Appendix A. The Faustmann rule.

As described in the main text we have the following maximization problem:

$$\underset{T_i}{Max}[P_i(T_i)e^{-rT_i} + L_i(T_i)e^{-rT_i}] \qquad \text{for } i = 1, 2 \qquad \text{(A.1)}$$

Now we get the following first-order condition:

$$MCTP_i(T_i) + MFTP_i(T_i) = 0 \qquad \text{for } i = 1, 2 \qquad \text{(A.2)}$$

From the definitions from in the main text we obtain:

$$MCTP_i(T_i) = (P_i'(T_i) - rP_i(T_i))e^{-rT_i} \qquad \text{for } i = 1, 2 \qquad \text{(A.3)}$$

$$MFTP_i(T_i) = (L_i'(T_i) - rL_i(T_i))e^{-rT_i} \qquad \text{for } i = 1, 2 \qquad \text{(A.4)}$$

By inserting (A.3) and (A.4) in (A.2) we get that:

$$\begin{aligned}&(P_i'(T_i) - rP_i(T_i))e^{-rT_i} + \\ &(L_i'(T_i) - rL_i(T_i))e^{-rT_i} = 0\end{aligned} \qquad \text{for } i = 1, 2 \qquad \text{(A.5)}$$

Reducing e^{-rT_i} imply that (A.5) can be written as:

$$P_i'(T_i) - rP_i(T_i) + L_i'(T_i) - rL_i(T_i) = 0 \qquad \text{for } i = 1, 2 \qquad \text{(A.6)}$$

From the definition of $L_i(T_i)$ in the main text we have that:

$$L_i(T_i) = \frac{P_i(T_i)}{(e^{rT_i} - 1)} \qquad \text{for } i = 1, 2 \qquad \text{(A.7)}$$

By differentiating (A.7) with respect to T_i we obtain that:

$$L_i'(T_i) = \frac{P_i'(T_i)(e^{rT_i} - 1) - re^{rT_i}P_i(T_i)}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(A.8)}$$

(A.8) can be written as:

$$L_i'(T_i) = \frac{P_i'(T_i)}{(e^{rT_i} - 1)} - \frac{re^{rT_i} P_i(T_i)}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(A.9)}$$

By inserting (A.7) and (A.9) into (A.6) we obtain that:

$$P_i'(T_i) - rP_i(T_i) + \frac{P_i'(T_i)}{(e^{rT_i} - 1)} - \frac{re^{rT_i} P_i(T_i)}{(e^{rT_i} - 1)^2} - \frac{rP_i(T_i)}{(e^{rT_i} - 1)} = 0 \qquad \text{for } i = 1, 2 \qquad \text{(A.10)}$$

Reorganizing (A.10) implies that:

$$P_i'(T_i)[1 + \frac{1}{(e^{rT_i} - 1)}] = rP_i(T_i)[1 + \frac{e^{rT_i}}{(e^{rT_i} - 1)^2} + \frac{1}{(e^{rT_i} - 1)}] \qquad \text{for } i = 1, 2 \qquad \text{(A.11)}$$

By rewriting (A.11) we obtain:

$$P_i'(T_i)[\frac{(e^{rT_i} - 1) + 1}{(e^{rT_i} - 1)}] = rP_i(T_i)[\frac{(e^{rT_i} - 1)^2 + e^{rT_i} + (e^{rT_i} - 1)}{(e^{rT_i} - 1)^2}] \qquad \text{for } i = 1, 2 \qquad \text{(A.12)}$$

(A.12) can written as:

$$P_i'(T_i)[\frac{e^{rT_i}}{(e^{rT_i} - 1)}] = rP_i(T_i)[\frac{(e^{rT_i})^2}{(e^{rT_i} - 1)^2}] \qquad \text{for } i = 1, 2 \qquad \text{(A.13)}$$

From (A.13) we obtain that:

$$\frac{P_i'(T_i)}{P_i(T_i)} = r[\frac{e^{rT_i}}{(e^{rT_i} - 1)}] \qquad \text{for } i = 1, 2 \qquad \text{(A.14)}$$

Now we have that $\frac{1}{e^{rT_i}} = e^{-rT_i}$ and $\frac{e^{rT_i}}{(e^{rT_i}-1)} = \frac{\frac{e^{rT_i}}{e^{rT_i}}}{(\frac{e^{rT_i}}{e^{rT_i}} - \frac{1}{e^{rT_i}})} = \frac{1}{(1-e^{-rT_i})}$. By using these definitions in (A.14) we get that:

$$\frac{P_i'(T_i)}{P_i(T_i)} = r[\frac{1}{(1-e^{-rT_i})}] \qquad \text{for } i = 1, 2 \qquad \text{(A.15)}$$

(A.15) is identical to the Fautsmann rule. In (A.15) the right-hand side will always be positive so the left-hand side must also be positive implying that an interior solution for the rotation period occur where $P_i'(T_i) > 0$.

Appendix B. The Hartman rule.

From (2) in the main text we have that:

$$MCTP_i(T_i) + MFTP_i(T_i) + MCPA(T_i) + MFPA(T_i) = 0 \quad \text{for } i = 1, 2 \qquad \text{(B.1)}$$

From the definitions from in the main text we have:

$$MCTP_i(T_i) = (P_i'(T_i) - rP_i(T_i))e^{-rT_i} \quad \text{for } i = 1, 2 \qquad \text{(B.2)}$$

$$MFTP_i(T_i) = (L_i'(T_i) - rL_i(T_i))e^{-rT_i} \quad \text{for } i = 1, 2 \qquad \text{(B.3)}$$

$$MCPA(T_i) = V(T_i)e^{-rT_i} \quad \text{for } i = 1, 2 \qquad \text{(B.4)}$$

$$MFPA(T_i) = (Q'(T_i) - rQ(T_i))e^{-rT_i} \quad \text{for } i = 1, 2 \qquad \text{(B.5)}$$

When inserting (B.2)-(B.4) in (B.1) we get that:

$$(P_i'(T_i) - rP_i(T_i))e^{-rT_i} + (L_i'(T_i) - rL_i(T_i))e^{-rT_i} + V(T_i)e^{-rT_i} + (Q'(T_i) - rQ(T_i))e^{-rT_i} = 0 \quad \text{for } i = 1, 2 \qquad \text{(B.6)}$$

Reducing e^{-rT_i} imply that (B.6) can be written as:

$$P_i'(T_i) - rP_i(T_i) + L_i'(T_i) - rL_i(T_i) + V(T_i) + Q'(T_i) - rQ(T_i) = 0 \quad \text{for } i = 1, 2 \qquad \text{(B.7)}$$

From the definition of $L_i(T_i)$ in the main text we have that:

$$L_i(T_i) = \frac{P_i(T_i)}{(e^{rT_i} - 1)} \quad \text{for } i = 1, 2 \qquad \text{(B.8)}$$

By differentiating (B.8) with respect to T_i we obtain that:

$$L_i'(T_i) = \frac{P_i'(T_i)(e^{rT_i} - 1) - re^{rT_i} P_i(T_i)}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(B.9)}$$

(B.10) can be written as:

$$L_i'(T_i) = \frac{P_i'(T_i)}{(e^{rT_i} - 1)} - \frac{re^{rT_i} P_i(T_i)}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(B.10)}$$

