Online Appendix

Understanding the Performance of Components in Betting Against Beta

A.1. Proof of the Return Decomposition

The excess return of an arbitrary portfolio can be written concisely in matrix terms as $R_p = \boldsymbol{r}'\boldsymbol{w}$, with the linear constraint that $\mathbf{1}'\boldsymbol{w} = c$. Here $\boldsymbol{r}$ denotes the column of the excess returns of the composite stocks, $\boldsymbol{w}$ denotes the column of the portfolio weights, and c is a constant. When $c = 0$, it is a zero-cost portfolio. When $c = 1$, it is the typical (unlevered) long-only portfolio. Actually, as long as $c > 0$, the portfolio represents an overall net-long position of the composite stocks.

In matrix terms, the excess return of $BAB^{Selection}$ portfolio can be written as

$$BAB^{Selection} = \boldsymbol{r}'\boldsymbol{w}^{EW},$$

where $\boldsymbol{w}^{EW} = \boldsymbol{w}_L^{EW} - \boldsymbol{w}_H^{EW}$. Note $\mathbf{1}'\boldsymbol{w}^{EW} = 0$, as it is a long-and-short, zero-cost portfolio with equal weighting scheme.

The excess return of the zero-cost, rank-weighted low-minus-high beta portfolio, BAB^{RW}, can be written as

$$BAB^{RW} = \boldsymbol{r}'\boldsymbol{w}^{RW},$$

where $\boldsymbol{w}^{RW} = \boldsymbol{w}_L^{RW} - \boldsymbol{w}_H^{RW}$. Similarly, $\mathbf{1}'\boldsymbol{w}^{RW} = 0$.

The excess return of the betting-against-beta portfolio can be written as

$$BAB = \boldsymbol{r}'\boldsymbol{w}^{BAR},$$

where $\boldsymbol{w}^{BAR} = \frac{1}{\beta_L}\boldsymbol{w}_L^{RW} - \frac{1}{\beta_H}\boldsymbol{w}_H^{RW}$. Note BAB leverages the low-beta stocks and deleverages the high-beta stocks, producing a net-long position in stocks. This can be shown as follows:

$$position = \mathbf{1}'\boldsymbol{w}^{BAB} = \frac{1}{\beta_L}\mathbf{1}'\boldsymbol{w}_L^{RW} - \frac{1}{\beta_H}\mathbf{1}'\boldsymbol{w}_H^{RW} = \frac{1}{\beta_L} - \frac{1}{\beta_H} > 0, \qquad \text{[A.1]}$$

where the last equality utilized the fact: $\mathbf{1}'\boldsymbol{w}_L^{RW} = \mathbf{1}'\boldsymbol{w}_H^{RW} = 1$. Note the excess returns of BAB is calculated by netting off $\left(\frac{1}{\beta_L} - \frac{1}{\beta_H}\right) \times RF_t$ to ensure the zero-investment requirement.

Based on the above equations, the decomposition of the excess return of the betting-against-beta portfolio can be rewritten concisely in matrix form:

$$\begin{aligned} BAB &= BAB^{Parity} + BAB^{Rank} + BAB^{Selection} \qquad \text{[A.2]} \\ &= (BAB - BAB^{RW}) + \left(BAB^{RW} - BAB^{Selection}\right) + \left(BAB^{Selection}\right) \\ &= \underbrace{\boldsymbol{r}'(\boldsymbol{w}^{BAB} - \boldsymbol{w}^{RW})}_{\substack{contribution \\ due\ to \\ beta\ parity}} + \underbrace{\boldsymbol{r}'(\boldsymbol{w}^{RW} - \boldsymbol{w}^{EW})}_{\substack{contribution \\ due\ to \\ weighting\ scheme}} + \underbrace{\boldsymbol{r}'\boldsymbol{w}^{EW}}_{\substack{contribution \\ due\ to \\ stock\ selection}} \end{aligned}$$

where $BAB^{Parity} \equiv BAB - BAB^{RW}$, and $BAB^{Rank} \equiv BAB^{RW} - BAB^{Selection}$.

