

Internet Appendix for:
**“High Aversion to Stochastic Time Preference Shocks and
Counterfactual Long-Run Risk in the Albuquerque et al.
Valuation Risk Model”**

A Solution details

This section of the appendix provides details and derivations for results discussed in the main text of the paper.

A.1 General pricing equations

The representative agent has the augmented Epstein-Zin preferences described by equation (1):

$$U_t = \left[\lambda_t C_t^{1-1/\psi} + \delta (\mathbf{E}_t [U_{t+1}^{1-\gamma}])^{\frac{1-1/\psi}{1-\gamma}} \right]^{1/(1-1/\psi)}.$$

Optimization is subject to a budget constraint of

$$W_{t+1} = R_{w,t+1} (W_t - C_t) \tag{IA.1}$$

where W_t is wealth at time t and $R_{w,t+1}$ is the return on the overall wealth portfolio, which is a claim to all future consumption.

[Albuquerque et al. \(2016\)](#) use standard techniques from the Epstein-Zin preference literature to show that the preferences represented by equation (1) imply the log stochastic discount factor expressed by equation (2):

$$m_{t+1} = \theta \log(\delta) + \theta \Lambda_{t+1} - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{w,t+1}.$$

This is the same as the standard Epstein-Zin stochastic discount factor except that discounting is time-varying (i.e., $\delta \frac{\lambda_{t+1}}{\lambda_t}$ instead of δ).

Using $0 = E_t [m_{t+1} + r_{i,t+1}] + \frac{1}{2} (\sigma_m^2 + \sigma_i^2 + 2\sigma_{mi})$ (the log version of $1 = E_t [M_{t+1}R_{i,t+1}]$), the expected return for any asset can be expressed as

$$\begin{aligned}
E_t [r_{i,t+1}] + \frac{1}{2}\sigma_i^2 &= -\theta \log \left(\delta \frac{\lambda_{t+1}}{\lambda_t} \right) + \frac{\theta}{\psi} E_t [\Delta c_{t+1}] + (1 - \theta) E_t [r_{w,t+1}] \\
&\quad - \frac{1}{2} \left(\frac{\theta}{\psi} \right)^2 \sigma_c^2 - \frac{1}{2} (1 - \theta)^2 \sigma_w^2 + \frac{\theta}{\psi} (\theta - 1) \sigma_{wc} \\
&\quad + \frac{\theta}{\psi} \sigma_{ic} + (1 - \theta) \sigma_{iw}.
\end{aligned} \tag{IA.2}$$

The $\frac{1}{2}\sigma_i^2$ on the left hand side of equation (IA.2) is the Jensen's inequality correction for log returns.

The resulting risk-free rate is

$$\begin{aligned}
r_{f,t+1} &= -\theta \log \left(\delta \frac{\lambda_{t+1}}{\lambda_t} \right) + \frac{\theta}{\psi} E_t [\Delta c_{t+1}] + (1 - \theta) E_t [r_{w,t+1}] \\
&\quad - \frac{1}{2} \left(\frac{\theta}{\psi} \right)^2 \sigma_c^2 - \frac{1}{2} (1 - \theta)^2 \sigma_w^2 + \frac{\theta}{\psi} (\theta - 1) \sigma_{wc}.
\end{aligned} \tag{IA.3}$$

Differencing equations (IA.2) and (IA.3) yields the risk premia of equation (6):

$$E_t [r_{i,t+1}] - r_{f,t+1} + \frac{1}{2}\sigma_i^2 = \frac{\theta}{\psi} \sigma_{ic} + (1 - \theta) \sigma_{iw},$$

which is exactly the same expression as in standard Epstein-Zin models. Substituting $E_t [r_{w,t+1}]$ into equation (IA.3), yields equation (5):

$$r_{f,t+1} = -\log(\delta) - \Lambda_{t+1} + \frac{1}{\psi} E_t [\Delta c_{t+1}] - \frac{1 - \theta}{2} \sigma_w^2 - \frac{\theta}{2\psi^2} \sigma_c^2,$$

which is the same as standard Epstein-Zin models except that δ is replaced by $\delta \frac{\lambda_{t+1}}{\lambda_t}$.

