ONLINE APPENDIX

Fund Alphas Depend on the Return Horizon

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Appendix A: Instantaneous and Long-Horizon Alphas and Betas

In this appendix, we derive the relationship between instantaneous alpha and long-horizon alpha in a continuous time CAPM. Let the market price P^m and asset i's idiosyncratic return V^ϵ follow Brownian motions:

$$\frac{dP^m}{P^m} = \mu_m dt + \sigma_m dZ^m$$

$$\frac{dV^\epsilon}{V^\epsilon} = \sigma_\epsilon dZ^\epsilon.$$

Applying Ito's lemma yields:

$$d\log P^m = \left(\mu_m - \frac{\sigma_m^2}{2}\right)dt + \sigma_m dZ^m$$

$$d\log V^\epsilon = -\frac{\sigma_\epsilon^2}{2}dt + \sigma_\epsilon dZ^\epsilon.$$

Long-horizon market return and return variance are as follows:

$$\log P_T^m - \log P_0^m = \left(\mu_m - \frac{\sigma_m^2}{2}\right)T + \sigma_m \int_0^T dZ^m$$

$$E\left(\frac{P_T^m}{P_0^m}\right) = E\left[e^{\left(\mu_m - \frac{1}{2}\sigma_m^2\right)T + \sigma_m \int_0^T dZ^m}\right] = e^{\mu_m T}$$

$$Var\left(\frac{P_T^m}{P_0^m}\right) = E\left[\left(\frac{P_T^m}{P_0^m}\right)^2\right] - E\left[\left(\frac{P_T^m}{P_0^m}\right)\right]^2 = E\left[e^{(2\mu_m - \sigma_m^2)T + 2\sigma_m \int_0^T dZ^m}\right] - e^{2\mu_m T}$$

$$= e^{(2\mu_m + \sigma_m^2)T} - e^{2\mu_m T} = \left(e^{\sigma_m^2 T} - 1\right)e^{2\mu_m T}$$

The asset's instantaneous return follows:

$$\frac{dP_i}{P_i} = \alpha_i dt + \beta_i \frac{dP^m}{P^m} + \frac{dV^\epsilon}{V^\epsilon} = \alpha_i dt + \beta_i(\mu_m dt + \sigma_m dZ^m) + \sigma_\epsilon dZ^\epsilon$$

$$= (\alpha_i + \beta_i \mu_m)dt + \beta_i \sigma_m dZ^m + \sigma_\epsilon dZ^\epsilon$$

Applying Ito's lemma to the last equation yields:

$$d\log P_i = \left(\alpha_i + \beta_i \mu_m - \frac{\beta_i^2 \sigma_m^2}{2} - \frac{\sigma_\epsilon^2}{2}\right)dt + \beta_i \sigma_m dZ^m + \sigma_\epsilon dZ^\epsilon$$

The asset's long-horizon return and return variance are as follows:

$$\log P_T^i - \log P_0^i = \left(\alpha_i + \beta_i \mu_m - \frac{\beta_i^2 \sigma_m^2}{2} - \frac{\sigma_\epsilon^2}{2}\right) T + \beta_i \sigma_m \int_0^T dZ^m + \sigma_\epsilon \int_0^T dZ^{\epsilon \frac{dV^\epsilon}{V^\epsilon}}$$

$$E\left(\frac{P_T^i}{P_0^i}\right) = E\left[e^{\left(\alpha_i + \beta_i \mu_m - \frac{\beta_i^2 \sigma_m^2}{2} - \frac{\sigma_\epsilon^2}{2}\right) T + \beta_i \sigma_m \int_0^T dZ^m + \sigma_{\epsilon \int_0^T dZ^\epsilon}}\right]$$

$$= e^{\left(\alpha_i + \beta_i \mu_m - \frac{\beta_i^2 \sigma_m^2}{2} - \frac{\sigma_\epsilon^2}{2}\right) T + \left(\frac{\beta_i^2 \sigma_m^2}{2} + \frac{\sigma_\epsilon^2}{2}\right) T} = e^{(\alpha_i + \beta_i \mu_m) T}$$