With a bit of algebraic manipulation, it can be shown that the excess return of the beta-parity component, denoted as BAB^{Parity}, is

$$BAB^{Parity} = \boldsymbol{r}'(\boldsymbol{w}^{BAB} - \boldsymbol{w}^{RW}) = \boldsymbol{r}'\left[\left(\frac{1}{\beta_L} - 1\right)\boldsymbol{w}_L^{RW}\right] + \boldsymbol{r}'\left[\left(1 - \frac{1}{\beta_H}\right)\boldsymbol{w}_H^{RW}\right]. \quad \text{[A.3]}$$

The last equality utilizes the definitions of $\boldsymbol{w}^{BAR} = \frac{1}{\beta_L}\boldsymbol{w}_L^{RW} - \frac{1}{\beta_H}\boldsymbol{w}_H^{RW}$ and $\boldsymbol{w}^{RW} = \boldsymbol{w}_L^{RW} - \boldsymbol{w}_H^{RW}$. For the last equality, both terms in the squared brackets have non-negative weights, indicating a net-long position in stocks (*i.e.*, a positive market exposure/beta).

A.2. Variable Definitions

Notation	**Definition**
ME and lnME	The market capitalization and the natural logarithm of the market capitalization of a stock, defined as the (natural logarithm of) firm's total market capitalization measured at the end of June in year t.
BTM and lnBTM	The book-to-market ratio and the natural logarithm of the book-to-market ratio, defined as the (natural logarithm of) firm's book-to-market equity measured at the fiscal year ending in $t-1$.
OP	Operational profitability, defined as the ratio of operational profits and book equity measured at the fiscal year ending in $t-1$, which follows from Fama and French (2017).
INV	Asset investments, defined as the growth rate of total assets for the fiscal year ending in $t-1$, which follows from Fama and French (2017).
$\mathrm{RET}^{\mathrm{MOM}}$	Intermediate-term return momentum, defined as the cumulative returns over the past 12-month rolling window, skipping the most recent month according to Fama and French (2012).
SSKEW	Systematic skewness (also known as coskewness), defined as in Harvey and Siddique (2000), is calculated as the slope coefficient on the squared market terms in the following regression. $$R_i - RF = \alpha_i + \beta_i RMRF + \gamma_i RMRF^2 + \varepsilon_i$$

The above regression is performed using daily observations over the past 12-month rolling window. The estimation procedure is repeated each month to obtain the *ex ante* SSKEW measure for each month.

ISKEW	Idiosyncratic skewness, defined as the skewness of the daily residual terms obtained from the same regression used to calculate the (monthly) SSKEW measure.
IVOL	Idiosyncratic volatility, defined similarly as in Ang *et al.* (2006), which is the standard deviation of the residuals from the following regression. $R_i - RF = \alpha_i + \beta_i^{RMRF} RMRF + \beta_i^{SMB} SMB + \beta_i^{HML} HML + \varepsilon_i$ The *ex ante* IVOL measure is constructed using the above Fama-French three-factor model using daily observations over the prior month, which requires at least 10 observations to run the regression.
MAX5	The lottery demand measure, defined as the average of the largest five daily returns in the prior month (Bali *et al.* 2011; Bali *et al.* 2017).
ILLIQ	Amihud illiquidity ratio, defined as the 12-month rolling average of the ratio of absolute return and the dollar trading volume (Amihud 2002).
RET^{STREV}	Short-term return reversal, defined as the one-month stock returns in the prior month (Jegadeesh & Titman 1993).
TURN	Turnover ratio, defined as the average daily turnover ratio over the past 12-month rolling window. A minimum number of 100 daily observations is required in order to compute the statistics.
FVIX	The traded aggregated volatility factor, defined as the factor-mimicking portfolio which tracks the daily changes in the VIX index. To ensure a longer sample period, I adopt the old CBOE VIX index on S&P 100, which starts from 1 January 1986.

