt-uncertainty relation is using the JLN measure is explored further. The positive relation with uncertainty changes is not masking a negative relation with uncertainty levels. The relation is found in both expected and unexpected components of uncertainty changes.

Another way of quantifying the response of debt to orthogonalized uncertainty shocks is via impulse response functions in vector autoregressions. Figure C1 shows two response functions, each computed in non-overlapping quarterly specifications, using four lags. The left-hand panel is a bivariate specification in levels where debt is scaled by output (both in logs) to achieve stationarity. (This specification also includes linear trends.) The right-hand panel is from a trivariate system – uncertainty, output, and debt – in changes. In each system, the variables are ordered with the debt series last and the shocks are orthogonalized via Cholesky decomposition.[48]

Consistent with the change regressions above, the right panel affirms a significant positive debt change response at one and two quarters. Uncertainty shocks predict changes in debt. These responses then mean-revert as uncertainty itself mean-reverts. The left panel shows that the cumulative positive effect on leverage levels (which includes the negative response of the denominator to an uncertainty shock) remains positive for around two years.

[48]The data series are the same ones used in Table A4.

Figure C1: Impulse response functions



Description: The figure shows impulse responses to a one standard deviation shock to uncertainty in quarterly vector autoregressions. The left panel uses a bivariate specification of logs of debt-to-income $(b - y)$ and uncertainty (v) and includes linear trends. The right panel uses a trivariate system of log changes in uncertainty, output, and debt. Both specifications include four lags. The horizontal axis is in quarters. The dashed lines are bootstrapped 95% confidence intervals.

Interpretation: Uncertainty shocks produce statistically significant positive impulse responses of leverage levels and debt changes in quarterly vector autoregressions.