From the definition of $Q(T_i)$ in the main text we have that:

$$Q(T_i) = \frac{\int_0^{T_i} V(y_i) dy_i}{(e^{rT_i} - 1)} \qquad \text{for } i = 1, 2 \qquad \text{(B.11)}$$

By differentiating (B.11) with respect to T_i we obtain that:

$$Q'(T_i) = \frac{V(T_i)(e^{rT_i} - 1) - re^{rT_i} \int_0^{T_i} V(y_i) dy_i}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(B.12)}$$

(B.12) can be written as:

$$Q'(T_i) = \frac{V(T_i)}{(e^{rT_i} - 1)} - \frac{re^{rT_i} \int_0^{T_i} V(y_i) dy_i}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(B.13)}$$

By inserting (B.8), (B.10), (B.11) and (B.13) into (B.7) we obtain that:

$$\begin{aligned} & P_i'(T_i) - rP_i(T_i) + \frac{P_i'(T_i)}{(e^{rT_i} - 1)} - \\ & \frac{re^{rT_i} P_i(T_i)}{(e^{rT_{1i}} - 1)^2} - \frac{rP_i(T_i)}{e^{rT_i} - 1} + V(T_i) + \\ & \frac{V(T_i)}{(e^{rT_i} - 1)} - \frac{re^{rT_i} \int_0^{T_i} V(y_i) dy_i}{(e^{rT_i} - 1)^2} - \frac{r \int_0^{T_i} V(y_i) dy_i}{(e^{rT_i} - 1)} = 0 \end{aligned} \qquad \text{for } i = 1, 2 \qquad \text{(B.14)}$$

Reorganizing (B.14) implies that:

$$
\begin{aligned}
&P_i'(T_i)[1+\frac{1}{(e^{rT_i}-1)}] = \\
&rP_i(T_i)[1+\frac{e^{rT_i}}{(e^{rT_i}-1)^2}+\frac{1}{(e^{rT_i}-1)}]+ \\
&r\int_0^{T_i} V(y_i)dy_i\,\frac{e^{rT_i}}{(e^{rT_i}-1)^2}+\frac{1}{(e^{rT_i}-1)}]- \\
&V(T_i)[1+\frac{1}{(e^{rT_i}-1)}]
\end{aligned}
\qquad \text{for } i = 1, 2 \qquad \text{(B.15)}
$$

By rewriting (B.15) we obtain that:

$$
\begin{aligned}
&P_i'(T_i)[\frac{(e^{rT_i}-1)+1}{(e^{rT_i}-1)}] = \\
&rP_i(T_i)[\frac{(e^{rT_i}-1)^2+e^{rT_i}+(e^{rT_i}-1)}{(e^{rT_i}-1)^2}]+ \\
&r\int_0^{T_i} V(y_i)dy_i[\frac{e^{rT_i}+(e^{rT_i}-1)}{(e^{rT_i}-1)^2}]- \\
&V(T_i)[\frac{(e^{rT_i}-1)+1}{(e^{rT_i}-1)}]
\end{aligned}
\qquad \text{for } i = 1, 2 \qquad \text{(B.16)}
$$

(B.16) can be further rewritten as:

$$
\begin{aligned}
&P_i'(T_i)[\frac{e^{rT_i}}{(e^{rT_i}-1)}] = rP_i(T_i)[\frac{(e^{rT_i})^2}{(e^{rT_i}-1)^2}]+ \\
&r\int_0^{T_i} V(y_i)dy_i[\frac{e^{rT_i}}{(e^{rT_i}-1)^2}+\frac{1}{(e^{rT_i}-1)}]- \\
&V(T_i)[\frac{e^{rT_i}}{(e^{rT_i}-1)}]
\end{aligned}
\qquad \text{for } i = 1, 2 \qquad \text{(B.17)}
$$

Reorganizing (B.17) provides:

$$
\frac{P_i'(T_i)}{P_i(T_i)} = r[\frac{e^{rT_i}}{(e^{rT_i}-1)}+
$$

$$\frac{\int_0^{T_i} V(y_i)dy_i}{P_i(T_i)}(\frac{1}{(e^{rT_i}-1)^2}+\frac{1}{e^{rT_i}})]-\frac{V(T_i)}{P_i(T_i)} \quad \text{for } i = 1, 2 \qquad \text{(B.18)}$$

Now we have that $\frac{1}{e^{rT_i}} = e^{-rT_i}$ and $\frac{e^{rT_i}}{(e^{rT_i}-1)} = \frac{\frac{e^{rT_i}}{e^{rT_i}}}{(\frac{e^{rT_i}}{e^{rT_i}}-\frac{1}{e^{rT_i}})} = \frac{1}{1-e^{-rT_i}}$. By using these definitions in (B.18) we get that:

$$\frac{P_i'(T_i)}{P_i(T_i)} = r[\frac{1}{(1-e^{-rT_i})} + \frac{(\int_0^{T_i} e^{-ry_i}V(y_i)dy_i)}{P_i(T_i)(1-e^{-rT_i})}] - \frac{V(T_i)}{P_i(T_i)} \quad \text{for } i = 1, 2 \qquad \text{(B.19)}$$

(B.19) is exactly identical to the original Hartman rule. Note that an interior solution for the rotation period may occur where $P_i'(T_i) < 0$ if:

$$\frac{V(T_i)}{P_i(T_i)} > r[\frac{1}{(1-e^{-rT_i})} + \frac{(\int_0^{T_i} e^{-ry_i}V(y_i)dy_i)}{P_i(T_i)(1-e^{-rT_i})}] \quad \text{for } i = 1, 2 \qquad \text{(B.20)}$$

In relation to (B.20) it seems reasonable to assume that $\int_0^{T_i} e^{-ry_i}V(y_i)dy_i > V(T_i)$ because $\int_0^{T_i} e^{-ry_i}V(y_i)dy_i$ is the accumulated private amenity value from the initial point in time until the trees are harvested while $V(T_i)$ is the private amenity value at the point in time when the trees are harvested. This fact then to imply that (B.21) cannot hold. However, r is included on the right-hand side of (B.20) and if r is low enough" (B.21) may hold. Thus, with the Hartman rule we may obtain that an interior solution for the rotation period occur where $P_i'(T_i) < 0$.

Appendix C: Social optimum.

C.1. Type 1.