Following Ang *et al.* (2006), the traded factor is constructed by regressing the daily changes in VIX index on the daily excess returns of the basis assets (*i.e.*, the quintile portfolios sorted on past return sensitivities to VIX changes) with the full sample (to ensure precision):

$$\Delta VIX_t = \alpha + \sum_{i=1}^{5} \gamma_i \left(RET_{i,t}^{VIX} - RF_t\right) + \varepsilon_t,$$

where $RET_{i,t}^{VIX}$ is the *i*-the quintile portfolio sorted on past return sensitivities to VIX changes. The traded factor is the fitted part of the above regression less the intercept term. The daily FVIX factor is then cumulated to monthly level to generate the monthly time series.

FSENT The traded sentiment factor, defined as the factor-mimicking portfolio which tracks the monthly changes in the survey-based US sentiment index (*i.e.*, the Michigan consumer confidence index).

The traded sentiment factor is constructed (in a similar manner as the FVIX factor) by regressing the monthly changes in the consumer confidence index on the excess returns to the basis assets (*i.e.*, the quintile portfolios sorted on past return sensitivities to changes of the consumer confidence) with the full sample (to ensure precision).

$$\Delta SENT_t = \alpha + \sum_{i=1}^{5} \gamma_i \left(RET_{i,t}^{SENT} - RF_t\right) + \varepsilon_t,$$

where $RET_{i,t}^{SENT}$ is the *i*-the quintile portfolio sorted on past return sensitivities to changes of the consumer confidence index. The traded factor is the fitted part of the above regression less the intercept term.[16]

A.3. Portfolio Turnover of BAB, BAC, and their Component Portfolios

This subsection addresses the legitimate concern that whether the BA-type strategies (including their component portfolios) could survive reasonable transaction costs.

Panel A of Table A3 presents the *annualized* portfolio turnover for BAB, BAC, and their stand-alone component portfolios over the sample period. Following the classification of Novy-Marx and Velikov

[16] Note the coefficient γ_i serves effectively as the "weight" in the mimicking factor portfolio. Therefore, I rescale the γ_i coefficient by a factor of $|\sum \gamma_i|$, so that these weights add up to one. This helps conform the traded factor to a conventional portfolio (*i.e.*, bet \$1 on the sentiment-related risky assets and \$1 on the risk-free rate).

(2015), the BAB (BAC) strategy is a mid-turnover strategy as its annualized portfolio turnover is in between one and five times per year. The two zero-cost component portfolios of BAB (BAC), the stock selection and the rank weight components are also mid-turnover strategies. In contrast, the beta-parity component of BAB (BAC) is a low-turnover strategy with annualized portfolio turnover of 76% (48%), less than one time on a yearly basis. In general, the component portfolios of BAC have lower portfolio turnover than the counterparts of BAB.

Panel B of Table A3 provides a simple, back-of-the-envelope calculation of the transaction costs involved in implementing these investment strategies. We report the breakeven transaction costs that would eliminate the average excess returns and the risk-adjusted returns of BAB, BAC, and their component portfolios. Focusing on BAB and its three components, it seems that the beta-parity component is the most implementable strategy in practice, as its cut-off costs are 559, 339, 314, 393, 355 bps under the alternative factor models. For perspective, Korajczyk and Sadka (2004) estimate that the effective spread ranges from 0.16 to 141 bps with a mean of 5.59 bps for the US stocks over the period 1967 – 1999. Given the high portfolio turnover and close-to-zero (risk-adjusted) returns of the stock selection and rank weight components (see **Panel A**), their cut-off costs do not seem to withstand the requirements for practical implementation. The breakeven transaction costs for the overall BAB strategy are still economically meaningful when evaluated by the CAPM and FF3. However, when evaluated by FF5, FF6, and FF7 factor models, it could only withstand 87, 45, and 41 bps instead.