A.2 Intertemporal CAPM

Following [Campbell \(1993\)](#), the budget constraint can be log-linearized to generate equation (7):

$$r_{w,t+1} - \mathbf{E}_t [r_{w,t+1}] = (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=0}^{\infty} \rho^j \Delta c_{t+1+j} - (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j r_{w,t+1+j}$$

where $\rho = 1 - \exp(\overline{c - w})$ is a log-linearization constant ($\overline{c - w}$ is the average log consumption-wealth ratio). Rearranging, current consumption shocks can be expressed as

$$\begin{aligned} \Delta c_{t+1} - \mathbf{E}_t [\Delta c_{t+1}] &= r_{w,t+1} - \mathbf{E}_t [r_{w,t+1}] \\ &\quad + (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j r_{w,t+1+j} \\ &\quad - (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}. \end{aligned} \tag{IA.4}$$

So far, we have only made use of modified Epstein-Zin preferences and the budget constraint. We now use assumptions about consumption and time preference innovations. Due to our homoscedasticity assumption, risk premia do not change over time, and the risk-free rate only changes in response to time preference and consumption growth innovations. Thus, innovations to expected returns can be decomposed as

$$\begin{aligned} (\mathbf{E}_{t+1} - \mathbf{E}_t) r_{w,t+1+j} &= (\mathbf{E}_{t+1} - \mathbf{E}_t) r_{f,t+1+j} \\ &= (\mathbf{E}_{t+1} - \mathbf{E}_t) \log \left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}} \right) + \frac{1}{\psi} (\mathbf{E}_{t+1} - \mathbf{E}_t) [\Delta c_{t+j+1}] \end{aligned} \tag{IA.5}$$

for $j \geq 1$. Substituting equation [\(IA.5\)](#) into equation [\(IA.4\)](#) yields

$$\begin{aligned} \Delta c_{t+1} - \mathbf{E}_t [\Delta c_{t+1}] &= r_{w,t+1} - \mathbf{E}_t [r_{w,t+1}] \\ &\quad - \left(1 - \frac{1}{\psi} \right) (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \end{aligned}$$

$$+ (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j \log \left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}} \right). \quad (\text{IA.6})$$

Substituting out consumption shock covariance (σ_{ic}) from equation (6) yields risk premia as a function of covariance with market returns and innovations to future time preferences and consumption growth:

$$\begin{aligned} \mathbf{E}_t [r_{i,t+1}] - r_{f,t+1} + \frac{1}{2} \sigma_i^2 &= \gamma \sigma_{iw} + (\gamma - 1) \frac{1}{\psi} \text{cov}_t \left(r_{i,t+1}, (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \right) \\ &\quad + \frac{\theta}{\psi} \text{cov}_t \left(r_{i,t+1}, (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j \log \left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}} \right) \right). \end{aligned} \quad (\text{IA.7})$$

This can be alternatively expressed as

$$\mathbf{E}_t [r_{i,t+1}] - r_{f,t+1} + \frac{1}{2} \sigma_i^2 = \gamma \sigma_{iw} - \frac{\gamma - 1}{\psi - 1} \sigma_{ih(\lambda)} + (\gamma - 1) \sigma_{ih(c)} \quad (\text{IA.8})$$

where

$$\sigma_{ih(\lambda)} = \text{cov}_t \left(r_{i,t+1}, (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j \log \left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}} \right) \right)$$

and

$$\sigma_{ih(c)} = \frac{1}{\psi} \text{cov}_t \left(r_{i,t+1}, (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \right)$$

are the two different types of risk-free rate news covariance.

Equation (IA.8) is an intertemporal capital asset pricing model (ICAPM) pricing equation. As in [Campbell \(1993\)](#), risk premia are a function of covariance with the market return and covariance with shocks to investment opportunities. Market return risk (σ_{iw}) is priced by relative risk aversion (γ) as in other ICAPM models. Also consistent with other ICAPM models, future interest rate covariance ($\sigma_{ih(c)}$ and $\sigma_{ih(\lambda)}$) is priced only if $\gamma \neq 1$. Yet, the two components of interest rate risk have different prices. Whereas $\sigma_{ih(c)}$ is priced by $\gamma - 1$, $\sigma_{ih(\lambda)}$ is priced by $-\frac{\gamma-1}{\psi-1}$. When $\psi > 1$, the prices have opposite signs, and if ψ is close to 1, time-preference risk is amplified relative to consumption growth risk. The key distinction

between equation (IA.8) and more standard ICAPM models such as Campbell (1993) is that equation (IA.8) includes shocks to both consumption growth and time preferences. Because Campbell (1993) assumes preferences are constant, there is no $\sigma_{ih(\lambda)}$ in his model, and σ_{ih} is equivalent to $\sigma_{ih(c)}$.

