Internet Appendix

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I.1 Cox Model Regressions

In this section, we follow Cox [1972] and choose to not impose a specific functional form to the pattern of duration dependence (γ_j). Instead, we include individual duration period dummies together with controls **x** (which cannot contain an intercept). We estimate the following semi-parametric cloglog model:

$$\log\left(-\log\left[1-h_{j}\left(\mathbf{x}\right)\right]\right) = \beta'\mathbf{x} + \gamma_{1}D_{1} + \gamma_{2}D_{2} + \dots + \gamma_{j}D_{j}.$$
(I.1)

Cox Regression – Baseline	model
	(1) End of Fraud
log(Total Assets)	-0.138*** (0.036)
RoE	-0.104** (0.043)
Market-to-Book	-0.005 (0.038)
Leverage	0.783** (0.358)
Soft Assets	-0.317 (0.409)
Abnormal Stock Return	-0.649** (0.262)
Market Return	-0.913 (0.816)
Large Earnings Miss Dummy	0.868*** (0.190)
Stock price crash	0.175 (0.288)
Industry Dummies	YES
Time Period Dummies	YES
No. of AAERs	191
Ν	1,372

TABLE I.1
Cox Regression – Baseline Model

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. * p < 0.1; ** p < 0.05; *** p < 0.01

	b		s			
	(1) End of Fraud	(2) End of Fraud	(3) End of Fraud	(4) End of Fraud	(5) End of Fraud	(6) End of Fraud
4 th Quarter	0.796^{**} (0.231)	0.494* (0.261)	0.965^{***} (0.336)	0.652^{*} (0.336)	$1.118^{***}_{(0.252)}$	1.095^{**} (0.449)
4^{th} Quarter $ imes$ Audit Explanation		0.865*** (0.274)				0.831^{***} (0.301)
log(Auditor Tenure)			-0.335*** (0.127)			-0.350*** (0.128)
$4^{th.}$ Quarter $\times \log(Auditor Tenure)$			0.159 (0.184)			0.235 (0.192)
Big N				-0.633** (0.248)		-0.254 (0.326)
4^{th} Quarter $ imes$ Big N				0.241 (0.352)		-0.619 (0.458)
Auditor Industry Specialist					-0.148 (0.320)	-0.127 (0.324)
4^{th} Quarter × Industry Specialist					-0.266 (0.497)	-0.357 (0.511)
Control Variables	YES	YES	YES	YES	YES	YES
Industry Dummies	YES	YES	YES	YES	YES	YES
Time Period Dummies	YES	YES	YES	YES	YES	YES
No. of AAERs	179	179	150	179	152	150
Ν	1,303	1,303	1,112	1,303	1,130	1,112

TABLE I.2Cox Regression – The Role of Auditors

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. * p < 0.1; ** p < 0.05; *** p < 0.01

C04 N	1101662182	in more of	MC IOINNIL	incress.		
	(1) End of Fraud	(2) End of Fraud	(3) End of Fraud	(4) End of Fraud	(5) End of Fraud	(6) End of Fraud
4 th Quarter	0.497* (0.260)	0.490* (0.259)	0.478* (0.264)	0.487* (0.269)	0.521* (0.270)	0.477*(0.266)
4^{th} Quarter \times Audit Explanation	0.841^{***} (0.273)	0.828^{***} (0.273)	0.908*** (0.275)	0.838*** (0.274)	0.835*** (0.274)	$0.909^{***}_{(0.277)}$
Auditor Switch 1	0.525** (0.243)			0.496 (0.314)		
Auditor Switch 2		$0.711^{**}_{(0.286)}$			0.796** (0.354)	
Auditor Switch 3			2.026*** (0.612)			2.024*** (0.667)
4^{th} Quarter \times Auditor Switch 1				0.069 (0.481)		
4^{th} Quarter \times Auditor Switch 2					-0.217 (0.561)	
4^{th} Quarter \times Auditor Switch 3						0.016 (1.621)
Control Variables	YES	YES	YES	YES	YES	YES
Industry Dummies	YES	YES	YES	YES	YES	YES
Time Period Dummies	YES	YES	YES	YES	YES	YES
No. of AAERs	179	179	179	179	179	179
Ν	1,303	1,303	1,303	1,303	1,303	1,303
	JU -		-			

TABLE I.3 *Cox Regression – The Role of Auditor Switches* The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. p < 0.1; ** p < 0.05; *** p < 0.01

	(1)	(2)	(3)
	End of Fraud	End of Fraud	End of Fraud
Analyst Indicator	-0.211 (0.258)		
Analysts 1 ^{st.} Quintile		0.706*** (0.272)	0.624** (0.280)
Analysts 2 ^{nd.} Quintile		0.182 (0.316)	0.136 (0.316)
Analysts 3 ^{rd.} Quintile		0.443 (0.377)	0.387 (0.377)
Analysts 4 ^{th.} Quintile		-0.239 (0.441)	-0.281 (0.439)
Analysts 5 ^{th.} Quintile		-0.168 (0.525)	-0.259 (0.525)
Mean Forecast Error			3.487 (2.617)
Large Earnings Miss Dummy	0.871*** (0.207)	0.652*** (0.204)	0.602*** (0.208)
4 th Quarter	0.488* (0.264)	0.404 (0.267)	0.387 (0.267)
4^{th} Quarter × Audit Explanation	0.913*** (0.275)	0.950*** (0.281)	0.982*** (0.283)
Auditor Switch 3	2.035*** (0.613)	2.040*** (0.640)	2.042*** (0.630)
Control Variables	YES	YES	YES
Industry Dummies	YES	YES	YES
Time Period Dummies	YES	YES	YES
No. of AAERs	179	179	179
Ν	1,303	1,303	1,303

TABLE I.4Cox Regression – The Role of Analysts

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. * p < 0.1; ** p < 0.05; *** p < 0.01

TABLE I.5	Cox Regression – The Role of Analysts
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	(1)	(5)	(3)	(4)	(5)	(9)
	End of Fraud	End of Fraud	End of Fraud	End of Fraud	End of Fraud	End of Fraud
Analyst Indicator	-0.211 (0.258)	-0.171 (0.257)	-0.087 (0.257)	0.025 (0.281)	-0.065 (0.271)	-0.092 (0.268)
Estimate Drop Indicator		0.455** (0.196)				
log(1+No. of Dropped Estimates)			0.522^{***} (0.148)			0.527^{***} (0.173)
Estimate Addition Indicator				-0.391** (0.197)		
log(1+No. Added Estimates)					-0.269 (0.164)	$\begin{array}{c} 0.011 \\ (0.188) \end{array}$
Large Earnings Miss Dummy	0.871^{***} (0.207)	0.838^{***} (0.206)	0.811^{***} (0.206)	0.850^{***} (0.206)	0.852*** (0.206)	0.811^{***} (0.206)
4 th Quarter	0.488* (0.264)	0.523** (0.266)	0.526** (0.266)	0.525** (0.264)	0.515* (0.264)	0.526** (0.266)
4^{th} Quarter × Audit Explanation	0.913^{**} (0.275)	0.887*** (0.276)	0.863*** (0.278)	0.925*** (0.275)	0.925*** (0.275)	0.862*** (0.278)
Auditor Switch 3	2.035*** (0.613)	2.128^{***} (0.629)	2.188*** (0.631)	2.111^{**} (0.629)	2.122*** (0.637)	2.186*** (0.632)
Control Variables	YES	YES	YES	YES	YES	YES
Industry Dummies	YES	YES	YES	YES	YES	YES
Time Period Dummies	YES	YES	YES	YES	YES	YES
No. of Frauds	179	179	179	179	179	179
Ν	1,303	1,303	1,303	1,303	1,303	1,303
The table reports the results of implementing a ra	indom effects pa 382 to 2010 per	anel complement and The full se	ary log-log regr t of variables m	ession of the qui	arterly fraud terr of Table 4 is i	nination hazard

rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. * p < 0.1; ** p < 0.05; *** p < 0.01

	(1)	(2)	(3)	(4)
	End of Fraud	End of Fraud	End of Fraud	End of Fraud
1 st Quarter Start	-1.498***		-1.432***	-1.435***
-	(0.207)		(0.211)	(0.245)
log(Number of Areas)		-0.487***	-0.250	-0.170
-		(0.172)	(0.170)	(0.191)
Total Accruals				-2.111***
				(0.533)
4 th Quarter	0.178	0.360	0.172	0.313
-	(0.235)	(0.264)	(0.235)	(0.271)
4 th Quarter x Audit Explanation	1.017***	0.951***	1.006***	1.254***
	(0.285)	(0.282)	(0.285)	(0.315)
Auditor Switch 3	1.852***	2.030***	1.858***	1.597**
	(0.660)	(0.614)	(0.644)	(0.659)
Analysts 1 ^{st.} Quintile	0.824***	0.577**	0.787***	0.885***
	(0.271)	(0.270)	(0.270)	(0.318)
Analysts 2 nd . Quintile	0.303	0.109	0.263	0.469
	(0.302)	(0.309)	(0.301)	(0.351)
Analysts 3 ^{rd.} Quintile	0.455	0.312	0.408	0.746*
	(0.357)	(0.372)	(0.356)	(0.389)
Analysts 4 ^{th.} Quintile	-0.059	-0.332	-0.099	0.229
	(0.420)	(0.430)	(0.417)	(0.448)
Analysts 5 ^{th.} Quintile	-0.098	-0.302	-0.167	0.208
	(0.484)	(0.518)	(0.486)	(0.509)
Large Earnings Miss Dummy	0.619***	0.608***	0.591***	0.700***
	(0.206)	(0.206)	(0.208)	(0.224)
Control Variables	YES	YES	YES	YES
Industry Dummies	YES	YES	YES	YES
Time Period Dummies	YES	YES	YES	YES
No. of AAERs	179	179	179	160
Ν	1,303	1,303	1,303	1,177

TABLE I.6Cox Regression – The Role of Managerial Effort

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses.