$$Var\left(\frac{P_T^i}{P_0^i}\right) = E\left[\left(\frac{P_T^i}{P_0^i}\right)^2\right] - E\left[\left(\frac{P_T^i}{P_0^i}\right)\right]^2$$

$$= E\left[e^{2\left(\alpha_i + \beta_i \mu_m - \frac{\beta_i^2 \sigma_m^2}{2} - \frac{\sigma_\epsilon^2}{2}\right) T + 2\sigma_m \int_0^T dZ^m + 2\sigma_\epsilon \int_0^T dZ^\epsilon}\right] - e^{2(\alpha_i + \beta_i \mu_m) T}$$

$$= e^{2\left(\alpha_i + \beta_i \mu_m - \frac{\beta_i^2 \sigma_m^2}{2} - \frac{\sigma_\epsilon^2}{2}\right) T + 2\beta_i^2 \sigma_m^2 T + 2\sigma_\epsilon^2 T} - e^{2(\alpha_i + \beta_i \mu_m) T}$$

$$= \left(e^{(\beta_i^2 \sigma_m^2 + \sigma_\epsilon^2) T} - 1\right) e^{2(\alpha_i + \beta \mu_m) T}$$

$$Cov\left(\frac{P_T^i}{P_0^i}, \frac{P_T^m}{P_0^m}\right) = E\left[\left(\frac{P_T^i}{P_0^i} \frac{P_T^m}{P_0^m}\right)\right] - E\left[\frac{P_T^i}{P_0^i}\right] E\left[\frac{P_T^m}{P_0^m}\right]$$

$$= E\left[e^{\left(\alpha_i + \beta_i \mu_m - \frac{\beta_i^2 \sigma_m^2}{2} - \frac{\sigma_\epsilon^2}{2}\right) T + \beta_i \sigma_m \int_0^T dZ^m + \sigma_\epsilon \int_0^T dZ^\epsilon} e^{\left(\mu_m - \frac{\sigma_m^2}{2}\right) T + \sigma_m \int_0^T dZ^m}\right] - e^{(\alpha_i + \beta_i \mu_m) T} e^{\mu_m T}$$

$$= E\left[e^{\left[\alpha_i + (\beta_i + 1)\mu_m - \frac{1}{2}(\beta_i^2 \sigma_m^2 + \sigma_m^2 + \sigma_\epsilon^2)\right] T + (\beta_i + 1)\sigma_m \int_0^T dZ^m + \sigma_\epsilon \int_0^T dZ^\epsilon}\right] - e^{[\alpha_i + (\beta_i + 1)\mu_m] T}$$

$$= e^{\left[\alpha_i + (\beta_i + 1)\mu_m - \frac{1}{2}(\beta_i^2 \sigma_m^2 + \sigma_m^2 + \sigma_\epsilon^2)\right] T + \frac{1}{2}(\beta_i + 1)^2 \sigma_m^2 T + \frac{1}{2}\sigma_\epsilon^2 T} - e^{[\alpha_i + (\beta_i + 1)\mu_m] T}$$

$$= \left(e^{\beta_i \sigma_m^2 T} - 1\right) e^{[\alpha_i + (\beta_i + 1)\mu_m] T}$$

The asset's long-horizon beta is:

$$\beta_i^L = \frac{Cov\left(\frac{P_T^i}{P_0^i}, \frac{P_T^m}{P_0^m}\right)}{Var\left(\frac{P_T^m}{P_0^m}\right)} = \frac{\left(e^{\beta_i \sigma_m^2 T} - 1\right) e^{[\alpha_i + (\beta_i + 1)\mu_m]T}}{\left(e^{\sigma_m^2 T} - 1\right) e^{2\mu_m T}}$$

The relationship between long-horizon alpha and instantaneous alpha is:

$$E\left(\frac{P_T^i}{P_0^i}\right) - 1 = \alpha_i^L + \beta_i^L \left[E\left(\frac{P_T^m}{P_0^m}\right) - 1\right]$$

$$e^{(\alpha_i + \beta_i \mu_m)T} - 1 = \alpha_i^L + \frac{\left(e^{\beta_i \sigma_m^2 T} - 1\right) e^{[\alpha_i + (\beta_i + 1)\mu_m]T}}{\left(e^{\sigma_m^2 T} - 1\right) e^{2\mu_m T}} (e^{\mu_m T} - 1)$$

$$\alpha_i^L = e^{(\alpha_i + \beta_i \mu_m)T} - 1 - \frac{e^{[\alpha_i + (\beta_i + 1)\mu_m]T}\left(e^{\beta_i \sigma_m^2 T} - 1\right)}{\left(e^{\sigma_m^2 T} - 1\right) e^{2\mu_m T}} (e^{\mu_m T} - 1)$$