From the main text we have that:

$$\begin{aligned}&(1+\varepsilon)(MCTP_1(T_1)+MFTP_1(T_1))+MCSA(T_1)+MFSA(T_1)+\\&\varepsilon(MCPA(T_1)+MFPA(T_1))=0\end{aligned} \tag{C.1}$$

By using the definitions from in the main text we obtain:

$$MCTP_1(T_1)=(P_1'(T_1)-rP_1(T_1))e^{-rT_1} \tag{C.2}$$

$$MFTP_1(T_1)=(L_1'(T_1)-rL_1(T_1))e^{-rT_1} \tag{C.3}$$

$$MCSA(T_1)=W(T_1)e^{-rT_1} \tag{C.4}$$

$$MFSA(T_1)=(K'(T_1)-rK(T_1))e^{-rT_1} \tag{C.5}$$

$$MCPA(T_1)=-V(T_1)e^{-rT_1} \tag{C.6}$$

$$MFPA(T_1)=-(Q'(T_1)-rQ(T_1))e^{-rT_1} \tag{C.7}$$

Note that the sign in (C.6) and (C.7) follow from the fact that we assume that the marginal social cost of the subsidy payment is negative. When inserting (C.2) - (C.7) in (C.1) we get that:

$$\begin{aligned}&(1+\varepsilon)(P_1'(T_1)-rP_1(T_1))e^{-rT_1}+(L_1'(T_1)-rL_1(T_1))e^{-rT_1}+W(T_1)e^{-rT_1}+\\&(K'(T_1)-rK(T_1))e^{-rT_1}-\varepsilon(V(T_1)e^{-rT_1}+(Q'(T_1)-rQ(T_1))e^{-rT_1})=0\end{aligned} \tag{C.8}$$

Reducing e^{-rT_1} imply that (C.8) can be written as:

$$\begin{aligned}&(1+\varepsilon)(P_1'(T_1)-rP_1(T_1)+L_1'(T_1)-rL_1(T_1))+W(T_1)+\\&K'(T_1)-rK(T_1)-\varepsilon(V(T_1)+Q'(T_1)-rQ(T_1))=0\end{aligned} \tag{C.9}$$

From the definition of $L_1(T_1)$ in the main text we have that:

$$L_1(T_1) = \frac{P_1(T_1)}{(e^{rT_1} - 1)} \tag{C.10}$$

By differentiating (C.10) with respect to T_1 we obtain that:

$$L_1{'}(T_1) = \frac{P_1{'}(T_1)(e^{rT_1} - 1) - re^{rT_1} P_1(T_1)}{(e^{rT_1} - 1)^2} \tag{C.11}$$

(C.11) can be written as:

$$L_1{'}(T_1) = \frac{P_1{'}(T_1)}{(e^{rT_1} - 1)} - \frac{re^{rT_1} P_1(T_1)}{(e^{rT_1} - 1)^2} \tag{C.12}$$

From the definition of $K(T_1)$ in the main text we have that:

$$K(T_1) = \frac{\int_0^{T_1} W(x_1)dx_1}{(e^{rT_1} - 1)} \tag{C.13}$$

By differentiating (C.13) with respect to T_1 we obtain that:

$$K'(T_1) = \frac{W(T_1)(e^{rT_1} - 1) - re^{rT_1} \int_0^{T_1} W(x_1)dx_1}{(e^{rT_1} - 1)^2} \tag{C.14}$$

(C.14) can be written as:

$$K'(T_1) = \frac{W(T_1)}{(e^{rT_1} - 1)} - \frac{re^{rT_1} \int_0^{T_1} W(x_1)dx_1}{(e^{rT_1} - 1)^2} \tag{C.15}$$

From the definition of $Q(T_1)$ in the main text we have that:

$$Q(T_1) = \frac{\int_0^{T_1} V(y_1)dy_1}{(e^{rT_1} - 1)} \qquad (C.16)$$

By differentiating (C.16) with respect to T_1 we obtain that:

$$Q'(T_1) = \frac{V(T_1)(e^{rT_1} - 1) - re^{rT_1}\int_0^{T_1} V(y_1)dy_1}{(e^{rT_1} - 1)^2} \qquad (C.17)$$

(C.17) can be written as:

$$Q'(T_1) = \frac{V(T_1)}{(e^{rT_1} - 1)} - \frac{re^{rT_1}\int_0^{T_1} V(y_1)dy_1}{(e^{rT_1} - 1)^2} \qquad (C.18)$$

By inserting (C.10), (C.12), (C.13), (C.15), (C.16) and (C.18) into (C.9) we obtain that:

$$\begin{aligned}
&(1+\varepsilon)(P_1'(T_1) - rP_1(T_1) + \frac{P_1'(T_1)}{(e^{rT_1} - 1)} - \frac{re^{rT_1}P_1(T_1)}{(e^{rT_1} - 1)^2} - \frac{rP_1(T_1)}{e^{rT_1} - 1}) + \\
&W(T_1) + \frac{W(T_1)}{(e^{rT_1} - 1)} - \frac{re^{rT_1}\int_0^{T_1} W(x_1)dx_1}{(e^{rT_1} - 1)^2} - \frac{r\int_0^{T_1} W(x_1)dx_1}{(e^{rT_1} - 1)} - \\
&\varepsilon(V(T_1) + \frac{V(T_1)}{(e^{rT_1} - 1)} - \frac{re^{rT_1}\int_0^{T_1} V(y_1)dy_1}{(e^{rT_1} - 1)^2} - \frac{r\int_0^{T_1} V(y_1)dy_1}{(e^{rT_1} - 1)}) = 0
\end{aligned} \qquad (C.19)$$

Reorganizing (C.19) implies that:

$$\begin{aligned}
&(1+\varepsilon)P_1'(T_1)[1 + \frac{1}{(e^{rT_i} - 1)}] = (1+\varepsilon)rP_1(T_1)[1 + \frac{e^{rT_1}}{(e^{rT_1} - 1)^2} + \frac{1}{(e^{rT_1} - 1)}] + \\
&r\int_0^{T_1} W(x_1)dx_1[\frac{e^{rT_1}}{(e^{rT_1} - 1)^2} + \frac{1}{(e^{rT_1} - 1)}] - W(T_1)[1 + \frac{1}{(e^{rT_1} - 1)}] - \\
&\varepsilon r\int_0^{T_1} V(y_1)dy_1[\frac{e^{rT_1}}{(e^{rT_1} - 1)^2} + \frac{1}{(e^{rT_1} - 1)}] + \varepsilon V(T_1)[1 + \frac{1}{(e^{rT_1} - 1)}]
\end{aligned} \qquad (C.20)$$

By rewriting (C.20) we obtain that:

$$(1+\varepsilon)P_1'(T_1)[\frac{(e^{rT_1}-1)+1}{(e^{rT_1}-1)}]=(1+\varepsilon)rP_1(T_1)$$

$$[\frac{(e^{rT_1}-1)^2+e^{rT_1}+(e^{rT_1}-1)}{(e^{rT_1}-1)^2}]+r\int_0^{T_1}W(x_1)dx_1[\frac{e^{rT_1}+(e^{rT_1}-1)}{(e^{rT_1}-1)^2}]-$$

$$W(T_1)[\frac{(e^{rT_1}-1)+1}{(e^{rT_1}-1)}]-\varepsilon r\int_0^{T_1}V(y_1)dy_1[\frac{e^{rT_1}+(e^{rT_1}-1)}{(e^{rT_1}-1)^2}]+ \qquad \text{(C.21)}$$

$$\varepsilon V(T_1)[\frac{(e^{rT_1}-1)+1}{(e^{rT_1}-1)}]$$

(C.21) can be written as:

$$(1+\varepsilon)P_1'(T_1)[\frac{e^{rT_1}}{(e^{rT_1}-1)}]=(1+\varepsilon)rP_1(T_1)[\frac{(e^{rT_1})^2}{(e^{rT_1}-1)^2}]+$$

$$r\int_0^{T_1}W(x_1)dx_1[\frac{e^{rT_1}}{(e^{rT_1}-1)^2}+\frac{1}{(e^{rT_1}-1)}]-W(T_1)[\frac{e^{rT_1}}{(e^{rT_1}-1)}]- \qquad \text{(C.22)}$$

$$\varepsilon r\int_0^{T_1}V(y_1)dy_1[\frac{e^{rT_1}}{(e^{rT_1}-1)^2}+\frac{1}{(e^{rT_1}-1)}]+\varepsilon V(T_1)[\frac{e^{rT_1}}{(e^{rT_1}-1)}]$$