* p < 0.1; ** p < 0.05; *** p < 0.01

I.2 Model

In this appendix, we develop a stylized model of information production and fraud duration that guides our empirical analysis. A detailed presentation of the proofs, which do not affect the economic intuition of the results, is presented in Section I.3.

I.2.1 Basic model

Consider two types of risk-neutral, long-lived firms: Manipulators (*M*) and Non-Manipulators (*NM*). We assume that *NM*s never misrepresent their financial statements. Differently, *M*s regularly manipulate their financial statements to their own benefit. Even though later we endogenize the firm's choice of becoming a manipulator, let's initially denote the probability that any given firm is a manipulator by $\xi \in (0, 1)$.

Every time a financial statement is issued, a group of information producers and intermediaries scrutinize the accounting data. These are auditors, analysts, and institutional investors, among others. In this basic model, we assume a unique information producer – we generalize the results for multiple information producers in the next subsection. The signals detected by the information producers can be good (s = G) or bad (s = B). The probability that an information producer detects a bad signal while scrutinizing a manipulator is given by Pr(B|M) = p. On the other hand, information producers only detect good signals while screening non-manipulators' statements, i.e., Pr(B|NM) = 0. Signals across different financial statements are assumed to be i.i.d. in this section – we relax this assumption later.

Risk-neutral monitors – composed of regulators, institutional investors, and board members – observe the signals detected by the information producers and decide if they should intervene in the firm or not, i.e. a monitor's action space is $A = \{I, NI\}$, where I and NI represent intervention and non-intervention, respectively. In order to intervene in a firm and scrutinize it for accounting misbehavior, monitors must incur a cost $\mathcal{C} > 0$. Whenever a manipulator is caught, intervening monitors obtain a gain of $\mathcal{P} > \mathcal{C} > 0$. However, if they intervene in a non-manipulator, their return is normalized to zero. Both \mathcal{P} and \mathcal{C} may be monitor-specific, but, for ease of notation and because our results do not depend on such heterogeneity, we assume \mathcal{P} and \mathcal{C} are common for all monitors. Accordingly, in period t, a monitor's instantaneous expected utility is given by:

$$u(a_t, \mathscr{H}_t) = \begin{cases} \Pr(M|\mathscr{H}_t) \times \mathscr{P} - \mathscr{C}, & \text{if } a_t = I, \\ 0, & \text{if } a_t = NI, \end{cases}$$
(I.2)

where \mathcal{H}_t is the history of signals. The probability of a manipulator conditional on \mathcal{H}_t is

$$\Pr(M|\mathscr{H}_t) = \begin{cases} 1, & \text{if } h_i = B, \text{ for some } h_i \in \mathscr{H}_t, \\ \frac{\xi(1-p)^t}{(1-\xi)+\xi(1-p)^t}, & \text{otherwise.} \end{cases}$$
(I.3)

Based on the instantaneous utility function, the value function for monitors is given by

$$V(\mathscr{H}_t) = \max_{a_t \in A} \{ \Pr(M|\mathscr{H}_t) \times \mathscr{P} - \mathscr{C}, \delta E_t[V(\mathscr{H}_{t+1})] \},$$
(I.4)

where $\delta \in (0,1)$ is the discount rate.¹ Now, we can show a few results, but let's first define $\mathscr{H}_t(B) \equiv \{\mathscr{H}_t \text{ s.t. } \exists h_i = B \in \mathscr{H}_t\}$ as the set of histories in which a bad signal was observed at some point and denote the history at the beginning of the firm by $\mathscr{H}_{\emptyset} = \emptyset$.

LEMMA 1. If $\mathscr{H}_t \in \mathscr{H}_t(B)$, monitors should intervene, i.e., $V(\mathscr{H}_t) = \mathscr{P} - \mathscr{C}$.

Then, the following conclusion is a straightforward consequence:

COROLLARY 1. Monitors should immediately intervene if they observe a bad signal.

We can now state the main proposition in the monitor's problem.

PROPOSITION 1. If $\xi \mathcal{P} < \mathcal{C}$, then monitors only intervene if they observe a bad signal.

In other words, if the expected gain from intervening in a randomly drawn firm is smaller than the cost of intervening, then a bad signal is a necessary condition for intervention. Therefore, based on Proposition 1, if it is not optimal to immediately intervene in a firm – even before observing any signal – it is never optimal to intervene before observing a bad signal. From this point on, we keep the assumption $\xi \mathscr{P} < \mathscr{C}$, so monitors only intervene once they observe a bad signal.² Consequently, the length of a fraud is described by a geometric distribution, which leads to the following proposition:

PROPOSITION 2. The expected length of a fraud is given by $E[N] = \frac{1}{p}$.

As a result, the better the information producers are at spotting frauds, by detecting bad signals, the lower the life expectancy of a fraud. Before we move to the extensions, keep in mind that the hazard rate function, i.e., the probability that a fraud is detected in period t conditional on having survived until period t - 1, is given by p, a constant, as the geometric distribution is memoryless. In the extensions, we consider cases in which the hazard rate is time dependent, because longer frauds may become easier to catch.

I.2.2 Extensions

I.2.2.1 Multiple information producers

Independent signals

Let $\mathscr{I} = \{1, ..., I\}$ be the set of information producers. In order to study the case in which they are the most efficient, assume that they detect signals independently from each other. As before, assume that information producers never detect a bad signal while scrutinizing *NM* firms. Differently, we assume that information provider *i* detects a bad signal while scrutinizing a type *M* firm with probability p_i . Then, the probability that at least one information provider detects a bad signal is given by:

$$\Pr(B|M) = 1 - \prod_{i \in \mathscr{I}} (1 - p_i), \tag{I.5}$$

¹From equation (I.4), it is clear that \mathscr{P} includes the discounted difference between the value obtained by the monitor from correct intervention and from superfluous intervention, whereas \mathscr{C} includes his/her discounted value of needless intervention.

²Because monitors intervene whenever they observe a bad signal, it is also not optimal for firms that plan to engage in fraudulent behavior to build up their reputation by delaying the fraud start.

and the expected duration of a fraud is given by:

$$E[N] = \frac{1}{1 - \prod_{i \in \mathscr{I}} (1 - p_i)}.$$
(I.6)

As before, the better information providers are at spotting a fraud – i.e., the higher p_i for at least some $i \in \mathscr{I}$ – the shorter the expected duration of a fraud. Likewise, the introduction of an additional information producer increases the probability of fraud detection and reduces its expected length.

PROPOSITION 3. The introduction of a new information producer at any given period increases the likelihood of detecting a bad signal, thus shortening the fraud's expected length. The better the new information producer is at catching frauds – i.e., the higher his/her p – the larger the effect.

Correlated signals

In this case, since signal detection is not independent across information producers, we take into account the interactions among detected signals through their joint p.d.f.. Therefore, we have that the probability that at least one information producer detects a bad signal is $Pr(B|M) = 1 - Pr(s_1 = G, s_2 = G, ..., s_I = G)$, and the expected fraud duration is given by:

$$E[N] = \frac{1}{1 - \Pr(s_1 = G, s_2 = G, ..., s_{\mathbf{I}} = G)}.$$
(I.7)

As expected, as long as the signals are not perfectly correlated, in the sense that $Pr(s_i = G | s_1 = G, s_2 = G, ..., s_{i-1} = G, s_{i+1} = G, ..., s_I = G) < 1, \forall i \in \mathcal{I}$, all previous results are qualitatively the same, even though they are quantitatively weaker.

Because notation becomes cumbersome in the case of correlated signals across information producers, we focus on the case with independent signals. However, the reader should keep in mind that all results are preserved once we allow for partial correlation.

We can also consider the incentives for firms to exert effort to make frauds harder to detect. Before we discuss that, let's consider the case in which the probability of detection varies over time.