The relationship between long-horizon alpha and instantaneous alpha is non-linear and complicated. Long-horizon and instantaneous alphas could have different signs. They have the same sign when instantaneous beta is one. When $\beta_i = 1$, $\alpha_i^L = e^{\alpha_i T} - 1$ and has the same sign as α_i.

Appendix B: Sample Construction and Data Filters

We obtain data for the 1991 to 2020 period from the CRSP survivorship bias free Mutual Fund Database. We begin at 1991, as data regarding fund total net assets (TNA), which we use to aggregate fund returns across share classes, is not consistently available for earlier periods. We rely on the CRSP share class group number (*crsp_cl_grp*) in the fund names file. For funds without a CRSP share class group number, we identify share classes of the same fund based on fund names. When funds have multiple share classes CRSP fund names contain "/" or ";". The part of the fund name after the last "/" or ";" refers to the sub share class, while the prior part refers to the main fund name. For example, the fund named "MainStay Funds: MainStay Small Cap Growth Fund; Class A Shares" is Class A of the MainStay Small Cap Growth Fund; the fund named "Alliance Strategic Balanced Fund/A" is Class A of the Alliance Strategic Balanced Fund.

We study domestic equity funds (CRSP fund style code starting with "ED"), while excluding exchange traded funds, exchange traded notes (those with CRSP *et_flag* equal to "F" or "N"), funds that take short positions (CRSP fund style "EDYS"), commodity funds (CRSP fund style "EDSC") and real estate funds (CRSP fund style "EDSR"). We exclude target date funds, since these hold substantial non-equity positions. To exclude target date funds and college savings funds we remove all funds with names that contain a four-digit number between 1990 and 2050 and the word "target", except that we do not exclude funds with "Russell 2000" or "Russell2000" in their names.

We further exclude hedged funds (CRSP fund style of "EDYH" and Lipper fund style code of "LSE"), market neutral funds (CRSP fund style of "EDYH" and Lipper fund style code of "EMN") and absolute return funds (CRSP fund style of "EDYH" and Lipper fund style code of "ABR"). We also screen some funds within the CRSP style code starting with "ED", but with names that are inconsistent with this categorization. Specifically, we exclude a fund with "VIX" in its name, funds with "Long/Short", "Long-Short", and "OTC/Short" in their names, funds whose name includes "ETF" or

"ETN", leveraged funds with "1.25x", "1.5x", "2x", "2.5x", "3x", and "4x" in their names, and one fixed income fund with "Government Portfolio" in its name.

Prior studies (e.g., Elton, Gruber, and Blake, 2001) have documented the presence of errors in the CRSP mutual fund data. We mitigate the effect of potentially influential errors by comparing large reported returns to those contained in the Morningstar Mutual Fund database, or if Morningstar data is not available to the returns implied by percentage changes in CRSP-reported NAV or TNA. Specifically, we identify 836 extreme fund returns based on a deviation from the same-month CRSP value-weighted market return of 30% or more. With the help of Professor Shuaiyu Chen, we are able to match 633 of these to monthly return data in the Morningstar mutual fund database. For 524 of these cases, the CRSP and Morningstar returns differ by less than 1% and we retain the CRSP return. For the remaining 109 cases we retain the Morningstar return, which in every instance is less extreme than the CRSP return. For the 203 instances that cannot be matched to Morningstar, we focus on the percentage change in the CRSP reported net-asset-value (NAV) as well as the TNA. We retain observations where the reported return deviates from both the NAV and TNA-implied returns by less than 30%. For 75 observations from 53 funds the deviation exceeds 30%, and we delete the associated funds from the sample. Finally, we exclude funds that have fewer than twelve months of non-missing return data. The sample employed here is also used by Bessembinder, Cooper, and Zhang (2023).