Reorganizing (C.22) provides:

$$\frac{P_1'(T_1)}{P_1(T_1)}=r[\frac{e^{rT_1}}{(e^{rT_1}-1)}+(\frac{1}{1+\varepsilon})\frac{\int_0^{T_1}W(x_1)dx_1}{P_1(T_1)}(\frac{1}{(e^{rT_1}-1)^2}+\frac{1}{e^{rT_1}})- \qquad \text{(C.23)}$$

$$(\frac{\varepsilon}{1+\varepsilon})\frac{\int_0^{T_1}V(y_1)dy_1}{P_1(T_1)}(\frac{1}{(e^{rT_1}-1)^2}+\frac{1}{e^{rT_1}})]-(\frac{1}{1+\varepsilon})\frac{W(T_1)}{P_1(T_1)}+(\frac{\varepsilon}{1+\varepsilon})\frac{V(T_1)}{P_1(T_1)}$$

Now we have that $\frac{1}{e^{rT_1}}=e^{-rT_1}$ and $\frac{e^{rT_1}}{(e^{rT_1}-1)}=\frac{\frac{e^{rT_1}}{e^{rT_1}}}{(\frac{e^{rT_1}}{e^{rT_1}}-\frac{1}{e^{rT_1}})}=\frac{1}{1-e^{-rT_1}}$. By using these definitions in (C.23) we get that:

$$\frac{P_1'(T_1)}{P_1(T_1)} = r[\frac{1}{(1-e^{-rT_1})} + (\frac{1}{1+\varepsilon})\frac{(\int_0^{T_1} e^{-rx_1}W(x_1)dx_1)}{P_1(T_1)(1-e^{-rT_1})} - (\frac{\varepsilon}{1+\varepsilon})\frac{(\int_0^{T_1} e^{-ry_1}V(y_1)dy_1)}{P_1(T_1)(1-e^{-rT_1})}] - (\frac{1}{1+\varepsilon})\frac{W(T_1)}{P_1(T_1)} + (\frac{\varepsilon}{1+\varepsilon})\frac{V(T_1)}{P_1(T_1)} \quad (C.24)$$

By solving (C.24) we obtain the social optimal rotation period if the forest owner is of type 1 given by T_{1s}^{**} . (C.24) is a version of the Hartman rule when it is costly to collect tax revenue and can be interpreted by comparing it with the original Hartman rule in (B.20). In (C.24) the correction factors $\frac{1}{1+\varepsilon}$ and $\frac{\varepsilon}{1+\varepsilon}$ capture that it is costly to collect tax revenue. Furthermore, the social cost of the subsidy payment is based on the private amenity value and this is reflected in the terms $(\frac{\varepsilon}{1+\varepsilon})\frac{V(T_1)}{P_1(T_1)}$ and $(\frac{\varepsilon}{1+\varepsilon})\frac{(\int_0^{T_1} e^{-ry_1}V(y_1)dy_1)}{P_1(T_1)(1-e^{-rT_1})}$ in (C.24). From (C.24) the former tend to decrease the rotation period while the latter (which decrease the effective discount rate) tend to increase the rotation period. We will assume that the effect of $(\frac{\varepsilon}{1+\varepsilon})\frac{V(T_1)}{P_1(T_1)}$ dominates the effect of $(\frac{\varepsilon}{1+\varepsilon})\frac{(\int_0^{T_1} e^{-ry_1}V(y_1)dy_1)}{P_1(T_1)(1-e^{-rT_1})}$. However, despite this fact we will assume that the social optimal rotation period for type 1 is larger than the private optimal rotation period because we assume that $W(T_1) > V(T_1)$ and $\int_0^{T_1} e^{-rx_1}W(x_1)dx_1 > \int_0^{T_1} e^{-ry_1}V(y_1)dy_1$. Note, also, that the interior solution for the rotation period may occur when $P_1'(T_1) < 0$ if:

$$(\frac{1}{1+\varepsilon})\frac{W(T_1)}{P_1(T_1)}-(\frac{\varepsilon}{1+\varepsilon})\frac{V(T_1)}{P_1(T_1)}>r[\frac{1}{(1-e^{-rT_1})}+ \\ (\frac{1}{1+\varepsilon})\frac{(\int_0^{T_1}e^{-rx_1}W(x_1)dx_1)}{P_1(T_1)(1-e^{-rT_1})}-(\frac{\varepsilon}{1+\varepsilon})\frac{(\int_0^{T_1}e^{-ry_1}V(y_1)dy_1)}{P_1(T_1)(1-e^{-rT_1})}]. \tag{C.25}$$

Because $\int_0^{T_1}e^{-rx_1}W(x_1)dx_1)$ and $\int_0^{T_1}e^{-ry_1}V(y_1)dy_1$ is the accumulated private and social amenity value in the whole rotation period while $W(T_1)$ and $V(T_1)$ is the amenity values at the time where trees are harvested we would expect that $\int_0^{T_1}e^{-rx_1}W(x_1)dx_1>W(T_1)$ and $\int_0^{T_1}e^{-ry_1}V(y_1)dy_1>V(T_1)$. Now since we assume that $W(T_1)>V(T_1)$ and $\int_0^{T_1}e^{-rx_1}W(x_1)dx_1)>\int_0^{T_1}e^{-ry_1}V(y_1)dy_1$ it seems reasonable to assume that (C.25) cannot hold. However, r works in the opposite direction so the interior solution for T_1 may occur where $P_1'(T_1)<0$ provided the interest rate is low enough.

C.2. Type 2.

From the main text we have that:

$$(1+\varepsilon+\frac{\pi_1}{\pi_2}\varepsilon)(MCTP_2(T_2)+MFTP_2(T_2))+MCSA(T_2)+ \\ MFSA(T_2)+\varepsilon(MCPA(T_2)+MFPA(T_2)) \\ -\frac{\pi_1}{\pi_2}\varepsilon(MCTP_1(T_2)+MFTP_1(T_2))=0 \tag{C.26}$$

This expression can be written as:

$$(1+\varepsilon(\frac{\pi_1+\pi_2}{\pi_2}))(MCTP_2(T_2)+MFTP_2(T_2))+MCSA(T_2)+ \\ MFSA(T_2)+\varepsilon(MCPA(T_2)+MFPA(T_2))- \\ \frac{\pi_1}{\pi_2}\varepsilon(MCTP_1(T_2)+MFTP_1(T_2))=0 \tag{C.27}$$

(C.27) may be written as:

$$(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2})(MCTP_2(T_2)+MFTP_2(T_2))+MCSA(T_2)+$$
$$MFSA(T_2)+\varepsilon(MCPA(T_2)+MFPA(T_2))- \qquad (C.28)$$
$$\frac{\pi_1}{\pi_2}\varepsilon(MCTP_1(T_2)+MFTP_1(T_2))=0$$