A.3 Extended consumption CAPM

The budget constraint can also be used to substitute out wealth portfolio return covariance (σ_{iw}) from equation (6) by rearranging equation (IA.6) and using it to decompose σ_{iw} , thereby yielding equation (9):

$$E_t[r_{i,t+1}] - r_{f,t+1} + \frac{1}{2}\sigma_i^2 = \gamma\sigma_{ic} + (\gamma\psi - 1)\sigma_{ih(c)} - \frac{\gamma\psi - 1}{\psi - 1}\sigma_{ih(\lambda)}.$$

A.4 Augmented consumption

Another way to derive the ICAPM and extended CCAPM pricing equations is to change notation to consider time preference shocks in the same units as consumption. Specifically, consider augmented consumption, defined as

$$\tilde{C}_t \equiv \lambda_t^{1/(1-1/\psi)} C_t. \tag{IA.9}$$

With this notation change, equation (1) is transformed into standard Epstein-Zin preferences with respect to augmented consumption. All of Campbell's (1993) and Bansal and Yaron's (2004) results hold with respect to augmented consumption and returns measured in units of augmented consumption. In particular, the augmented risk-free rate is

$$\tilde{r}_{f,t+1} = -\log(\delta) + \frac{1}{\psi}E_t[\Delta\tilde{c}_{t+1}] - \frac{1-\theta}{2}\sigma_w^2 - \frac{\theta}{2\psi^2}\sigma_c^2 \tag{IA.10}$$

and the risk premium for any asset is given by

$$E_t [\tilde{r}_{i,t+1}] - \tilde{r}_{f,t+1} + \frac{1}{2}\sigma_i^2 = \gamma\sigma_{iw} + (\gamma - 1)\sigma_{ih(\tilde{c})} \quad (\text{IA.11})$$

where tildes represent augmented consumption and returns. Using the identities $\tilde{r}_{i,t+1} = r_{i,t+1} + \frac{1}{1-1/\psi} \log\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$ and $\Delta\tilde{c}_{t+1} = \Delta c_{t+1} + \frac{1}{1-1/\psi} \log\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$, equations (IA.10) and (IA.11) are equivalent to equations (5) and (IA.8).

A.5 Calibrated model solution

Albuquerque et al. (2016) solve the model using log-linear analytical approximations. Let portfolio w be the overall wealth portfolio, which represents a claim to aggregate consumption. Using Campbell and Shiller's (1988) approximation for the return on the overall wealth portfolio the log return to the wealth portfolio can be expressed as

$$r_{w,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1} \quad (\text{IA.12})$$

where z_t is the log wealth-consumption ratio at time t . Unknown linearization parameters κ_0 and κ_1 are given by

$$\kappa_1 = \frac{\exp(z)}{1 + \exp(z)} \quad (\text{IA.13})$$

$$\kappa_0 = \log(1 + \exp(z)) - \kappa_1 z \quad (\text{IA.14})$$

where z is the unconditional mean of z_t . Returns to the market portfolio, which is a claim to aggregate dividends, can be similarly approximated as

$$r_{m,t+1} = \kappa_{m0} + \kappa_{m1} z_{m,t+1} - z_{m,t} + \Delta d_{t+1} \quad (\text{IA.15})$$

with unknown parameters κ_{m0} and κ_{m1} constructed the same way.

Albuquerque et al. (2016) guess and verify that z_t and $z_{m,t}$ linearly depend on state variables, taking the form

$$z_t = A_0 + A_1 x_t + A_2 \eta_{t+1} + A_3 \sigma_t^2 + A_4 \Delta c_t \quad (\text{IA.16})$$

$$z_{m,t} = A_{m0} + A_{m1} x_t + A_{m2} \eta_{t+1} + A_{m3} \sigma_t^2 + A_{m4} \Delta c_t + A_{m5} \Delta d_t, \quad (\text{IA.17})$$

and solve for the unknown coefficients as functions of the model parameters and κ_0 , κ_1 , κ_{m0} , and κ_{m1} , which are functions of z and z_m . Closed-form solutions for these coefficients are reported in Albuquerque et al.'s internet appendix. One can then numerically iterate to find fixed points for z and z_m . Having solved for all coefficients, market returns in any period are given by equation (IA.15). To complete the solution, Albuquerque et al. use the stochastic discount factor (equation (2)), Euler equation, and equation (IA.12) to obtain the risk-free rate as a function of state variables.