I.2.2.2 Time-varying probability of a bad signal

As we mentioned previously, in the basic model the hazard rate is constant over time. This lack of memory is a feature of the geometric distribution that may not be particularly suited to our case. In this sense, we may consider that the probability of producing a bad signal may change over time, i.e.:

$$\Pr(B|M,t) = p(t). \tag{I.8}$$

A natural assumption would be p'(t) > 0, i.e., as time passes, the probability of obtaining a bad signal increases. For example, a longer fraud means that more financial statements are affected by the fraud and it may be easier to spot inconsistencies. We also assume that $p(t) < 1, \forall t \in \mathbb{N}$ and $\lim_{t\to\infty} p(t) = 1$, i.e., the probability of getting a bad signal increases but it is never 1 at a finite time. Then, the expected duration of the fraud is now:

$$E[N] = \sum_{t=1}^{\infty} t p(t) \prod_{t'=1}^{t-1} (1 - p(t')).$$
(I.9)

While the hazard rate is now h(t) = p(t).³

Moreover, even though we imagine that the probability of being detected has an upward trend, the actual probability may vary around the trend. In particular, we may expect that market and firm time-varying characteristics may affect the detection probability, pushing it above or below the long-term trend. For example, good or bad performance in the stock market may increase or decrease incentives to scrutinize, making it easier or harder for information producers to detect signs of manipulation. A similar argument can be made about the firm's own operational and stock market performance.

I.2.2.3 Firm-specific factors and the probability of a bad signal

Observable firm characteristics may influence the likelihood that an information producer may detect a bad signal. For example, firm size may be related to the duration of accounting misconduct in a few ways. Large firms have relatively richer information environments than small firms. A richer information environment should make the marginal cost of issuing an additional fraud signal lower for information producers and thus reduce the duration of accounting misconduct. Conversely, large firms also tend to have a wider scope of operations than small firms, which may make it easier for a manager to conceal misconduct. In this sense, we expect that the probability that an IP issues a bad signal for a manipulator *i* is given by $p(\mathbf{x}_{i,t},t)$, where $\mathbf{x}_{i,t}$ is a vector of firm *i* characteristics at time *t* that make it easier or harder for IPs to spot a bad signal.⁴

I.2.3 Firm's decision on commiting fraud and efforts to hide fraud

I.2.3.1 Firm's decision to commit fraud

Up to now, we consider the decision of whether to commit fraud or not as exogenous, representing the firm's type. In this section, we consider the firm's decision about committing fraud.

We assume that firms differ in their benefit in committing fraud or not, i.e., the firm's benefit in committing fraud \mathscr{B} is a draw in the distribution F(.) with support $(0,\overline{\mathscr{B}})$. We also assume that, if the firm is caught, it incurs a loss of $\mathscr{L} \equiv \overline{\mathscr{B}}$, independent of its type. Finally, a firm decides each moment if it will continue to commit fraud or if it decides to stop. For simplicity, we assume that only ongoing frauds can be discovered. In this sense, the firm can decide if it will commit (or continues) a fraud period by period.

Then, the period *t* expected benefit (or loss) of committing a fraud that has been ongoing for *t* periods for a type \mathscr{B} firm is given by:

$$\mathbf{Profit}(\mathscr{B}, \mathbf{t}) = (1 - p(t))\mathscr{B} + p(t)(-\mathscr{L}). \tag{I.10}$$

³In the Section I.4 we present a simple example in which p(t) is an increasing and concave function.

⁴We allow $\mathbf{x}_{i,t}$ to depend on *t* since several important firm characteristics – such as size, leverage, fraction of soft assets, among others – vary over time.

Even though firms live forever and the decision to start or continue a misrepresentation is a dynamic problem, proposition 4 below shows that the decision ultimately depends only on the current period's expected benefit or loss. Therefore, a firm decides to start or continue an ongoing fraud if **Profit**(\mathscr{B} ,t)>0.

PROPOSITION 4. In an economy in which firms choose optimally to commit fraud and frauds do not become harder to spot over time - i.e., $p'(t) \ge 0$ - the following is true:

1. Non-Manipulation is the optimal policy for all firms with $\mathscr{B} \leq \mathscr{B}^*$, where \mathscr{B}^* is given by:

$$(1-p(1))\mathscr{B}^* + p(1)(-\mathscr{L}) = 0.$$
 (I.11)

- 2. If p(t) = p for all t, then, if a firm decides to commit fraud, it will never stop until it gets *caught*.
- 3. If p'(t) > 0 and $\lim_{t\to\infty} p(t) = 1$ for every $\mathscr{B} > \mathscr{B}^*$, there is a $T(\mathscr{B}) < \infty$ in which, if the firm has not been caught up to that point, management decides that it is not profitable to continue the fraud anymore. $T(\mathscr{B})$ is defined by:

$$(1 - p(T(\mathscr{B})))\mathscr{B} + p(T(\mathscr{B}))(-\mathscr{L}) = 0.$$
(I.12)

From the implicit function theorem, notice that

$$\frac{dT(\mathscr{B})}{d\mathscr{B}} = \frac{(1 - p(T(\mathscr{B})))}{p'(T(\mathscr{B}))(\mathscr{B} + \mathscr{L})} > 0, \tag{I.13}$$

since p'(T) > 0 for all T. Based on this result, we have the following corollary:

COROLLARY 2. *Firms that benefit the most out of a fraud are more likely to get caught instead of stopping the fraud by themselves.*

Finally, based on the proof of proposition 4, we can also conclude that all results presented here are still true for time-varying benefit of fraud and loss due to detection $-\mathcal{B}(t)$ and $\mathcal{L}(t)$ – as long as $(1-p(t))\mathcal{B}(t) + p(t)\mathcal{L}(t)$ decreases over time. In this sense, as long as $\mathcal{B}(t)$ does not increase faster than $\mathcal{L}(t)$ over time, our results are still valid.

I.2.3.2 Fraudster's effort

Consider that the fraudster can exert an effort $e_M > 0$ in order to make it harder for information producers to spot irregularities. In order to simplify notation, let's initially assume that the probability of a bad signal does not change over time. Therefore, we assume that $\frac{\partial p_i(e_M)}{\partial e_M} < 0$, i.e., by exercising effort, the manipulator reduces the likelihood of a bad signal for any information provider $i \in \mathscr{I}$. We also assume that the cost of effort is given by a convex, strictly increasing function $C(e_M)$, while $\lim_{e_M \to e_M^*} C(e_M) = \infty$, where $p_i(e_M^*) = 0$, $\forall i \in \mathscr{I}$. In other words, it would be prohibitively expensive to completely eliminate the risk of getting caught.

Then, it is easy to see that the expected duration of the fraud is given by:

$$E[N|e_M] = \frac{1}{1 - \prod_{i \in \mathscr{I}} (1 - p_i(e_M))}.$$
 (I.14)

Therefore, as expected, $\frac{\partial E[N|e_M]}{\partial e_M} > 0.$

I.2.3.3 Optimal choice of effort

Now, let's consider that the firm committing fraud can optimally choose its effort to hide an ongoing fraud. As in the previous section, we consider that the firm chooses not only whether it will start or continue an ongoing fraud every period⁵ but also its efforts in hiding the fraud, paying a flow cost $C(e_M) > 0$. Then, if the firm decides to commit a fraud, the optimal choice of effort in period *t* is given by:

$$\max_{e_M}(1-p(t,e_M))\mathscr{B}+p(t,e_M)(-\mathscr{L})-C(e_M).$$
(I.15)

Then, from the first-order condition (F.O.C.), we have

$$-\frac{\partial p(t,e_M)}{\partial e_M}(\mathscr{B}+\mathscr{L})-C'(e_M)=0, \qquad (I.16)$$

where $\frac{\partial p(t,e_M)}{\partial e_M} < 0$. From the second-order condition, we have:

$$-\frac{\partial^2 p(t, e_M)}{\partial e_M^2} (\mathscr{B} + \mathscr{L}) - C''(e_M).$$
(I.17)

So, as long as $\frac{\partial^2 p(t,e_M)}{\partial e_M^2} > 0$, the problem is strictly concave and there is a unique optimal effort $e^*(t, \mathscr{B})$ pinned down by the F.O.C..

Notice that the firm's choice of committing or continuing a fraud is now given by:

$$(1 - p(t, e^*(t, \mathcal{B})))\mathcal{B} + p(t, e^*(t, \mathcal{B}))(-\mathcal{L}) - C(e^*(t, \mathcal{B})) > 0.$$
(I.18)

where $e^*(t, \mathcal{B})$ is pinned down by the F.O.C..

Finally, from the F.O.C., we also obtain the following results:

PROPOSITION 5. Based on a manipulator's optimal effort decision $e^*(t, \mathcal{B})$, the following is true:

- ^{∂e*(t,𝔅)}/_{∂𝔅} > 0, i.e., the firms that benefit the most from incurring fraud are also the ones that put more effort into hiding it.