By using the definitions from in the main text we obtain that:

$$MCTP_2(T_2)=(P_2'(T_2)-rP_2(T_2))e^{-rT_2} \qquad (C.29)$$

$$MFTP_2(T_2)=(L_2'(T_2)-rL_2(T_2))e^{-rT_2} \qquad (C.30)$$

$$MCSA(T_2)=W(T_2)e^{-rT_2} \qquad (C.31)$$

$$MFSA(T_2)=(K'(T_2)-rK(T_2))e^{-rT_2} \qquad (C.32)$$

$$MCPA(T_2)=-V(T_2)e^{-rT_2} \qquad (C.33)$$

$$MFPA(T_2)=-(Q'(T_2)-rQ(T_2))e^{-rT_2} \qquad (C.34)$$

$$MCTP_1(T_2)=(P_1'(T_2)-rP_1(T_2))e^{-rT_2} \qquad (C.35)$$

$$MFTP_1(T_2)=(L_1'(T_2)-rL_1(T_2))e^{-rT_2} \qquad (C.36)$$

Note that the sign in (C.33) and (C.34) follow from the fact that the marginal social cost of the subsidy payment is negative. When inserting (C.29)-(C.36) in (C.28) we get that:

$$(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2})((P_2'(T_2)-rP_2(T_2))e^{-rT_2}+(L_2'(T_2)-rL_2(T_2))e^{-rT_2})+$$
$$W(T_2)e^{-rT_2}+(K'(T_2)-rK(T_2))e^{-rT_2}-\varepsilon(V(T_2)e^{-rT_2}+ \qquad (C.37)$$
$$(Q'(T_2)-rQ(T_2))e^{-rT_2})-\frac{\pi_1}{\pi_2}\varepsilon((P_1'(T_2)-rP_1(T_2))e^{-rT_2}+$$
$$(L_1'(T_2)-rL_1(T_2))e^{-rT_2})=0$$

Reducing e^{-rT_2} imply that (C.37) can be written as:

$$\left(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2}\right)(P_2'(T_2)-rP_2(T_2)+L_2'(T_2)-rL_2(T_2))+W(T_2)+$$
$$K'(T_2)-rK(T_2)-\varepsilon(V(T_2)+Q'(T_2)-rQ(T_2)) \qquad \text{(C.38)}$$
$$-\frac{\pi_1}{\pi_2}\varepsilon(P_1'(T_2)-rP_1(T_2)+L_1'(T_2)-rL_1(T_2))=0$$

From the definition of $L_2(T_2)$ in the main text we have that:

$$L_2(T_2)=\frac{P_2(T_2)}{(e^{rT_2}-1)} \qquad \text{(C.39)}$$

By differentiating (C.39) with respect to T_2 we obtain that:

$$L_2'(T_2)=\frac{P_2'(T_2)(e^{rT_2}-1)-re^{rT_2}P_2(T_2)}{(e^{rT_2}-1)^2} \qquad \text{(C.40)}$$

(C.40) can be written as:

$$L_2'(T_2)=\frac{P_2'(T_2)}{(e^{rT_2}-1)}-\frac{re^{rT_2}P_2(T_2)}{(e^{rT_2}-1)^2} \qquad \text{(C.41)}$$

From the definition of $L_1(T_2)$ in the main text we have that:

$$L_1(T_2)=\frac{P_2(T_2)}{(e^{rT_2}-1)} \qquad \text{(C.42)}$$

By differentiating (C.42) with respect to T_2 we obtain that:

$$L_1'(T_2)=\frac{P_1'(T_2)(e^{rT_2}-1)-re^{rT_2}P_1(T_2)}{(e^{rT_2}-1)^2} \qquad \text{(C.43)}$$

(C.43) can be written as:

$$L_1'(T_2)=\frac{P_1'(T_2)}{(e^{rT_2}-1)}-\frac{re^{rT_2}P_1(T_2)}{(e^{rT_2}-1)^2} \qquad \text{(C.44)}$$

From the definition of $K(T_2)$ in the main text we have that:

$$K(T_2) = \frac{\int_0^{T_2} W(x_2)dx_2}{(e^{rT_2} - 1)} \tag{C.45}$$

By differentiating (C.45) with respect to T_2 we obtain that:

$$K'(T_2) = \frac{W(T_2)(e^{rT_2} - 1) - re^{rT_2}\int_0^{T_2} W(x_2)dx_2}{(e^{rT_2} - 1)^2} \tag{C.46}$$

(C.46) can be written as:

$$K'(T_2) = \frac{W(T_2)}{(e^{rT_2} - 1)} - \frac{re^{rT_2}\int_0^{T_2} W(x_2)dx_2}{(e^{rT_2} - 1)^2} \tag{C.47}$$

From the definition of $Q(T_2)$ in the main text we have that:

$$Q(T_2) = \frac{\int_0^{T_2} V(y_2)dy_2}{(e^{rT_2} - 1)} \tag{C.48}$$

By differentiating (C.48) with respect to T_2 we obtain that:

$$Q'(T_2) = \frac{V(T_2)(e^{rT_2} - 1) - re^{rT_2}\int_0^{T_2} V(y_2)dy_2}{(e^{rT_2} - 1)^2} \tag{C.49}$$

(C.49) can be written as:

$$Q'(T_2) = \frac{V(T_2)}{(e^{rT_2} - 1)} - \frac{re^{rT_2}\int_0^{T_2} V(y_2)dy_2}{(e^{rT_2} - 1)^2} \tag{C.50}$$

By inserting (C.39), (C.41), (C.42), (C.44), (C.45), (C.47), (C.48) and (C.50) into (C.38) we obtain that:

$$
\begin{aligned}
&(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2})(P_2'(T_2)-rP_2(T_2)+\frac{P_2'(T_2)}{(e^{rT_2}-1)}-\frac{re^{rT_2}P_2(T_2)}{(e^{rT_2}-1)^2}-\\
&\frac{rP_2(T_2)}{e^{rT_2}-1})+W(T_2)+\frac{W(T_2)}{(e^{rT_2}-1)}-\frac{re^{rT_2}\int_0^{T_2}W(x_2)dx_2}{(e^{rT_2}-1)^2}-\frac{r\int_0^{T_2}W(x_2)dx_2}{(e^{rT_2}-1)}-\\
&\frac{\pi_1}{\pi_2}\varepsilon(P_1'(T_2)-rP_1(T_2)+\frac{P_1'(T_2)}{(e^{rT_2}-1)}-\frac{re^{rT_2}P_1(T_2)}{(e^{rT_2}-1)^2}-\frac{rP_1(T_2)}{e^{rT_2}-1})=0
\end{aligned}
\tag{C.51}
$$