B Valuation risk in the cross section

I analyze cross-sectional valuation risk by sorting stocks based on their past return sensitivity to risk-free rate shocks. Ideally, we would like to separately measure consumption growth and time preference risk-free rate shocks. Given the unobservability of time preferences and the imprecise and low-frequency nature of consumption data, measuring aggregate risk-free rate shocks is probably the best we can do. While this does not formally test the model, it assesses whether there is support in the cross section for valuation risk. If exposure to risk-free rate shocks is not priced in the cross-section, this suggests that valuation risk is not a major factor for explaining asset prices. The model informs how we measure risk-free rate shocks. In particular, it highlights that investors care about shocks to both current and expected future risk-free rates. Thus, instead of considering just $cov_t(r_{i,t+1}, r_{f,t+2} - E_t[r_{f,t+2}])$, I focus on $\sigma_{ih} = cov_t\left(r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{w,t+1+j}\right)$.

To assess sensitivity to valuation shocks, we need to estimate $(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j}$.

This estimation has two challenges. First, the focus is on real interest rates. This is the risk-free rate in the model, and it is the relevant quantity for economic decisions. Unfortunately, the real risk-free rate is not directly observable. To overcome this challenge, I model expected Consumer Price Index (CPI) inflation and estimate the monthly real risk-free rates as the difference between the nominal 1-month Treasury bill yield and expected inflation over the next month. Baseline estimates focus on the 1983 to 2012 time period because monetary policy is more consistent and inflation is less volatile during this period than in previous periods.

A second empirical challenge is that valuation risk involves shocks to expectations. Thus, we need to estimate interest rate expectations. I do this with a vector autoregression (VAR) of interest rates, inflation, and other state variables. From the VAR, we extract an estimate for the time series of $(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j}$ innovations, which I then use to estimate σ_{ih} for individual stocks.

B.1 Vector autoregression

The VAR model is

$$Y_t = AY_{t-1} + \epsilon_t. \tag{IA.18}$$

Y_t is a $k \times 1$ vector with the nominal 1-month Treasury bill log yield and seasonally adjusted log CPI inflation over the past month as its first two elements. The remaining elements of Y_t are state variables useful for forecasting these two variables. The assumption that the VAR model has only one lag is not restrictive because lagged variables can be included in Y_t . Before estimating the VAR, Y_t is demeaned to avoid the need for a constant in equation (IA.18).

Vector ei is defined to be the i th column of a $k \times k$ identity matrix. Using this notation, expectations and shocks to current and future expectations can be extracted from Y_t , A , and ϵ_t . The real risk-free interest rate is estimated as the nominal 1-month Treasury bill yield

less expected inflation:

$$\widehat{r_{f,t+1}} = (e1' - e2'A) Y_t. \quad (\text{IA.19})$$

Similarly, expected future risk-free rates are

$$E_t [\widehat{r_{f,t+j}}] = (e1' - e2'A) A^{j-1} Y_t. \quad (\text{IA.20})$$

Shocks to current and expected risk-free rates are

$$(E_{t+1} - E_t) \widehat{r_{f,t+1+j}} = (e1' - e2'A) A^{j-1} \epsilon_{t+1}. \quad (\text{IA.21})$$

Total interest rate news is

$$\begin{aligned} News_{h,t+1} &= (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \widehat{r_{f,t+1+j}} \\ &= (e1' - e2'A) \sum_{j=1}^{\infty} \rho^j A^{j-1} \omega_{t+1} \\ &= (e1' - e2'A) \rho (I - \rho A)^{-1} \omega_{t+1} \end{aligned} \quad (\text{IA.22})$$

where I is the identity matrix and ρ is a log linearization coefficient equal to $1 - \exp(\overline{c - w})$ where $\overline{c - w}$ is the average log consumption-wealth ratio. I use a monthly coefficient value of $\rho = 0.996$ for the analysis.

To select state variables to include in Y_t , I first follow [Campbell \(1996\)](#) and include the relative Treasury bill rate, defined as the difference between the current one-month Treasury bill yield and the average one-month Treasury bill yield over the previous 12 months. I also include the relative monthly CPI inflation rate, defined the same way. Next, I include the yield spread between 10-year Treasury bonds and 3-month Treasury bonds because the slope of the yield curve is known to predict interest rate changes. Finally, I include the CRSP value-weighted market return and the log dividend-price ratio (defined as dividends over the past year divided by current price), which is known to predict market returns.