 ^{∂e*(t,𝔅)}/_{∂t} depends on ^{∂²p(t,e_M)}/_{∂e_M∂t}. In particular, if ^{∂²p(t,e_M)}/_{∂e_M∂t} > 0, the effect of the fraudster's efforts
 to conceal the misconduct decreases over time, so $\frac{\partial e^*(t,\mathscr{B})}{\partial t} < 0$.

Information producers' decision to monitor a firm or not I.2.4

In this section, we extend the model in order to consider the information producers' decision to monitor a company or not. Our goal is to understand how the analysts' decision to follow a company or not may impact the information revealed to market participants. Since the cost of

⁵We assume here that only ongoing frauds can be detected in order to simplify our expressions. Results are still true if we assume that stopped frauds see a significant decrease in their likelihood of detection.

following a company is higher the higher the probability a company is a manipulator, the simple fact that an analyst decides to start or to continue following a company is seen as good news by market participants. Similarly, the decision to stop following a company is perceived as bad news. In particular, let's consider that there is a cost C_{IP} for the information producer to follow a firm. Imagine that the cost of following can be high (*H*) or low (*L*). Moreover, assume that this cost depends on the firm being *M* or *NM*, i.e., $Pr(C_{IP} = L|NM) \equiv \gamma_{NM} > Pr(C_{IP} = L|M) \equiv \gamma_M$. Then, assume that the benefit of following a firm is constant \tilde{B} with $\tilde{B} - L > 0$ and $\tilde{B} - H < 0$.

In terms of timing, we consider that the information producer first observes how costly it is to follow a given company. Given that the benefit of following a company is given by $\tilde{B} - C_{IP}$, the analyst decides to follow the company if $C_{IP} = L$.

Then, let's consider how market participants adjust their beliefs about the likelihood that a given firm is a manipulator based on the analyst's decision to follow the company or not. Assume that ξ_0 is the initial probability of a firm being a manipulator. Then, imagine that there is only one possible analyst. In this case, the probability of a firm being a manipulator given that the analyst decided to follow the firm is given by:

$$\xi_1 = \frac{\Pr(L|M) \times \Pr(M)}{\Pr(L|M) \times \Pr(M) + \Pr(L|NM) \times \Pr(NM)} = \frac{\gamma_M \xi_0}{\gamma_M \xi_0 + \gamma_{NM} (1 - \xi_0)},$$
(I.19)

where ξ_1 is the probability that the firm is a manipulator given that it is followed by 1 out of 1 potential analyst. Notice that the posterior depends on the number of potential analysts that could follow the firm.

Then, consider the probability of N analysts deciding to follow the company out of \mathcal{M} potential analysts. Let's initially assume that the costs of following the firm observed by analysts are independent. Then, the probability of N out of \mathcal{M} analysts deciding to follow the firm, conditional on the firm being a manipulator, is given by a binomial probability, i.e.:

$$\Pr(N; \mathscr{M}|M) = \binom{N}{\mathscr{M}} \gamma_M^N (1 - \gamma_M)^{\mathscr{M} - N}.$$
 (I.20)

Then, the posterior probability that a firm is a manipulator, given that N analysts out of \mathcal{M} follow the firm, is:

$$\xi_{N} = \frac{\Pr(N; \mathscr{M}|M) \times \Pr(M)}{\Pr(N; \mathscr{M}|M) \times \Pr(M) + \Pr(N; \mathscr{M}|NM) \times \Pr(NM)}.$$
(I.21)

Substituting (I.20) into (I.21), we obtain:

$$\xi_{N} = \frac{\left[\binom{N}{\mathscr{M}}\gamma_{M}^{N}(1-\gamma_{M})^{\mathscr{M}-N}\right]\xi_{0}}{\left[\binom{N}{\mathscr{M}}\gamma_{M}^{N}(1-\gamma_{M})^{\mathscr{M}-N}\right]\xi_{0} + \left[\binom{N}{\mathscr{M}}\gamma_{M}^{N}(1-\gamma_{M})^{\mathscr{M}-N}\right](1-\xi_{0})}.$$
(I.22)

Figure 1 illustrates this result graphically.

However, notice that this effect only matters for duration if p is affected by the posterior, i.e., $p'(\xi) > 0$, showing that the likelihood of termination is increasing in the posterior probability of being a manipulator. Otherwise, apart from increasing the probability of immediately stopping a



Figure 1. Posterior probability as a function of no. of analysts

fraud, results should not change.

Finally, consider the case in which the costs of following a firm observed by different analysts are not independent. Then, the probability of N out of \mathcal{M} analysts deciding to follow the firm, conditional on the firm being a manipulator, is given by a correlated binomial probability. We follow Witt [2004] in order to come up with a specification for the correlated binomial model. In particular, we consider a universe of \mathcal{M} analysts that have identically distributed probabilities of following a given company with these two assumptions:

Assumption (1): Each analyst has a probability γ of following the company. Assumption (2): Each pair of analysts has correlation ρ between them.

In order to specify the joint probability distribution, this correlated binomial also relies on a third assumption.

Assumption (3): The correlation between analyst j + 1 and analyst j + 2 remains equal to ρ regardless of the number of known analysts following the firm among the other analysts.

Mathematically, assumption (3) can be written as $\gamma_{j+1} = \gamma_j + (1 - \gamma_j)\rho$, for $j = 1, ..., \mathcal{M} - 1$. Assumption (3) implies that in the correlated binomial, the default probability of analyst j + 1 following the firm conditional on j analysts following is increasing as j increases. This increasing probability given other analysts following is one aspect of the fatter tails of the correlated binomial distribution.

This simplified version of the correlated binomial has the benefit of having a closed-form distribution. In particular, for k > 0, the probability that k analysts decide to follow the firm, while $\mathcal{M} - k$ decide not to follow the company, has the probability distribution:

$$C(\mathcal{M},k)\sum_{j=0}^{\mathcal{M}-k} \left[(-1)^{j} C(\mathcal{M}-k,j) \prod_{i=1}^{j+k} \gamma_{i} \right], \qquad (I.23)$$

and the probability of no analysts following is:

$$1 + \sum_{j=1}^{j} (-1)^{j} C(\mathcal{M}, j) \prod_{i=1}^{j} \gamma_{i}.$$
(I.24)

where $C(\cdot, \cdot)$ represents the combinatorial function.

Finally, considering the posterior probability that a firm is a manipulator, given that N analysts out of \mathcal{M} follow it, we notice that the correlation reduces the informativeness of added analysts in signaling the likelihood of a firm being a manipulator. In particular, in Figure 2, we replicate the graphical example presented in Figure 1, adding the case of positive correlation. We can see that, when there is correlation among analysts, it is strictly convex, indicating that marginal increments in analyst following have ever decreasing returns when it comes to conveying information on whether a firm is likely to manipulate its financial statements or not.



Figure 2. Posterior probability as a function of no. of analysts

I.2.4.1 Dropping Analysts

Similarly, consider that, at any given period, there is a probability δ_M that an analyst decides to stop following a firm if the firm is a manipulator. For example, an increase in the financial statements' complexity can be seen as an increase in the analyst's cost of following the firm, which we refer to as a "cost shock" from now on. Assume that the probability of a cost shock in the case of a non-manipulator is given by $\delta_{NM} < \delta_M$. Then, let's consider that \tilde{N} analysts follow the firm in a given period. Initially, let's assume that cost shocks are independent and identically distributed. Then, the likelihood that n_1 out of \tilde{N} stop following the firm is given by:

$$\Pr(n_1|\tilde{M}) = \binom{n_1}{\tilde{N}} \delta_M^{n_1} (1 - \delta_M)^{\tilde{N} - n_1}.$$
 (I.25)

Then, the posterior once n_1 drop out is given by:

$$\xi_{N-n_{1}|\mathscr{M}} = \frac{\left[\binom{n_{1}}{N}\delta_{M}^{n_{1}}(1-\delta_{M})^{N-n_{1}}\right]\xi_{N|\mathscr{M}}}{\left[\binom{n_{1}}{N}\delta_{M}^{n_{1}}(1-\delta_{M})^{N-n_{1}}\right]\xi_{N|\mathscr{M}} + \left[\binom{n_{1}}{N}\delta_{NM}^{n_{1}}(1-\delta_{NM})^{N-n_{1}}\right](1-\xi_{N|\mathscr{M}})}.$$
 (I.26)

As we can see in Figure 3, the posterior probability of being a manipulator increases as the number of dropped analysts goes up. Moreover, the posterior also depends on the number of analysts that initially decided to follow the company. Hence, the impact of a analyst dropping coverage is different if the firm was initially followed by 15 or 5 analysts, for instance.