Appendix C: Additional Tables

Table A1: Fund turnover ratio and fund beta/alpha

We compute each fund's monthly beta by regressing excess monthly fund return on excess return to the SPDR S&P 500 ETF (SPY) and compute its long-horizon beta against SPY over three investment horizons (1 year, 5 years, and 10 years) using the modified Levhari and Levy (LL) approach detailed in Section 2.2. Lastly, we compute each fund's long-horizon alpha using its long-horizon returns and the long-horizon beta. We also compute each fund's turnover ratio as its median quarterly turnover ratio reported in CRSP, and sort the funds into four groups depending on whether the fund's monthly SPY beta is above 1 and whether its monthly SPY alpha is positive. For each group of funds, we divide them into terciles based on their turnover ratio. The table reports the average turnover ratio, the average monthly, annual, 5-year, and decade betas, and the average monthly, annual, 5-year, and decade alphas for each tercile of funds with at least 120 monthly returns. This table shows that investment horizon has significant effects on fund alpha estimates regardless of the fund's turnover rate.

Tercile, turnover ratio	N	Average turnover	Average SPY beta				Average SPY alpha (%, monthly rate)			
			Monthly	1-year	5-year	10-year	Monthly	1-year	5-year	10-year
Funds with monthly SPY beta > 1 & SPY alpha > 0										
1	248	0.259	1.106	1.122	1.252	1.455	0.152	0.131	0.025	-0.058
2	249	0.666	1.114	1.139	1.273	1.487	0.150	0.127	-0.030	-0.130
3	242	1.471	1.150	1.171	1.321	1.568	0.156	0.160	-0.036	-0.163
All	739	0.793	1.123	1.144	1.282	1.503	0.153	0.139	-0.014	-0.117
Funds with monthly SPY beta > 1 & SPY alpha < 0										
1	340	0.261	1.107	1.089	1.054	1.018	-0.169	-0.181	-0.231	-0.299
2	329	0.677	1.122	1.108	1.074	1.043	-0.182	-0.195	-0.270	-0.341
3	332	1.772	1.204	1.147	1.114	1.095	-0.256	-0.234	-0.334	-0.450
All	1001	0.899	1.144	1.115	1.080	1.052	-0.202	-0.203	-0.278	-0.363
Funds with monthly SPY beta < 1 & SPY alpha > 0										
1	218	0.185	0.837	0.853	0.862	0.888	0.139	0.118	0.136	0.257
2	208	0.457	0.860	0.874	0.902	0.957	0.155	0.144	0.139	0.232
3	213	1.155	0.878	0.890	0.929	0.997	0.172	0.153	0.141	0.205
All	639	0.597	0.858	0.872	0.897	0.947	0.155	0.138	0.139	0.232
Funds with monthly SPY beta < 1 & SPY alpha < 0										
1	204	0.190	0.892	0.880	0.808	0.733	-0.112	-0.112	-0.106	-0.065
2	202	0.491	0.894	0.879	0.798	0.715	-0.140	-0.147	-0.161	-0.119
3	195	1.756	0.871	0.870	0.784	0.702	-0.158	-0.171	-0.187	-0.151
All	601	0.799	0.886	0.876	0.797	0.717	-0.136	-0.143	-0.151	-0.111

Table A2: Long-horizon alpha versus short-horizon alpha for the portfolio of mutual funds

We form a portfolio of domestic equity mutual funds and compute its monthly beta by regressing excess monthly (equal-weighted or value-weighted) portfolio return on excess return to the SPDR S&P 500 ETF (SPY) and compute its long-horizon beta against SPY over three investment horizons (1 year, 5 years, and 10 years) using the modified Levhari and Levy (LL) approach detailed in Section 2.2. Lastly, we compute the portfolio's long-horizon alpha using its long-horizon returns and the long-horizon beta. This table compares the monthly versus long-horizon alphas for the portfolio of all funds and for four sub-groups of funds depending on whether their monthly SPY beta is above 1 and whether their monthly SPY alpha is positive. Panel A presents the results based on equal-weighted portfolio returns; Panel B presents the results based on value-weighted portfolio returns. This table shows that investment horizon has significant effects on alpha estimates for portfolio of mutual funds, alleviating the concern that the effects are driven by endogenous fund survival.