Similar pattern also holds for BAC and its components. The portfolio turnover of BAC and its components is less than that of their counterparts in BAB. For example, the breakeven transaction costs, which wipe out the FF7 alphas, for the three component portfolios in BAC are 180, 74, 616 bps, respectively. The cut-off costs for the overall BAC strategy ranges from 385 to 524 bps for the alphas under alternative model specification, indicating it remains beneficial to implement the BAC strategy when transaction costs are taken into account.

When interpreting the evidence with **Sections 4** collectively, it becomes clear that the time-series component (*i.e.*, the beta-parity component) is the most robust source for the profits of the BA-type strategies, both before and after taking transaction costs into account.

Table A1. A Numerical Example of the Portfolio Weights with Eight Stocks

Description: The table depicts the portfolio weights in a hypothetical investment universe with eight stocks of different market betas (ranging from 0.65 to 1.35). It illustrates the beta ranking of the stocks, and the portfolio weights in BAB and its three components: the stock selection component (Selection), the rank weight component (Rank), and the beta-parity component (Parity).

Interpretation: The stock selection component (Selection) and the rank weight component (Rank) are zero-cost investments with a total sum of portfolio weights of zero. The beta-parity component (Parity) has a net-long position. The BAB portfolio also has a net-long position.

Stock	Beta	Beta Ranking	Selection Weights	Rank Weights	Parity Weights	BAB Weights
1	0.65	1	0.25	0.19	0.16	0.59
2	0.75	2	0.25	0.06	0.11	0.42
3	0.85	3	0.25	–0.06	0.07	0.25
4	0.95	4	0.25	–0.19	0.02	0.08
5	1.05	5	–0.25	0.19	0.01	–0.05
6	1.15	6	–0.25	0.06	0.04	–0.15
7	1.25	7	–0.25	–0.06	0.06	–0.25
8	1.35	8	–0.25	–0.19	0.09	–0.35
Sum			**0.00**	**0.00**	**0.56**	**0.56**

Table A2. Performance Attribution of BAB and BAC under Alternative Asset Pricing Models

Description: The table reports the mean excess returns, Sharpe ratio, and the risk-adjusted returns of the stock-selection component, the rank-weight component, and the beta-parity component in BAB (Panel A) and in BAC (Panel B). HXZ4 Alpha and M4 Alpha are the intercept terms estimated by the regression of the Investment q-factor model (HXZ4) and the Mispricing four-factor model (M4), respectively. Newey-West adjusted *t*-statistics are reported in brackets. The sample period is between July 1963 and December 2016. ***, **, and * denotes the statistical significance at the 1%, 5%, and 10% level, respectively.

Interpretation: The beta-parity component remains the sole driver of the outperformance of BAB, while all three components contributes to BAC on a risk-adjusted basis.

	Panel A: Decomposition of BAB				**Panel B: Decomposition of BAC**			
	Selection	Rank	Parity	BAB	Selection	Rank	Parity	BAC
Excess Return	0.08	0.05	0.75***	0.88***	0.36***	0.14***	0.49***	0.99***
[*t*-stat.]	[0.53]	[0.82]	[4.05]	[4.43]	[3.82]	[3.93]	[3.85]	[4.77]
Sharpe	0.07	0.11	0.65	0.90	0.46	0.49	0.64	0.75
Proportion	9.02%	5.69%	85.29%		36.32%	13.74%	49.93%	
HXZ4 Alpha	0.06	0.02	0.28***	0.36*	0.44***	0.15***	0.32***	0.91***
[*t*-stat.]	[0.41]	[0.34]	[2.94]	[1.71]	[4.22]	[4.13]	[3.53]	[4.85]
Proportion	16.67%	5.55%	77.78%		48.35%	16.49%	35.16%	
M4 Alpha	0.09	0.04	0.26***	0.39*	0.32***	0.12***	0.28***	0.72***
[*t*-stat.]	[0.65]	[0.66]	[3.41]	[1.91]	[2.83]	[3.06]	[3.45]	[3.63]
Proportion	22.69%	9.84%	67.48%		44.94%	16.08%	38.98%	