Figure 3. Posterior prob. as a function of no of dropped analysts

Consequently, if again we assume that $p'(\xi_{N-n_1|\mathcal{M}}) > 0$, we should expect that the likelihood of termination goes up as the number of analysts following the firm goes down. The result with correlated signals should be similar to the correlated binomial discussion we presented in Section I.2.4. Finally, if we consider that the arrival rates are the same, i.e. $\gamma_M = \delta_M$ and $\gamma_{NM} = \delta_{NM}$, we can combine the different binomial distributions and jointly describe the analysts' decisions about starting and stopping to follow a given firm.

I.3 Proofs

Proof of Lemma 1: If $\mathcal{H}_t \in \mathcal{H}_t(B)$, we have that $Pr(M|\mathcal{H}_t) = 1$. But then, it is not optimal to wait to intervene in the company, since $\delta < 1$ and $\mathcal{H}_{t+1} \in \mathcal{H}_t(B)$.

Proof of Proposition 1: If $\xi P < \mathscr{C}$, we have that at $\mathscr{H}_{\emptyset} = \emptyset$ it's optimal to wait for a signal instead of immediately intervening in the firm. But then at t = 1, if monitors observe a bad signal, as seen in *Corollary 1*, they should intervene in the firm, since $\Pr(M|\mathscr{H}_1) = 1$. On the other hand, if $s_1 = G$, then $\Pr(M|\mathscr{H}_1) = \frac{(1-p)\xi}{(1-\xi)+(1-p)\xi} < \xi$. More generally, we have that, $\forall \mathscr{H}_t \notin \mathscr{H}_t(B), \Pr(M|\mathscr{H}_t) = \frac{(1-p)^t\xi}{(1-\xi)+(1-p)^t\xi} < \xi$. Therefore, $\Pr(M|\mathscr{H}_t)P - \mathscr{C} < 0$, $\forall \mathscr{H}_t \notin \mathscr{H}_t(B)$. Since $\delta E_t[V(\mathscr{H}_{t+1}) \ge 0$, it is not optimal to intervene until a bad signal is observed.

Proof of Proposition 2:

$$E[N] = \sum_{n=1}^{\infty} np(1-p)^{n-1} = p \sum_{n=1}^{\infty} \frac{d}{d(1-p)} (1-p)^n$$

= $p \frac{d}{d(1-p)} \sum_{n=1}^{\infty} (1-p)^n = p \frac{d}{d(1-p)} \left[\frac{1-p}{1-(1-p)} \right] = \frac{1}{p}.$ (I.27)

Proof of Proposition 3: Consider that the current number of information providers is **I**. Then, the probability of a bad signal for a manipulator is

$$\Pr(B|M) = 1 - \prod_{i=1}^{I} (1 - p_i).$$
(I.28)

Now let's introduce an additional information provider; then the probability of a bad signal becomes:

$$\Pr(B|M) = 1 - \prod_{i=1}^{I+1} (1 - p_i).$$
(I.29)

Therefore, the likelihood of a bad signal increases by:

$$1 - (1 - p_{\mathbf{I}+1}) = p_{\mathbf{I}+1}.$$
 (I.30)

Therefore, the better the new information producer, the higher the likelihood of a bad signal for a manipulator.

Similarly, the new expected duration of a fraud is given by

$$E[N] = \frac{1}{1 - \prod_{i \in \mathscr{I}+1} (1 - p_i)}.$$
(I.31)

While the expected length of a fraud has been reduced by

$$\frac{1}{1-\prod_{i\in\mathscr{I}+1}(1-p_i)} - \frac{1}{1-\prod_{i\in\mathscr{I}}(1-p_i)} = \\
= \frac{[1-\prod_{i\in\mathscr{I}}(1-p_i)] - [1-\prod_{i\in\mathscr{I}+1}(1-p_i)]}{[1-\prod_{i\in\mathscr{I}+1}(1-p_i)] \times [1-\prod_{i\in\mathscr{I}}(1-p_i)]} \\
= \frac{-p_{\mathbf{I}+1}\prod_{i\in\mathscr{I}}(1-p_i)}{[1-\prod_{i\in\mathscr{I}+1}(1-p_i)] \times [1-\prod_{i\in\mathscr{I}}(1-p_i)]}.$$
(I.32)

As before, the better the new information provider is at spotting a fraud, the shorter the expected length of the fraud. $\hfill \Box$

Proof of Proposition 4: We initially present the proofs for items 1 and 3.

Proof of 1. and 3.:

The optimal decision about starting/continuing a fraud at period $t \in \{1, 2, ...\}$ is given by:

$$\Pi(\mathscr{B},t) = \max\{0 + \delta\Pi(\mathscr{B},t), (1-p(t))[\mathscr{B} + \delta\Pi(\mathscr{B},t+1)] + p(t)(-L)\}.$$
 (I.33)

If $0 + \delta \Pi(\mathcal{B}, t) > (1 - p(t))[\mathcal{B} + \delta \Pi(\mathcal{B}, t+1)] + p(t)(-L)$, then we have that:

$$\Pi(\mathscr{B},t) = 0 + \delta \Pi(\mathscr{B},t). \tag{I.34}$$

Rearranging it, we have:

$$\Pi(\mathscr{B},t) = \frac{0}{1-\delta} = 0. \tag{I.35}$$

Therefore, $\Pi(\mathscr{B}, t) > 0$ implies that the fraud has started or has continued. Consequently:

$$(1 - p(t))[\mathscr{B} + \delta \Pi(\mathscr{B}, t+1)] + p(t)(-L) > 0.$$
 (I.36)

Rearranging it, we have:

$$(1-p(t))\mathscr{B}+p(t)(-L) > -\delta\Pi(\mathscr{B},t+1).$$
(I.37)

By definition $\Pi(\mathscr{B}, t+1) \ge 0$. If $\Pi(\mathscr{B}, t+1) = 0$, the above expression becomes $(1 - p(t))\mathscr{B} + p(t)(-L) > 0$, which concludes the proof. On the other hand, imagine that $(1 - p(t))\mathscr{B} + p(t)(-L) < 0$ but $(1 - p(t))\mathscr{B} + p(t)(-L) > -\delta\Pi(\mathscr{B}, t+1)$. Notice that $\Pi(\mathscr{B}, t+1)$ is given by

$$\Pi(\mathscr{B},t+1) = \left\{ \begin{array}{c} (1-p(t+1))\mathscr{B} + p(t+1)(-L) + \\ +\sum_{j=1}^{T-t-1} [(1-p(t+1+j))\mathscr{B} + p(t+1+j)(-L)] \delta^j \prod_{i=0}^{j-1} (1-p(t+1+i)) \end{array} \right\}.$$
(I.38)

where *T* is the optimal time to stop the fraud (if there is no optimal time to stop the fraud, then we can take $T \rightarrow \infty$ without changing the argument).

Since p(.) is strictly increasing in its argument, we would have that $\Pi(\mathcal{B}, t+1) < 0$, since all its arguments would be negative. As a result, we have a contradiction.

Once we have this result, it is easy to see that, as t increases, $(1 - p(t))\mathscr{B} + p(t)(-L)$ decreases and eventually crosses the zero threshold.

Proof of 2.:

Now we have $p(t) \equiv p$. In this case the problem becomes stationary. Then $\Pi(\mathscr{B}, t) \equiv \Pi(\mathscr{B})$

$$\Pi(\mathscr{B}) = \max\{0 + \delta\Pi(\mathscr{B}), (1 - p)[\mathscr{B} + \delta\Pi(\mathscr{B})] + p(-L)\}.$$
(I.39)

in which we assume that if the fraud is discontinued, the firm still has the right to continue with the fraud next period, but the duration of the fraud is considered frozen at period t. As we will see, our result is independent of this particular assumption.

So, if the first term in the max operator is the highest, we can easily see that $\Pi(\mathscr{B}) = 0$. Similarly, if starting the fraud is optimal, we have that $\Pi(\mathscr{B}) = \frac{(1-p)\mathscr{B}+p(-L)}{1-\delta}$ which is positive if $1-p)\mathscr{B}+p(-L) > 0$. But once the problem is stationary, the value of continuing the fraud the next period is still the same, so it will be optimal to continue the fraud. So the fraud will continue until the firm is caught.

Proof of Proposition 5: Both items are proved by applying the implicit function theorem to the

F.O.C.. For item 1., we have:

$$\frac{\partial e^*(t,\mathscr{B})}{\partial \mathscr{B}} = \frac{-\frac{\partial p(t,e_M)}{\partial e_M}}{\frac{\partial^2 p(t,e_M)}{\partial e_M^2} + C''(e_M)} > 0.$$
(I.40)

While, for item 2, applying IFT we have:

$$\frac{\partial e^*(t,\mathscr{B})}{\partial t} = \frac{-\frac{\partial^2 p(t,e_M)}{\partial e_M \partial t}(\mathscr{B} + L)}{\frac{\partial^2 p(t,e_M)}{\partial e_M^2}(\mathscr{B} + L) + C''(e_M)}.$$
(I.41)

Therefore, the sign of $\frac{\partial e^*(t,\mathscr{B})}{\partial t}$ depends on $\frac{\partial^2 p(t,e_M)}{\partial e_M \partial t}$, i.e., if $\frac{\partial^2 p(t,e_M)}{\partial e_M \partial t} > 0$ we must have $\frac{\partial e^*(t,\mathscr{B})}{\partial t} < 0$. 0. Similarly, if $\frac{\partial^2 p(t,e_M)}{\partial e_M \partial t} < 0$ we must have $\frac{\partial e^*(t,\mathscr{B})}{\partial t} > 0$.