These variables are useful to the extent that equity returns are related to expected future interest rates. Equation (IA.18) can also be estimated with lags of Y_t . Because the Bayesian Information Criteria is insensitive to adding lags, I do not include lagged variables in Y_t .

Table IA.2 reports coefficient estimates and standard errors for the elements of A related to predicting nominal interest rates and inflation. The first two columns report results for the 1983 to 2012 time period, which is the primary focus. Nominal interest rate shocks are highly persistent with lag coefficient of 0.96. Inflation shocks are much less persistent and have a lag coefficient of 0.07. Inflation is increasing in lagged nominal yields. The VAR explains 95% of the variation in nominal yields over time. Inflation changes are less predictable with an R-squared of 0.24.

Figure IA.1 plots the estimated real risk-free rate from the VAR model along with the nominal one-month Treasury bill yield and the Federal Reserve Bank of Cleveland's real risk-free rate estimate.¹ As one would expect in a stable inflation environment, real interest rates generally follow the same pattern as nominal interest rates. Nonetheless, inflation expectations do change over time, particularly late in the sample. The VAR real risk-free rate estimate closely tracks the Federal Reserve Bank of Cleveland's estimate.

As a robustness check, I also estimate real risk-free rates and real risk-free rate news over a longer time period, starting in 1927. The methodology for the longer time period is the same as before except that the CPI is unadjusted because the seasonally adjusted CPI is only available starting in 1947. Columns (3) and (4) of Table IA.2 report the VAR results. In the extended time sample, inflation shocks are more persistent (inflation's lagged coefficient is 0.78, compared to 0.07 before). The results are otherwise similar to the original VAR.

B.2 Cross-sectional results

To assess whether valuation risk is priced in the cross section, I sort stocks into portfolios based on past covariance with risk-free rate news. Risk-free rate news covariance,

¹The Federal Reserve Bank of Cleveland's real risk-free rate estimates are described by [Haubrich, Penacchi, and Ritchken \(2008, 2012\)](#).

$\sigma_{ih} = cov_t \left(r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{w,t+1+j} \right)$, is estimated on a rolling basis for all NYSE, AMEX, and NASDAQ common stocks using returns and risk-free rate news over the past three years, with the requirement that included stocks must have at least two years of historical data. Value-weighted decile portfolios are formed monthly by sorting stocks according to those estimates.

Table IA.3 reports market capitalization, average excess returns, and $\beta_{ih} = \frac{\sigma_{ih}}{\sigma_h^2}$ estimates for each portfolio. The table also reports pricing errors (alphas) relative to the CAPM and Fama and French (1993) three factor model and factor loadings (betas) for the three factor model. Panel A reports results for the baseline 1985-2012 time period.² Risk-free rate news betas increase across the portfolios, and decile 10's news beta is a significant 0.58 higher than decile 1's news beta. Monthly excess returns are 42 bps lower in the 10th decile than in the 1st decile, but this return difference is not statistically significant, and there is no clear pattern to excess returns across the decile portfolios other than a drop in returns in decile 10. CAPM and 3 Factor alphas follow the same basic pattern. Factor loadings are also similar across the portfolios. The one exception is that decile 10 has a large negative loading on the value factor (HML). In short, there is no evidence that valuation risk is priced in the cross section of stock returns.

Results are similar in the extended 1929-2012 sample, reported in Panel B. Once again, average excess returns and alpha estimates decrease with interest rate news exposure, but the differences are not significant. The biggest difference between Panel A and Panel B is that β_{ih} differences across the portfolios are not significant in the extended sample. This suggests that stock-level valuation risk was not stable over time early in the sample, undercutting our ability to form valuation risk portfolios.

²Portfolio formation is based on at least two years of historical data, which causes the sample to start in 1985 instead of 1983.

Appendix References

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C Supplemental tables and figure

Table IA.1. Predictive regression coefficients

Description: This table reports simulated regression coefficients for the predictive regressions summarized in Table 6. The reported results are from regressing future log excess equity returns (panel A), consumption growth (panel B), dividend growth (panel C), and real risk-free rates (panel D) on the current log price-dividend ratio. Regressions in the data are based on 1930–2008 annual historical data from the Bureau of Economic analysis and CRSP. Simulation regressions are based on 100,000 simulations with time periods equal to the historical data. The models are simulated monthly and then annualized for comparability with the historical data. For the simulations, the table reports the median price-dividend ratio regression coefficient and the % of simulated regression coefficients that are larger than the comparable regression coefficient in the data. Standard errors are Newey-West with $2*(\text{horizon}-1)$ lags.