Table A3. Portfolio Turnover and Breakeven Transaction Costs

Description: Panel A reports the annualized portfolio turnover of the stock-selection component (Selection), the rank-weight component (Rank), and the beta-parity component (Parity) in the BAB and BAC strategies. For a long-and-short portfolio, the turnover is summed over the long and short sides. Panel B reports the breakeven transaction costs that would zero out the average excess returns and the risk-adjusted returns (*i.e.*, alphas) under the CAPM model, the Fama-French three-factor model (FF3), the Fama-French five-factor model (FF5), the Fama-French six-factor model (FF6), and the augmented seven-factor model (FF7). - indicates that the breakeven transaction cost is either below the threshold of 10 basis points (bps), or undefined as the pre-cost average (risk-adjusted) return is negative. The sample period is from July 1963 to December 2016.

Interpretation: The beta-parity component in BAB (or BAC) remains the most cost-effective component with low portfolio turnover.

	Selection	Rank	Parity	BAB	Selection	Rank	Parity	BAC
Panel A: Annualized Portfolio Turnover: 196307 - 201612								
	182.58%	189.19%	76.45%	206.70%	151.46%	151.40%	47.93%	175.11%
Panel B: Break-even Transaction Costs (in bps): 196307 - 201612								
Excess Return	11.14	-	1,065.64	285.61	238.31	86.43	1,001.44	558.29
CAPM Alpha	130.48	36.16	559.48	300.25	294.38	117.65	620.81	524.62
FF3 Alpha	129.59	38.78	339.15	215.56	224.61	86.50	524.18	384.52
FF5 Alpha	26.02	-	313.93	86.77	225.01	86.05	591.25	403.77
FF6 Alpha	-	-	393.34	44.79	188.06	72.49	652.72	389.25
FF7 Alpha	-	-	355.09	41.24	180.26	73.55	615.74	372.26

Table A4. Portfolio Turnover and the Breakeven Transaction Costs for the Betting Against Beta and Betting Against Correlations Strategies

Description: Panel A reports the annualized portfolio turnover of the stock-selection component (Selection), the rank-weight component (Rank), and the beta-parity component (Parity) in the alternative BAB and BAC strategies. The alternative BAB and BAC strategies are formed by stocks with market capitalization above the median value in the cross section. For a long-and-short portfolio, the turnover is averaged over the long and short sides. Panel B reports the breakeven transaction costs that would zero out the average excess returns and the risk-adjusted returns (*i.e.*, alphas) under the augmented seven-factor model (FF7). - indicates that the breakeven transaction cost is either below the threshold of 10 basis points (bps), or undefined as the pre-cost average (risk-adjusted) return is negative. The sample period is from July 1963 to December 2016.

Interpretation: The beta-parity component in BAB (or BAC) remains the most cost-effective component with low portfolio turnover.

	Selection	**Rank**	**Parity**	**BAB**	**Selection**	**Rank**	**Parity**	**BAC**
				Panel A: Annualized Portfolio Turnover: 196307 - 201612				
All Firms	182.58%	189.19%	76.45%	206.70%	151.46%	151.40%	47.93%	175.11%
Top 50%	187.21%	202.97%	67.39%	206.97%	163.84%	169.63%	44.40%	185.95%
Bottom 50%	255.04%	260.47%	117.62%	296.07%	239.87%	246.78%	92.75%	283.59%
				Panel B: Break-even Transaction Costs (in bps): 196307 - 201612				
Excess Return								
All Firms	11.14	-	1,065.64	285.61	238.31	86.43	1,001.44	558.29
Top 50%	57.44	-	608.57	171.75	-	-	437.27	75.94
Bottom 50%	-	-	1,157.60	256.32	116.28	12.20	1,107.52	434.55
FF7 Alpha								
All Firms	-	-	355.09	41.24	180.26	73.55	615.74	372.26
Top 50%	-	-	-	-	-	-	-	-
Bottom 50%	-	-	528.52	55.56	113.41	28.00	647.78	307.24