I.4 Example of Time-Varying Hazard Function

Assume that the probability of a bad signal for a manipulator that has an ongoing fraud for *t* periods is given by:

$$p(t) = 1 - \frac{\alpha}{t}.\tag{I.42}$$

Naturally

$$\frac{\partial p(t)}{\partial \alpha} = -\frac{1}{t} < 0 \quad \text{and} \quad \frac{\partial^2 p(t)}{\partial \alpha \partial t} = \frac{1}{t^2} > 0.$$
 (I.43)

The figure below presents a couple of examples for p(t) as we vary α



Figure 4. Probability of a bad signal for a manipulator

Notice also that $(1 - p(t)) = \frac{\alpha}{t}$. In this case, the expected duration of the fraud is given by

$$E[N] = \sum_{t=1}^{\infty} t\left(1 - \frac{\alpha}{t}\right) \prod_{t'=1}^{t-1} \frac{\alpha}{t'}.$$
(I.44)

Rearranging it, we have:

$$E[N] = \sum_{t=0}^{\infty} (t+1) \frac{\alpha^{t}}{t!} - \alpha \sum_{t=0}^{\infty} \frac{\alpha^{t}}{t!}.$$
 (I.45)

Solving it, we obtain:

$$E[N] = (1+\alpha)e^{\alpha} - \alpha e^{\alpha} = e^{\alpha}.$$
 (I.46)

Therefore, the higher α , the longer the duration of the fraud.

Moreover, even though we imagine that the probability of being detected has an upward trend, the actual probability may vary around the trend. In particular, we may expect that market and firm time-varying characteristics may affect the probability of detection, pushing it above or below the long-term trend. For example, good or bad performance in the stock market may increase or decrease incentives to scrutinize, making it easier or harder for information producers to detect signs of manipulation. A similar argument can be made about the firm's own operational and stock market performance. Going back to the example presented above, we would have that the graph for p(t) over time would look more like the one in the figure below:



Figure 5. Evolution of p(t): Trend vs. actual

I.5 Graphs – Marginal Effects



Figure 6A. Firm size and end of fraud hazards. The figure shows the estimated hazards of end of fraud as a function of quarters elapsed since the start of the misconduct. The hazards are estimated at the 25th percentile and 75th percentile sample values of book value of total assets, holding all other variables constant at their median sample values. The hazard estimates are based on column 2 of Table 4.



Figure 6C. Profitability and end of fraud hazards. The figure shows the estimated hazards of end of fraud as a function of quarters elapsed since the start of the fraud. The hazards are estimated at the 25th percentile and 75th percentile sample values for return on equity (RoE), holding all other variables constant at their median sample values. The hazard estimates are based on column 2 of Table 4.



Figure 6B. Leverage and end of fraud hazards. The figure shows the estimated hazards of end of fraud as a function of quarters elapsed since the start of the fraud. The hazards are estimated at the 25th percentile and 75th percentile sample values for leverage, holding all other variables constant at their median sample values. The hazard estimates are based on column 2 of Table 4.



Figure 6D. Large earnings miss and end of fraud hazards. The figure shows the estimated hazards of end of fraud as a function of quarters elapsed since the start of the fraud. The hazards are estimated based on the firm having missed earnings forecast by more than 5 percent or not, holding all other variables constant at their median sample values. The hazard estimates are based on column 2 of Table 4.



Figure 7A. Fourth fiscal quarter and end of misconduct hazards. The figure shows the estimated hazards of end of fraud as a function of quarters elapsed since the start of the fraud. The hazards are estimated based on the quarter not being the 4th fiscal quarter, being the 4th fiscal quarter without an audit explanation, and being the 4th fiscal quarter with an audit explanation. The hazard estimates are based on column 3 of Table 5.



Figure 7B. Auditor Switch and end of misconduct hazards. The figure shows the estimated hazards of end of fraud as a function of quarters elapsed since the start of the fraud. The hazards are estimated based on the quarter being the 4th fiscal quarter, with and without a new auditor evaluating the fiscal year-end financial statements. The hazard estimates are based on column 4 of Table 5.



Figure 8A. First fiscal quarter fraud start and end of fraud hazards. The started their fraud in the first fiscal quarter and firms that started their fraud any other fiscal quarter, holding all other variables constant at their median figure shows the estimated hazards of end of fraud as a function of quarters elapsed since start of the fraud. The hazards are estimated for firms that sample values. The hazard estimates are based on column 3 of Table 9.



start of the fraud. The hazards are estimated at the 25th percentile and 75th percentile sample values of total accruals, holding all other variables constant at their median sample values. The hazard estimates are based on column 4 of Table 6. Figure 8C. Total accruals and end of fraud hazards. The figure shows the estimated hazards of end of fraud as a function of quarters elapsed since the



ards. The figure shows the estimated hazards of end of fraud as a function of quarters elapsed since the start of the fraud. The hazards are estimated at the 25th percentile and 75th percentile sample values of affected accounting at their median sample values. The hazard estimates are based on column 3 Figure 8B. Number of affected accounting areas and end of fraud hazareas (one and three areas, respectively), holding all other variables constant of Table 9.

I.6 Market Reaction to Audit Explanation

In this section, we present evidence on how the market reacts to audit explanations. An explanatory paragraph is added to an unqualified audit opinion to provide additional information to investors. According to Auditing Standard AU 508, explanatory language describes four categories of information: (1) inconsistencies with prior reports such as newly adopted accounting principles, or changes in accounting method; (2) emphasis of matters in financial reports including significant transactions; (3) audit-related information such as how the audit was conducted, the auditor's responsibility, and scope limitations; and (4) supplemental information, including information about financial distress and the assumption that the firm will remain a going concern. The presence of an explanatory paragraph is not intended to imply that the financial reports are unfairly stated. However, Czerney et al. [2014] find that explanatory paragraph are more likely to appear in subsequently restated financial statements and Beasley et al. [2010] find that explanatory language is more likely to be found in fraudulent firm audit reports.

In Table I.7 below, we shed light on the market reaction to audit explanation. If explanations are innocuous, then we shouldn't observe a market reaction. However, investors could react negatively if explanatory language conveys bad news about the financial reporting process, which could lead to scrutiny by investors. Alternatively, explanatory language could be issued in response to concerns already present in the market to comfort investors. In this case, an audit explanation should cause a positive market response. The results presented below are consistent with the former reasoning: explanatory language in unqualified audit reports is associated with a negative market response, suggesting that investors consider the additional language bad news.

Market Reaction to	o Audit Expl	anation
	(1) CAR [-1,+1]	(2) CAR [-2,+2]
Unexpected Earnings	0.001*** (0.000)	0.001*** (0.000)
Audit Explanation	-0.154** (0.074)	-0.168* (0.092)
Size	0.027 (0.017)	0.005 (0.020)
Book-to-Market	0.062 (0.070)	0.190** (0.088)
Leverage	0.151 (0.152)	0.478^{***} (0.184)
Industry Dummies	YES	YES
Year Dummies	YES	YES
Ν	54,957	54,943

TABLE I.7

The table reports results from estimating the market response around 10K filing dates available on EDGAR (1994-2019). The dependent variable in the first column is the 3-day cumulative abnormal return (CAR) centered around the 10K filing date (-1,+1). The dependent variable in the second column is the 5-day CAR centered around the 10K filing date (-2,+2). The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. * p < 0.1; ** p < 0.05; *** p < 0.01

I.7 Additional Analyst Results

I.7.1 Instrumental variables

In this section, we replicate the results from Table 7, but applying an instrumental variable framework suggested by Yu [2008]. In particular, we construct estimated coverage variables that take into account the expected changes in analyst coverage due to changes in the size of brokerage houses.⁶ According to Yu [2008], when a brokerage house reduces its size, it employs fewer analysts and tends to drop some of its existing coverage to reduce total workload. Following Yu [2008], we treat this change of coverage driven by the change in size of brokerage houses as an exogenous variation. To capture this variation, Yu [2008] constructs an "expected coverage" variable, using the following equations:

Expected Coverage_{it j} =
$$\left(\frac{\text{Brokerage Size}_{jt}}{\text{Brokerage Size}_{j0}}\right) \times Coverage_{i0}$$
 (I.47)

and

Expected Coverage_{it} =
$$\sum_{j=1}^{n} Expected Coverage_{itj}$$
. (I.48)

*Expected Coverage*_{*it j*} is the expected coverage of firm *i* from broker *j* in year *t*. Brokerage Size_{*jt*} and Brokerage Size_{*j*0} are the number of analysts employed by broker *j* in year *t* and year 0, respectively. *Coverage*_{*i*0} is the size of the coverage of firm *i* in year 0. *Expected Coverage*_{*it*} is the expected coverage of firm *i* in year *t*. Notice that this measure constrains to one the maximum number of analysts that a broker sends to cover a firm. As a result, the expected coverage from a certain broker can be interpreted as the probability this broker will continue to cover a firm after its size changes. Since we focus on the expected coverage, we avoid the selection problem that would arise in the realized coverage.