Reorganizing (C.51) implies that

$$
\begin{aligned}
&(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2})P_2'(T_2)[1+\frac{1}{(e^{rT_2}-1)}]=\\
&(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2})rP_2(T_2)[1+\frac{e^{rT_2}}{(e^{rT_2}-1)^2}+\frac{1}{(e^{rT_2}-1)}]+\\
&r\int_0^{T_2}W(x_2)dx_2[\frac{e^{rT_2}}{(e^{rT_2}-1)^2}+\frac{1}{(e^{rT_2}-1)}]-\frac{\pi_1}{\pi_2}\varepsilon rP_1(T_2)[1+\frac{e^{rT_2}}{(e^{rT_2}-1)^2}+\\
&\frac{1}{(e^{rT_2}-1)}]-r\varepsilon\int_0^{T_2}V(y_2)dy_2[\frac{e^{rT_2}}{(e^{rT_2}-1)^2}+\frac{1}{(e^{rT_2}-1)}]-W(T_2)[1+\\
&\frac{1}{(e^{rT_2}-1)}]+\varepsilon V(T_2)[1+\frac{1}{(e^{rT_2}-1)}]+\frac{\pi_1}{\pi_2}\varepsilon P_1'(T_2)[1+\frac{1}{(e^{rT_2}-1)}]
\end{aligned}
\tag{C.52}
$$

By rewriting (C.52) we obtain that:

$$(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2})P_2'(T_2)[\frac{(e^{rT_2}-1)+1}{(e^{rT_2}-1)}]=(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2})rP_2(T_2)$$
$$[\frac{(e^{rT_2}-1)^2+e^{rT_2}+(e^{rT_2}-1)}{(e^{rT_2}-1)^2}]+r\int_0^{T_2}W(x_2)dx_2[\frac{e^{rT_2}+(e^{rT_2}-1)}{(e^{rT_2}-1)^2}]-$$
$$\frac{\pi_1}{\pi_2}\varepsilon rP_1(T_2)[\frac{(e^{rT_2}-1)^2+e^{rT_2}+(e^{rT_2}-1)}{(e^{rT_2}-1)^2}]- \tag{C.53}$$
$$r\varepsilon\int_0^{T_2}V(y_2)dy_2[\frac{e^{rT_2}+(e^{rT_2}-1)}{(e^{rT_2}-1)^2}]-W(T_2)[\frac{(e^{rT_2}-1)+1}{(e^{rT_2}-1)}]+$$
$$\varepsilon V(T_2)[\frac{(e^{rT_2}-1)+1}{(e^{rT_2}-1)}]+\frac{\pi_1}{\pi_2}\varepsilon P_1'(T_2)[\frac{(e^{rT_2}-1)+1}{(e^{rT_2}-1)}]$$

(C.53) can be further rewritten as:

$$(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2})P_2'(T_2)[\frac{e^{rT_2}}{(e^{rT_{12}}-1)}]=$$
$$(\frac{\pi_2+\varepsilon(\pi_1+\pi_2)}{\pi_2})rP_2(T_2)[\frac{(e^{rT_2})^2}{(e^{rT_2}-1)^2}]+$$
$$r\int_0^{T_2}W(x_2)dx_2[\frac{e^{rT_2}}{(e^{rT_2}-1)^2}+\frac{1}{(e^{rT_2}-1)}]-\frac{\pi_1}{\pi_2}\varepsilon rP_1(T_2)[\frac{(e^{rT_2})^2}{(e^{rT_2}-1)^2}]- \tag{C.54}$$
$$r\varepsilon\int_0^{T_2}V(y_2)dy_2[\frac{e^{rT_2}}{(e^{rT_2}-1)^2}+\frac{1}{(e^{rT_2}-1)}]-W(T_2)[\frac{e^{rT_2}}{(e^{rT_2}-1)}]+$$
$$\frac{\pi_1}{\pi_2}\varepsilon P_1'(T_2)[\frac{e^{rT_2}}{(e^{rT_2}-1)}]+\varepsilon V(T_2)[\frac{e^{rT_2}}{(e^{rT_2}-1)}]$$

Reorganizing (C.54) provides:

$$\frac{P_2'(T_2)}{P_2(T_2)}=r[\frac{e^{rT_2}}{(e^{rT_2}-1)}+(\frac{\pi_2}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{\int_0^{T_2}W(x_2)dx_2}{P_2(T_2)}(\frac{1}{(e^{rT_2}-1)^2}+$$

$$\frac{1}{e^{rT_2}})-(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1(T_2)}{P_2(T_2)}\frac{e^{rT_2}}{(e^{rT_2}-1)}-$$

$$\frac{\pi_2\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)}\int_0^{T_2}V(y_2)dy_2(\frac{e^{rT_2}}{(e^{rT_2}-1)^2}+\frac{1}{(e^{rT_2}-1)})]-$$
$$(\frac{\pi_2}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{W(T_2)}{P_2(T_2)}+(\frac{\pi_2\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{V(T_2)}{P_2(T_2)}+ \qquad (C.55)$$
$$(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1´(T_2)}{P_2(T_2)}$$

Now we have that $\frac{1}{e^{rT_2}}=e^{-rT_2}$ and $\frac{e^{rT_2}}{(e^{rT_2}-1)}=\frac{\frac{e^{rT_2}}{e^{rT_2}}}{(\frac{e^{rT_2}}{e^{rT_2}}-\frac{1}{e^{rT_2}})}=\frac{1}{1-e^{-rT_2}}$. By using these definitions in (C.55) we get that:

$$\frac{P_2´(T_2)}{P_2(T_2)}=r[\frac{1}{(1-e^{-rT_2})}+(\frac{\pi_2}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{(\int_0^{T_2}e^{-rx_2}W(x_2)dx_2)}{P_2(T_2)(1-e^{-rT_2})}-$$
$$(\frac{\pi_2\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{(\int_0^{T_2}e^{-ry_2}V(y_2)dy_2)}{P_2(T_2)(1-e^{-rT_2})}- \qquad (C.56)$$
$$(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1(T_2)}{P_2(T_2)(1-e^{-rT_2})}]-(\frac{\pi_2}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{W(T_2)}{P_2(T_2)}+$$
$$(\frac{\pi_2\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{V(T_2)}{P_2(T_2)}+(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1´(T_2)}{P_2(T_2)}$$

By solving (C.56) we obtain the social optimal rotation period if the forest owner is of type 2 given by T_{2s}^* and (C.56) is a version of the Hartman rule with imperfect information. It is useful to interpret (C.56) by comparing with (C.20). Now $(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1´(T_2)}{P_2(T_2)}$ tend to reduce the rotation period while $(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1(T_2)}{P_2(T_2)(1-e^{-rT_2})}$ tend to increase the rotation period. We assume that the former effect dominates the latter so we get that $T_{1s}^*>T_{2s}^*$. However, despite this fact we assume that $T_{2s}^*>T_{2p}^*$ because we assume that $W(T_2)>V(T_2)$ and $\int_0^{T_2}e^{-rx_2}W(x_2)dx_2)>$

$\int_0^{T_2} e^{-ry_2}V(y_2)dy_2$. This effect is assumed to dominate the counteracting effect of $(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1'(T_2)}{P_2(T_2)}$ and $(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1(T_2)}{P_2(T_2)(1-e^{-rT_2})}$. Finally, the interior solution for T_2 may occur where $P_2'(T_2)<0$ if:

$$
\begin{aligned}
&(\frac{\pi_2}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{W(T_2)}{P_2(T_2)}-(\frac{\pi_2\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{V(T_2)}{P_2(T_2)}+\\
&r(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1(T_2)}{P_2(T_2)(1-e^{-rT_2})}>\\
&r[\frac{1}{(1-e^{-rT_2})}+(\frac{\pi_2}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{(\int_0^{T_2} e^{-rx_2}W(x_2)dx_2)}{P_2(T_2)(1-e^{-rT_2})}-\\
&(\frac{\pi_2\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{(\int_0^{T_2} e^{-ry_2}V(y_2)dy_2)}{P_2(T_2)(1-e^{-rT_2})}+(\frac{\pi_1\varepsilon}{\pi_2+\varepsilon(\pi_1+\pi_2)})\frac{P_1'(T_2)}{P_2(T_2)}
\end{aligned}
\qquad \text{(C.57)}
$$