Interpretation: Predictive regressions show the extent to which the price-dividend ratios predicts subsequent returns, consumption growth, dividend growth, and risk-free rates in the data and model.

| | Data | | | Benchmark model $\hat{\beta}$ | | Extended model $\hat{\beta}$ | |
|-----------------------------|---------------|-------|-------|-------------------------------|----------|------------------------------|----------|
| | $\hat{\beta}$ | t | R^2 | Median | % > data | Median | % > data |
| Panel A. Excess returns | | | | | | | |
| 1 year | -0.09 | -1.80 | 0.04 | -0.04 | 74.7% | -0.04 | 80.9% |
| 3 years | -0.26 | -3.23 | 0.17 | -0.13 | 74.9% | -0.12 | 80.7% |
| 5 years | -0.41 | -3.78 | 0.27 | -0.21 | 74.6% | -0.20 | 80.4% |
| Panel B. Consumption growth | | | | | | | |
| 1 year | 0.01 | 1.59 | 0.06 | 0.00 | 9.9% | 0.03 | 97.5% |
| 3 years | 0.01 | 0.59 | 0.01 | 0.00 | 33.6% | 0.09 | 99.5% |
| 5 years | 0.00 | -0.06 | 0.00 | 0.00 | 51.2% | 0.13 | 99.5% |
| Panel C. Dividend growth | | | | | | | |
| 1 year | 0.07 | 1.98 | 0.09 | 0.01 | 0.5% | 0.06 | 32.7% |
| 3 years | 0.11 | 1.33 | 0.06 | 0.01 | 5.7% | 0.19 | 89.7% |
| 5 years | 0.09 | 1.21 | 0.04 | 0.01 | 19.8% | 0.30 | 96.0% |
| Panel D. Risk-free rate | | | | | | | |
| 1 year | 0.01 | 1.15 | 0.03 | -0.13 | 0.0% | -0.01 | 19.9% |
| 3 years | 0.03 | 0.82 | 0.03 | -0.34 | 0.0% | -0.02 | 24.6% |
| 5 years | 0.05 | 1.06 | 0.05 | -0.49 | 0.0% | -0.03 | 24.1% |

Table IA.2. Vector autoregression results

Description: This table reports results from the vector autoregression (VAR) described by equation (IA.18). The nominal log yield on a one-month Treasury bill is $y1$. Inflation is one-month log seasonally-adjusted CPI inflation. Relative $y1$ and relative inflation are the difference between current yields and inflation and average values over the past twelve months. The yield spread between 10-year Treasury bonds and 3-month Treasury bills is $y120 - y3$. The excess return of the CRSP value weighted market return over the risk-free rate is $r_m - r_f$. The log dividend-price ratio, $d - p$, is calculated for the CRSP value-weighted market index using current prices and average dividends over the past twelve months. Results are for a 1-lag VAR of demeaned $y1$, inflation, relative $y1$, relative inflation, $r_m - r_f$, and $d - p$. Coefficients for dependent variables $y1$ and inflation are reported. The other dependent variables are omitted for brevity. Bootstrapped standard errors are in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance.

Interpretation: Nominal interest rate shocks are highly persistent. Inflation shocks are less persistent and less predictable.

| | 1983–2012 | | 1927–2012 | |
|--------------------------------------|-----------------------|---------------------|------------------------|------------------------|
| | <i>y1</i> | <i>inflation</i> | <i>y1</i> | <i>inflation</i> |
| Lagged Variables | | | | |
| <i>y1</i> | 0.9639*** (0.0202) | 0.1939* (0.1003) | 0.9741*** (0.0116) | 0.0631 (0.0773) |
| <i>inflation</i> | 0.0314 (0.0297) | 0.0737 (0.1734) | 0.0102* (0.0062) | 0.7762*** (0.0709) |
| <i>relative y1</i> | -0.0976** (0.0457) | 0.1295 (0.1585) | -0.1752*** (0.0407) | 0.5909*** (0.1599) |
| <i>relative inflation</i> | -0.0136 (0.0281) | 0.3268* (0.1767) | -0.003 (0.0056) | -0.4554*** (0.0837) |
| <i>y120 - y3</i> | -0.0032 (0.0036) | -0.002 (0.0155) | -0.0062** (0.0024) | 0.0014 (0.0122) |
| <i>r_m - r_f</i> | 0.0013* (0.0007) | 0.0083* (0.0042) | 0.0008** (0.0004) | 0.0061* (0.0034) |
| <i>d - p</i> | 0.0001 (0.0001) | 0.0002 (0.0005) | 0.0000 (0.0000) | -0.0002 (0.0003) |
| <i>R</i> ² | 0.95 | 0.24 | 0.95 | 0.32 |