Using the data from I/B/E/S, we calculate *Expected Coverage_{it}* setting the year immediately before the start of the fraud as the base year. To compute the expected coverage for any given firm, we need the firm to be covered by at least one brokerage house in the benchmark year. Consequently, an important limitation of this method is that expected coverage can capture only the reduction in coverage but not the initiation of coverage. On the bright side, Table 8 indicates that results for analyst drops are stronger than the counterparts for analyst addition. Still, due to this limitation, we focus on Table 7 for our current exercise. In particular, we estimate a 2-stage least squares (2SLS) model in which in the first stage we run the following specification:

$$Coverage_{it} = c_t + \phi Expected \ Coverage_{it} + \theta Controls_{it} + \varepsilon_{it}, \tag{I.49}$$

where *Controls_{it}* includes all controls used in Table 4's column 2, 4^{th} *Quarter*, 4^{th} *Quarter* × *Audit Explanation, Auditor Switch 3*, as well as industry and time fixed effects. We cluster the standard

⁶ Yu [2008] uses residual coverage, in which the residuals are derived from a regression of the number of covering analysts on firm size, past performance, growth, external financing activities, and cash flow volatility. The reason for using the residual is that some of these variables are not included in the second stage. Since we do not have the same issue, we use the traditional approach for 2SLS. In particular, we use the actual coverage as dependent variable in the first stage and the estimated value in the second stage.

errors by firm. Finally, $Coverage_{it}$ shifts with the model, being either a dummy variable (*Coverage Indicator*) or the number of analysts following the firm that we use to construct the estimated coverage quintiles that we apply in the second stage. Results for the second stage are presented in Table I.8. As we can see, they are qualitatively equivalent to the ones in Table 7, with only the coefficient for the first quintile being statistically significant. This result reinforces the idea that having analyst coverage is likely to reduce fraud duration, but additional coverage is unlikely to be beneficial.

TABLE I.8	
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Role of Analyst	s – IV Appro	pach	
	(1)	(2)	(3)
	End of Fraud	End of Fraud	End of Fraud
Estimated Coverage Indicator	-0.226 (0.515)		
Estimated Coverage 1 st Quintile		0.580** (0.267)	0.511* (0.271)
Estimated Coverage 2 nd Quintile		0.200 (0.291)	0.145 (0.293)
Estimated Coverage 3 rd Quintile		-0.254 (0.336)	-0.323 (0.338)
Estimated Coverage 4 th Quintile		-0.571 (0.426)	-0.637 (0.424)
Estimated Coverage 5 th Quintile		-0.722	-0.828 (0.526)
Mean Forecast Error			3.908 (2.528)
4 th Quarter	0.396*	0.367* (0.204)	0.346*
4^{th} Quarter \times Audit Explanation	0.961*** (0.261)	0.987*** (0.264)	1.030*** (0.266)
Auditor Switch 3	2.069***	2.209***	2.226*** (0.589)
Large Earnings Miss Dummy	0.890*** (0.220)	0.716*** (0.201)	0.651*** (0.205)
log(Period)	0.202**	0.280***	0.293***
	(0.102) NEC	(0.108) NEC	(0.108) NEC
Control variables	IES VEC	IES VEG	IES
Industry Dummies	YES	YES	YES
Time Period Dummies	YES	YES	YES
No. of Misconducts	179	179	179
Ν	1,370	1,370	1,370

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. * p < 0.1; ** p < 0.05; *** p < 0.01

I.8 Robustness Tests

I.8.1 Robustness test – Affected areas

In this section, we evaluate the impact of different affected areas on the fraud termination hazard rate. Towards that goal, we focus on the specification presented in Table 9's column 1, adding dummies for the affected areas. Results are presented in Table I.9. As we can see in column 1, indicators for balance-sheet-related indicators as well as gross-profit-related indicators are not statistically significant. Consequently, once controls in our main analysis are factored out, frauds that affect the balance sheet are not significantly longer or shorter than frauds affecting the income statement. Similarly, column 2 presents the dummies for affected areas following the definitions presented in Dechow et al. [2011]. Notice that, apart from *Other*, which represents misstatements that could not be classified in an income, expense, or equity account, and marginally *Reserves*, no other area dummy coefficient is statistically different from zero. Hence, apart from the case of frauds that affect an area outside the key financial areas, the affected area does not seem to significantly impact the fraud duration. Keep in mind that areas are not mutually exclusive, so frauds can affect multiple areas simultaneously, as we control with the variable *log(Number of Fraud Areas)*.

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тприст ој Ајјестей Атеиз						
	(1) End of Fraud	(2) End of Fraud				
Balance-Sheet-Related Indicator	-0.171 (0.221)					
Gross-Profit-Related Indicator	0.217 (0.215)					
Misstatement: Revenues		-0.227 (0.244)				
Misstatement: Receivables		0.256 (0.212)				
Misstatement: COGS		-0.189 (0.317)				
Misstatement: Inventory		0.134 (0.291)				
Misstatement: Reserves		-0.630* (0.349)				
Misstatement: Debt		-0.567 (0.600)				
Misstatement: Mkt Securities		0.769 (1.215)				
Misstatement: Pay		-0.356 (0.463)				
Misstatement: Assets		0.049 (0.283)				
Misstatement: Liabilities		-0.043 (0.319)				
Misstatement: Other		-0.734*** (0.232)				
1 st Quarter Start	-1.369*** (0.199)	-1.424*** (0.217)				
4 th Quarter	0.253	0.222				
4^{th} Quarter × Audit Explanation	(0.267) 1.048*** (0.269)	(0.207) 1.079*** (0.273)				
Auditor Switch 3	(0.20) 1.991*** (0.654)	(0.275) 1.867*** (0.661)				
Analysts 1 st Quintile	0.729***	0.689**				
log(Period)	0.664***	0.771***				
Other Analysts Quintiles	YES	YES				
Control Variables	YES	YES				
Industry Dummies	YES	YES				
Time Period Dummies	YES	YES				
	1 270	1 270				
IN	1,370	1,370				

Impact of Affected Areas

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. * p < 0.1; ** p < 0.05; *** p < 0.01

I.8.2 Robustness test – Add and drop Analysts

TABLE I.10

Robustness Check – Timing of Information Revelation				
	(1)	(2)	(3)	
	End of	End of	End of	
	Fraud	Fraud	Fraud	
	All	Reveal Pre	Reveal Post	
4 th Quarter	0.147	0.613	-0.074	
	(0.355)	(0.680)	(0.493)	
4^{th} Quarter \times Audit Explanation	1.509***	1.906**	1.576**	
	(0.458)	(0.800)	(0.676)	
Auditor Switch 2	2.143***	3.459***	2.632***	
	(0.547)	(1.171)	(0.773)	
Analyst Indicator	0.445	1.234	0.233	
	(0.520)	(1.005)	(0.746)	
log(1+No. of Dropped Estimates)	0.684***	0.558	0.289	
	(0.257)	(0.454)	(0.354)	
log(1+No. Added Estimates)	0.190	0.036	-0.097	
	(0.252)	(0.470)	(0.353)	
Large Earnings Miss Dummy	0.316	-0.612	0.904**	
	(0.345)	(0.609)	(0.388)	
Amendment	1.265***	1.332**	1.525***	
	(0.272)	(0.576)	(0.370)	
Class Lawsuit	1.626***	3.332***	2.464*	
	(0.451)	(0.740)	(1.312)	
log(Period)	0.517	0.202	0.668**	
	(0.315)	(0.346)	(0.291)	
Control Variables	YES	YES	YES	
Industry Dummies	YES	YES	YES	
Time Period Dummies	YES	YES	YES	
No. of Misconducts	87	37	50	
Ν	765	387	378	

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. * p < 0.1; ** p < 0.05; *** p < 0.01