Panel A: Monthly and long-horizon beta/alpha for the equal-weighted portfolio of mutual funds

All funds (N = 7883)			
Monthly beta	1.014	Monthly alpha (%)	-0.073
1-year beta	1.013	1-year alpha (%, monthly rate)	-0.105
5-year beta	0.984	5-year alpha (%, monthly rate)	-0.189
10-year beta	0.948	10-year alpha (%, monthly rate)	-0.105
Funds with monthly SPY beta > 1 & SPY alpha > 0 (N = 1535)			
Monthly beta	1.133	Monthly alpha (%)	0.155
1-year beta	1.172	1-year alpha (%, monthly rate)	0.100
5-year beta	1.314	5-year alpha (%, monthly rate)	-0.088
10-year beta	1.545	10-year alpha (%, monthly rate)	-0.054
Funds with monthly SPY beta > 1 & SPY alpha < 0 (N = 2911)			
Monthly beta	1.156	Monthly alpha (%)	-0.253
1-year beta	1.144	1-year alpha (%, monthly rate)	-0.284
5-year beta	1.085	5-year alpha (%, monthly rate)	-0.393
10-year beta	1.015	10-year alpha (%, monthly rate)	-0.405
Funds with monthly SPY beta < 1 & SPY alpha > 0 (N = 1310)			
Monthly beta	0.831	Monthly alpha (%)	0.134
1-year beta	0.829	1-year alpha (%, monthly rate)	0.121
5-year beta	0.828	5-year alpha (%, monthly rate)	0.102
10-year beta	0.834	10-year alpha (%, monthly rate)	0.307
Funds with monthly SPY beta < 1 & SPY alpha < 0 (N = 2127)			
Monthly beta	0.854	Monthly alpha (%)	-0.186
1-year beta	0.825	1-year alpha (%, monthly rate)	-0.191
5-year beta	0.712	5-year alpha (%, monthly rate)	-0.214
10-year beta	0.590	10-year alpha (%, monthly rate)	-0.137

Panel B: Monthly and long-horizon beta/alpha for the value-weighted portfolio of mutual funds

All funds (N = 7883)			
Monthly beta	1.008	Monthly alpha (%)	-0.055
1-year beta	1.000	1-year alpha (%, monthly rate)	-0.071
5-year beta	0.982	5-year alpha (%, monthly rate)	-0.154
10-year beta	0.955	10-year alpha (%, monthly rate)	-0.118
Funds with monthly SPY beta > 1 & SPY alpha > 0 (N = 1535)			
Monthly beta	1.100	Monthly alpha (%)	0.002
1-year beta	1.115	1-year alpha (%, monthly rate)	-0.035
5-year beta	1.167	5-year alpha (%, monthly rate)	-0.191
10-year beta	1.209	10-year alpha (%, monthly rate)	-0.186
Funds with monthly SPY beta > 1 & SPY alpha < 0 (N = 2911)			
Monthly beta	1.091	Monthly alpha (%)	-0.167
1-year beta	1.076	1-year alpha (%, monthly rate)	-0.185
5-year beta	1.034	5-year alpha (%, monthly rate)	-0.281
10-year beta	0.985	10-year alpha (%, monthly rate)	-0.304
Funds with monthly SPY beta < 1 & SPY alpha > 0 (N = 1310)			
Monthly beta	0.875	Monthly alpha (%)	0.056
1-year beta	0.863	1-year alpha (%, monthly rate)	0.054
5-year beta	0.856	5-year alpha (%, monthly rate)	0.029
10-year beta	0.831	10-year alpha (%, monthly rate)	0.174
Funds with monthly SPY beta < 1 & SPY alpha < 0 (N = 2127)			
Monthly beta	0.883	Monthly alpha (%)	-0.127
1-year beta	0.861	1-year alpha (%, monthly rate)	-0.127
5-year beta	0.773	5-year alpha (%, monthly rate)	-0.146
10-year beta	0.677	10-year alpha (%, monthly rate)	-0.083