Again we would expect that $\int_0^{T_2} e^{-rx_2}W(x_2)dx_2 > W(T_2)$ and $\int_0^{T_2} e^{-ry_2}V(y_2)dy_2 > V(T_2)$ and because we assume that we assume that $W(T_2)>V(T_2)$ and $\int_0^{T_2} e^{-rx_2}W(x_2)dx_2) > \int_0^{T_2} e^{-ry_2}V(y_2)dy_2$ this tends to imply that (C.57) cannot hold. However, for the profit function for type 1 we assume that ordinary effects dominates first-order effects implying that $\frac{P_1(T_2)}{P_2(T_2)(1-e^{-rT_2})} > \frac{P_1'(T_2)}{P_2(T_2)}$ and this tend to imply that (C.57) cannot hold. Finally, as for type 1 we would expect that a low value of r tend to imply that (C.57). Thus, an interior solution for the rotation period may occur where $P_2'(T_2)<0$.

Appendix D. Participation and self-selection restrictions.

D.1. Type 1.

For type 1 the two restrictions are:

$$S_1(T_1) \geq U - \int_0^{T_1} V(y_1)e^{-ry_1}dy_1 - Q(T_1)e^{-rT_1} - P_1(T_1)e^{-rT_1} - L_1(T_1)e^{-rT_1} \quad \text{(D.1)}$$

$$\begin{aligned} S_1(T_1) \geq & -\int_0^{T_1} V(y_1)e^{-ry_1}dy_1 - Q(T_1)e^{-rT_1} - P_1(T_1)e^{-rT_1} - L_1(T_1)e^{-rT_1} + \\ & (\int_0^{T_2} V(y_2)e^{-ry_2}dy_2 + Q(T_2)e^{-rT_2} + P_1(T_2)e^{-rT_2} + L_1(T_2)e^{-rT_2} + S_2(T_2)) \end{aligned} \quad \text{(D.2)}$$

Since $S_1(T_1)$ enters with a negative sign in the objective function of the regulator, one of these restrictions must be binding. To investigate whether (D.1) or (D.2) is binding we use the participation restriction for type 2:

$$\int_0^{T_2} V(y_2)e^{-ry_2}dy_2 + Q(T_2)e^{-rT_2} + P_2(T_2)e^{-rT_2} + L_2(T_2)e^{-rT_2} + S_2(T_2) \geq U \quad \text{(D.3)}$$

From the assumptions in section 2.1 we have that:

$$P_1(T_2)e^{-rT_2} + L_1(T_2)e^{-rT_2} > P_2(T_2)e^{-rT_2} + L_2(T_2)e^{-rT_2} \quad \text{(D.4)}$$

When adding $S_2(T_2)$, $\int_0^{T_2} V(y_2)e^{-ry_2}dy_2$ and $Q(T_2)e^{-rT_2}$ on both sides of (D.4) and using (D.3) we get that:

$$\begin{aligned} & \int_0^{T_2} V(y_2)e^{-ry_2}dy_2 + Q(T_2)e^{-rT_2} + P_1(T_2)e^{-rT_2} + L_1(T_2)e^{-rT_2} + \\ & S_2(T_2) > \int_0^{T_2} V(y_2)e^{-ry_2}dy_2 + Q(T_2)e^{-rT_2} + \\ & P_2(T_2)e^{-rT_2} + L_2(T_2)e^{-rT_2} + S_2(T_2) \geq U \end{aligned} \quad \text{(D.5)}$$

Thus, the expression in the bracket of (D.5) is larger than the reservation payoff. This implies that it must be (D.2), and not (D.1), that is binding. The binding self-selection restriction for type 1 can be written is:

$$\begin{aligned} S_1(T_1) = & -\int_0^{T_1} V(y_1)e^{-ry_1}dy_1 - Q(T_1)e^{-rT_1} - P_1(T_1)e^{-rT_1} - L_1(T_1)e^{-rT_1} + \\ & \left(\int_0^{T_2} V(y_2)e^{-ry_2}dy_2 + Q(T_2)e^{-rT_2} + P_1(T_2)e^{-rT_2} + L_1(T_2)e^{-rT_2} + S_2(T_2)\right) \end{aligned} \quad \text{(D.6)}$$

D.2. Type 2.

One of the restrictions of type 2 must also be binding. Can it be the self-selection restriction? If this restriction is binding we have that:

$$\begin{aligned} & \int_0^{T_2} V(y_2)e^{-ry_2}dy_2 + Q(T_2)e^{-rT_2} + P_2(T_2)e^{-rT_2} + L_2(T_2)e^{-rT_2} + S_2(T_2) = \\ & \int_0^{T_1} V(y_1)e^{-ry_1}dy_1 + Q(T_1)e^{-rT_1} + P_2(T_1)e^{-rT_1} + L_2(T_1)e^{-rT_1} + S_1(T_1) \end{aligned} \quad \text{(D.7)}$$

Substituting (D.6) into (D.7) gives:

$$\begin{aligned} & (P_1(T_2) + L_1(T_2))e^{-rT_2} - (P_1(T_1) + L_1(T_1))e^{-rT_1} = \\ & (P_2(T_2) + L_2(T_2))e^{-rT_2} - (P_2(T_1) + L_2(T_1))e^{-rT_1} \end{aligned} \quad \text{(D.8)}$$

The left-hand side of (D.8) is the total current and future incremental timber profit for type 1 while the right-hand side is the total current and future incremental timber profit for type 2. From section 2 the total current and future incremental timber profit of type 1 must be larger than for type 2. Thus, (D.8) can never hold and, therefore, the participation restriction of type 2 is binding. The binding self-selection restriction for type 1 and participation restriction for type 2 can be written as:

$$S_1(T_1) = -\int_0^{T_1} V(y_1)e^{-ry_1}dy_1 - Q(T_1)e^{-rT_1} - P_1(T_1)e^{-rT_1} - L_1(T_1)e^{-rT_1} + \\ (\int_0^{T_2} V(y_2)e^{-ry_2}dy_2 + Q(T_2)e^{-rT_2} + P_1(T_2)e^{-rT_2} + L_1(T_2)e^{-rT_2} + S_2(T_2)) \quad \text{(D.9)}$$

$$S_2(T_2) = U - \int_0^{y_2} V(y_2)e^{-ry_2}dy_2 - Q(T_2)e^{-rT_2} - P_2(T_2)e^{-rT_2} - L_2(T_2)e^{-rT_2} \quad \text{(D.10)}$$

By substituting (D.10) into (D.9) we get that:

$$S_1(T_1) = U - \int_0^{T_1} V(y_1)e^{-ry_1}dy_1 - Q(T_1)e^{-rT_1} - P_1(T_1)e^{-rT_1} - \\ L_1(T_1)e^{-rT_1} + (P_1(T_2) + L_1(T_2) - (P_2(T_2) + L_2(T_2)))e^{-rT_2} \quad \text{(D.11)}$$

(D.10) and (D.11) is used in the main text in section 2.2.