Table IA.3. Valuation risk pricing in the cross section of stock returns

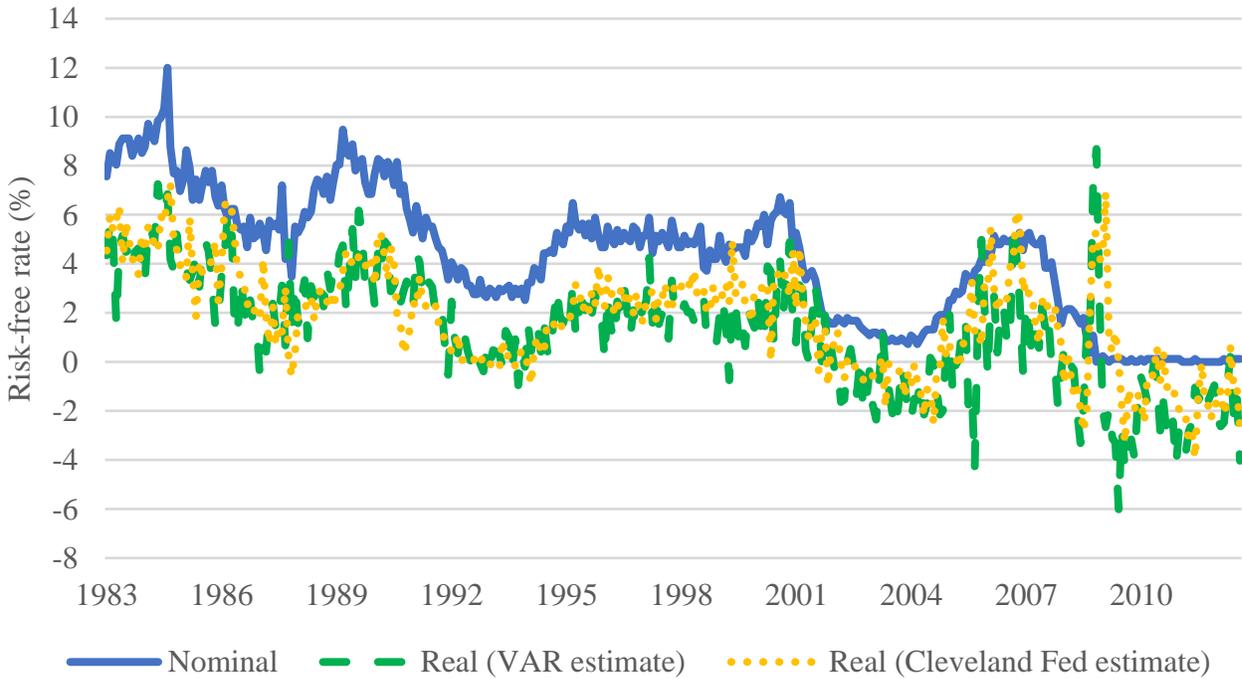
Description: Value-weighted decile portfolios are formed at the end of each month by sorting stocks based on covariance with risk-free rate news over the past three years. The table reports betas with respect to risk free rate news, average size, and average excess returns for each portfolio. The table also reports results for time series regressions of excess returns on excess market returns (the CAPM regression) and excess market returns (*rmrf*), the Fama-French size factor (*smb*), and the Fama-French value factor (*hml*) (the 3 Factor regression). The sample is NYSE, AMEX, and NASDAQ common stocks. Standard errors for the 10-1 portfolio difference are reported in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance.