I.8.3 Robustness test – Managerial effort

	(1)	(2)	(3)	(4)
	End of	End of	End of	End of
	Fraud	Fraud	Fraud	Fraud
1 st Quarter Start	-1.562*** (0.357)		-1.556*** (0.369)	-1.786*** (0.663)
log(Number of Areas)		-0.737* (0.379)	-0.586** (0.258)	-0.904* (0.510)
Total Accruals				-2.011** (0.952)
4 th Quarter	0.066	0.164	0.085	0.414
	(0.352)	(0.351)	(0.351)	(0.409)
4^{th} Quarter \times Audit Explanation	1.367***	1.320***	1.406***	1.289***
	(0.437)	(0.436)	(0.436)	(0.496)
Auditor Switch 3	2.291***	2.704***	2.360***	2.887**
	(0.722)	(0.920)	(0.712)	(1.233)
Analysts 1 st Quintile	1.472***	1.482***	1.530***	1.929**
	(0.505)	(0.571)	(0.518)	(0.766)
Analysts 2 nd Quintile	0.643	0.677	0.621	0.558
	(0.541)	(0.625)	(0.549)	(0.761)
Analysts 3 rd Quintile	0.917	0.950	0.705	0.850
	(0.591)	(0.667)	(0.603)	(0.827)
Analysts 4 th Quintile	0.519	0.274	0.383	0.343
	(0.616)	(0.690)	(0.612)	(0.864)
Analysts 5 th Quintile	0.871	0.334	0.439	0.819
	(0.670)	(0.851)	(0.704)	(0.988)
Large Earnings Miss Dummy	0.309	0.344	0.244	0.317
	(0.307)	(0.330)	(0.308)	(0.388)
Amendment	1.172***	1.188***	1.138***	1.282***
	(0.254)	(0.260)	(0.254)	(0.312)
Class Lawsuit	1.483***	1.532***	1.478***	1.837***
	(0.330)	(0.418)	(0.337)	(0.533)
log(Period)	0.748***	0.657*	0.913***	1.464**
	(0.210)	(0.386)	(0.232)	(0.631)
Control Variables	YES	YES	YES	YES
Industry Dummies	YES	YES	YES	YES
Time Period Dummies	YES	YES	YES	YES
No. of AAERs	87	87	87	86
Ν	765	765	765	749

TABLE I.11

The Role of Managerial Effort

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses.

* p < 0.1; ** p < 0.05; *** p < 0.01

TA	BL	Æ	L	.1	2
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	(1)	(2)	(3)
	End of	End of	End of
	Fraud	Fraud	Fraud
	All	Reveal Pre	Reveal Post
1 st Quarter Start	-1.551***	-3.251**	-2.813***
	(0.600)	(1.264)	(0.763)
log(Number of Areas)	-0.681	-0.256	-1.343**
	(0.441)	(0.725)	(0.576)
Total Accruals	-2.049**	-1.804	-3.416**
	(0.933)	(1.936)	(1.384)
4 th Quarter	0.217	0.465	0.450
	(0.397)	(0.830)	(0.568)
4^{th} Quarter $ imes$ Audit Explanation	1.533***	3.205**	0.625
	(0.501)	(1.294)	(0.789)
Auditor Switch 2	1.902***	3.900*	2.230**
	(0.627)	(2.030)	(0.972)
Analysts 1 st Quintile	2.276***	2.517*	2.350
	(0.770)	(1.512)	(1.753)
Analysts 2 nd Quintile	0.761	-0.516	1.093
	(0.758)	(1.484)	(1.965)
Analysts 3 rd Quintile	1.126	0.121	0.447
	(0.804)	(1.508)	(2.105)
Analysts 4 th Quintile	0.591	0.514	-2.231
	(0.820)	(1.297)	(2.522)
Analysts 5 th Quintile	0.892	-0.016	-2.316
	(0.935)	(1.238)	(2.606)
Large Earnings Miss Dummy	0.152	-0.945	1.222**
	(0.403)	(0.848)	(0.495)
Amendment	1.255***	1.667**	1.578^{***}
	(0.305)	(0.733)	(0.429)
Class Lawsuit	1.942***	4.972***	0.855
	(0.526)	(1.269)	(1.363)
log(Period)	1.231**	1.465*	1.823***
	(0.513)	(0.755)	(0.447)
Control Variables	YES	YES	YES
Industry Dummies	YES	YES	YES
Time Period Dummies	YES	YES	YES
No. of AAERs	86	36	49
Ν	749	379	369

The Role of Managerial Effort - Split

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using a sample of SEC AAERs over the 1982 to 2010 period. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses. * p < 0.1; ** p < 0.05; *** p < 0.01

I.8.4 Robustness test - Outliers

We conduct three robustness tests to confirm that outliers are not driving our results. First, we examine whether there is anything intrinsically different about short frauds that may bias our results by limiting our sample to frauds that last at least three quarters. Second, we check whether our main results are driven by very small or very large firms. To do this, we consider a sample trimmed at the 10^{th} and 90^{th} size percentiles, calculated based on the log(TotalAssets) at the last quarter before the fraud starts. Finally, we investigate how sensitive our results are to the fraudulent firm's size by dividing our total sample into two subsamples. One subsample (small firms) is composed of firms that are below the median of log(TotalAssets) at the last quarter before the fraud starts, and the other subsample (large firms) with firms above the median. Results are depicted in Table I.13 following a specification similar to the one presented in column 3 of Table 9. The only needed change was to use Auditor Switch 2 instead of Auditor Switch 3, since the latter was dropped from some specifications.

Column 1 of Table I.13 shows that restricting our sample to frauds that last three quarters or more does not qualitatively change our results compared to the ones presented for the unrestricted sample in Table 9. Results for the trimmed sample, presented in column 2, are also quite similar to the ones presented in Table 9, although Auditor Switch 2 and Analyst 1st. quintile lose statistical significance while keeping the correct signal. There are just a few distinctions. Finally, the results for the subsamples of large and small firms, presented in columns 3 and 4 of Table I.13, respectively, are also fairly consistent with the ones obtained for the overall sample, although statistical significance goes down for many variables, as the sample size decreases significantly.

In summary, the results discussed here show that our findings are robust to removing very short frauds and to differences in initial firm size.

	(1)	(2)	(3)	(4)		
	End of	End of	End of	End of		
	Fraud	Fraud	Fraud	Fraud		
1 st Quarter Start	-1.573***	-1.180***	-1.509***	-1.268***		
	(0.271)	(0.221)	(0.309)	(0.313)		
log(Number of Areas)	-0.411*	-0.283	-0.200	-0.522*		
	(0.218)	(0.185)	(0.248)	(0.271)		
4 th Quarter	0.187	0.163	0.293	0.214		
	(0.261)	(0.228)	(0.281)	(0.324)		
4^{th} Quarter \times Audit Explanation	1.227***	1.117***	0.806*	1.163***		
	(0.317)	(0.314)	(0.439)	(0.383)		
Auditor Switch 2	0.856**	0.053	1.131***	-0.831		
	(0.364)	(0.324)	(0.397)	(0.598)		
Analysts 1 st Quintile	0.929***	0.496	0.097	-0.442		
	(0.342)	(0.322)	(0.403)	(0.631)		
Analysts 2 nd Quintile	0.165 (0.357)	0.321 (0.349)		-0.338 (0.617)		
Analysts 3 rd Quintile	-0.369	0.024	0.236	-0.451		
	(0.439)	(0.398)	(0.503)	(0.618)		
Analysts 4 th Quintile	-0.813	-0.033	0.066	0.022		
	(0.497)	(0.434)	(0.496)	(0.608)		
Analysts 5 th Quintile	-0.909	-0.103	-0.070	0.459		
	(0.569)	(0.575)	(0.548)	(0.669)		
Large Earnings Miss Dummy	0.305	0.659***	0.964***	0.376		
	(0.251)	(0.226)	(0.344)	(0.283)		
log(Period)	1.980***	0.600***	0.784^{***}	0.860***		
	(0.237)	(0.143)	(0.197)	(0.206)		
Control Variables	YES	YES	YES	YES		
Industry Dummies	YES	YES	YES	YES		
Time Period Dummies	YES	YES	YES	YES		
No. of AAERs	128	144	90	88		
Ν	1,280	1,046	558	811		

 TABLE I.13

 Robustness Tests - Trimmed Samples

The table reports the results of implementing a random effects panel complementary log-log regression of the quarterly fraud termination hazard rate using different subsamples based on our sample of SEC AAERs over the 1982 to 2010 period. Column 1 restricts the sample to the subsample of frauds that last longer than 3 quarters. Column 2 restricts the sample to a trimmed subsample in which we eliminate both firms at the 1^{st} and 9^{th} deciles in terms of log(TotalAssets) at the fraud's onset. Finally, columns 3 and 4 restrict the sample to the subsamples of fraudulent firms below and above the median log(Total Assets) at the fraud's onset, respectively. The full set of variables used in column 2 of Table 4 is included but not reported. Full estimation results are available from the authors upon request. The definitions of all variables are presented in Appendix A. Standard errors are reported in parentheses.

* p < 0.1; ** p < 0.05; *** p < 0.01