Interpretation: Expected returns do not vary significantly across the decile portfolios.

| Panel A. 1985–2012 | | | | | | | | |
|--------------------|--------------------|---------------------|-------------------|-------------------|-------------------|-------------------------|-----------------|--------------------|
| Decile | r_f News Beta | Market Cap (\$B) | Excess Return | CAPM Alpha | 3 Factor Alpha | Factor Loadings (Betas) | | |
| | | | | | | <i>rmrf</i> | <i>smb</i> | <i>hml</i> |
| 1 | -0.17 | 0.72 | 0.63% | -0.19% | -0.16% | 1.27 | 0.61 | -0.06 |
| 2 | 0.07 | 1.36 | 0.94% | 0.24% | 0.30% | 1.10 | 0.22 | -0.15 |
| 3 | -0.04 | 1.94 | 0.87% | 0.25% | 0.23% | 1.04 | 0.07 | 0.04 |
| 4 | 0.13 | 2.42 | 0.65% | 0.06% | 0.03% | 1.00 | -0.04 | 0.09 |
| 5 | 0.00 | 2.74 | 0.51% | -0.03% | -0.05% | 0.94 | -0.10 | 0.03 |
| 6 | 0.02 | 2.76 | 0.48% | -0.06% | -0.08% | 0.93 | -0.14 | 0.05 |
| 7 | 0.03 | 2.58 | 0.54% | -0.02% | -0.04% | 0.97 | -0.11 | 0.03 |
| 8 | 0.15 | 2.21 | 0.68% | 0.06% | 0.08% | 1.04 | -0.13 | -0.07 |
| 9 | 0.14 | 1.69 | 0.61% | -0.06% | -0.04% | 1.10 | 0.01 | -0.06 |
| 10 | 0.41 | 0.85 | 0.21% | -0.62% | -0.44% | 1.21 | 0.55 | -0.47 |
| 10-1 | 0.58** (0.23) | 0.13** (0.06) | -0.42% (0.33%) | -0.42% (0.34%) | -0.27% (0.34%) | -0.06 (0.08) | -0.07 (0.11) | -0.41*** (0.12) |

| Panel B. 1929–2012 | | | | | | | | |
|--------------------|--------------------|---------------------|-------------------|-------------------|-------------------|-------------------------|-----------------|----------------|
| Decile | r_f News Beta | Market Cap (\$B) | Excess Return | CAPM Alpha | 3 Factor Alpha | Factor Loadings (Betas) | | |
| | | | | | | <i>rmrf</i> | <i>smb</i> | <i>hml</i> |
| 1 | -0.01 | 0.17 | 0.66% | -0.05% | -0.12% | 1.15 | 0.52 | -0.03 |
| 2 | 0.00 | 0.48 | 0.66% | 0.04% | 0.03% | 1.04 | 0.20 | -0.06 |
| 3 | 0.03 | 0.69 | 0.70% | 0.13% | 0.12% | 0.99 | 0.08 | -0.01 |
| 4 | 0.06 | 0.86 | 0.71% | 0.15% | 0.15% | 0.96 | 0.02 | 0.00 |
| 5 | 0.01 | 0.98 | 0.60% | 0.04% | 0.02% | 0.97 | -0.03 | 0.06 |
| 6 | 0.03 | 1.05 | 0.56% | -0.01% | -0.03% | 0.98 | -0.03 | 0.09 |
| 7 | 0.06 | 1.08 | 0.58% | -0.01% | -0.02% | 1.03 | -0.08 | 0.08 |
| 8 | 0.06 | 1.05 | 0.56% | -0.07% | -0.10% | 1.08 | 0.00 | 0.11 |
| 9 | 0.10 | 0.83 | 0.61% | -0.07% | -0.12% | 1.15 | 0.04 | 0.17 |
| 10 | 0.11 | 0.38 | 0.58% | -0.18% | -0.27% | 1.23 | 0.50 | 0.03 |
| 10-1 | 0.13 (0.09) | 0.21*** (0.02) | -0.09% (0.18%) | -0.13% (0.18%) | -0.14% (0.18%) | 0.07** (0.03) | -0.02 (0.06) | 0.05 (0.05) |

Figure IA.1. Risk-free rate, 1983–2012



Description: This figure plots the monthly nominal and real risk-free rate from 1983 to 2012. The nominal risk-free rate is the yield on a one-month nominal treasury bill. The real risk-free rate is estimated using VAR analysis. For comparison purposes, the Federal Reserve Bank of Cleveland’s real risk-free rate estimate is also plotted.

Interpretation: Real risk-free rate estimates from the VAR model closely track estimates from the Federal Reserve Bank of Cleveland and generally follow the same pattern as nominal interest rates.