Appendix E. Incorrect believes about the private amenity value.

E. 1. The rotation period.

From the main text we have that:

$$MCPA(T_i) + MFPA(T_i) + MCTP_i(T_i) + MFTP_i(T_i) + S_i´(T_i) = 0 \qquad \text{for } i = 1, 2 \qquad (E.1)$$

By using the definitions from the main text we obtain that:

$$MCTP_i(T_i) = (P_i´(T_i) - rP_i(T_i))e^{-rT_i} \qquad \text{for } i = 1, 2 \qquad (E.2)$$

$$MFTP_i(T_i) = (L_i´(T_i) - rL_i(T_i))e^{-rT_i} \qquad \text{for } i = 1, 2 \qquad (E.3)$$

$$MCPA(T_i) = -V(T_i)e^{-rT_i} \qquad \text{for } i = 1, 2 \qquad (E.4)$$

$$MFPA(T_i) = -(Q´(T_i) - rQ(T_i))e^{-rT_i} \qquad \text{for } i = 1, 2 \qquad (E.5)$$

Note the sign in (E.4) and (E.5) take into account that the marginal social cost of the subsidy payment is assumed to be negative. When inserting (E.2)- (E.5) in (E.1) we get that:

$$(P_i´(T_i) - rP_i(T_i))e^{-rT_i} + (L_i´(T_i) - rL_i(T_i))e^{-rT_i} - V(T_i)e^{-rT_i} - (Q´(T_i) - rQ(T_i))e^{-rT_i} + S_i´(T_i) = 0 \qquad \text{for } i = 1, 2 \qquad (E.6)$$

Reducing e^{-rT_i} imply that (E.6) can be written as:

$$P_i´(T_i) - rP_i(T_i) + L_i´(T_i) - rL_i(T_i) - V(T_i) - Q´(T_i) + rQ(T_i) + \frac{S_i´(T_i)}{e^{-rT_i}} = 0 \qquad \text{for } i = 1, 2 \qquad (E.7)$$

From the definition of $L_i(T_i)$ in the main text we have that:

$$L_i(T_i) = \frac{P_i(T_i)}{(e^{rT_i} - 1)} \qquad \text{for } i = 1, 2 \qquad (E.8)$$

By differentiating (E.8) with respect to T_i we obtain that:

$$L_i'(T_i) = \frac{P_i'(T_i)(e^{rT_i} - 1) - re^{rT_i} P_i(T_i)}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(E.9)}$$

(E.9) can be written as:

$$L_i'(T_i) = \frac{P_i'(T_i)}{(e^{rT_i} - 1)} - \frac{re^{rT_i} P_i(T_i)}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(E.10)}$$

From the definition of $Q(T_i)$ in the main text we have that:

$$Q(T_i) = \frac{\int_0^{T_i} V(y_i) dy_i}{(e^{rT_i} - 1)} \qquad \text{for } i = 1, 2 \qquad \text{(E.11)}$$

By differentiating (E.11) with respect to T_i we obtain that:

$$Q'(T_i) = \frac{V(T_i)(e^{rT_i} - 1) - re^{rT_i} \int_0^{T_2} V(y_i) dy_i}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(E.12)}$$

(E. 12) can be written as:

$$Q'(T_i) = \frac{V(T_i)}{(e^{rT_i} - 1)} - \frac{re^{rT_i} \int_0^{T_i} V(y_i) dy_i}{(e^{rT_i} - 1)^2} \qquad \text{for } i = 1, 2 \qquad \text{(E.13)}$$

By inserting (E.8), (E.10), (E.11) and (E.13) into (E.7) we obtain that:

$$P_i'(T_i) - rP_i(T_i) + \frac{P_i'(T_i)}{(e^{rT_i} - 1)} - \frac{re^{rT_i} P_i(T_i)}{(e^{rT_i} - 1)^2} -$$

$$\frac{rP_i(T_i)}{(e^{rT_i} - 1)} - V(T_i) - \frac{V(T_i)}{(e^{rT_i} - 1)} + \frac{re^{rT_i} \int_0^{T_i} V(y_i) dy_i}{(e^{rT_i} - 1)^2} +$$

$$\frac{r\int_0^{T_i} V(y_i)dy_i}{(e^{rT_i}-1)}+\frac{S_i'(T_i)}{e^{-rT_i}}=0 \qquad \text{for } i = 1, 2 \quad \text{(E.14)}$$

Reorganizing (E.14) implies that:

$$\begin{aligned}&P_i'(T_i)[1+\frac{1}{(e^{rT_i}-1)}]=\\&rP_i(T_i)[1+\frac{e^{rT_i}}{(e^{rT_i}-1)^2}+\frac{1}{(e^{rT_i}-1)}]-\\&r\int_0^{T_i} V(y_i)dy_i\,\frac{e^{rT_i}}{(e^{rT_i}-1)^2}+\frac{1}{(e^{rT_i}-1)}]+\\&V(T_i)[1+\frac{1}{(e^{rT_i}-1)}]-\frac{S_i'(T_i)}{e^{-rT_i}}\end{aligned} \qquad \text{for } i = 1, 2 \quad \text{(E.15)}$$

By rewriting (E.15) we obtain that:

$$\begin{aligned}&P_i'(T_i)[\frac{(e^{rT_i}-1)+1}{(e^{rT_i}-1)}]=\\&rP_i(T_i)[\frac{(e^{rT_i}-1)^2+e^{rT_i}+(e^{rT_i}-1)}{(e^{rT_i}-1)^2}]-\\&r\int_0^{T_i} V(y_i)dy_i[\frac{e^{rT_i}+(e^{rT_i}-1)}{(e^{rT_i}-1)^2}]+\\&V(T_i)[\frac{(e^{rT_i}-1)+1}{(e^{rT_i}-1)}]-\frac{S_i'(T_i)}{e^{-rT_i}}\end{aligned} \qquad \text{for } i = 1, 2 \quad \text{(E.16)}$$

(E.16) can be further rewritten as:

$$\begin{aligned}&P_i'(T_i)[\frac{e^{rT_i}}{(e^{rT_i}-1)}]=rP_i(T_i)[\frac{(e^{rT_i})^2}{(e^{rT_i}-1)^2}]-\\&r\int_0^{T_i} V(y_i)dy_i[\frac{e^{rT_i}}{(e^{rT_i}-1)^2}+\frac{1}{(e^{rT_i}-1)}]+\\&V(T_i)[\frac{e^{rT_i}}{(e^{rT_i}-1)}]-\frac{S_i'(T_i)}{e^{-rT_i}}\end{aligned} \qquad \text{for } i = 1, 2 \quad \text{(E.17)}$$

Reorganizing (E.17) provides:

$$\frac{P_i'(T_i)}{P_i(T_i)} = r[\frac{e^{rT_i}}{(e^{rT_i}-1)} - \frac{\int_0^{T_i} V(y_i)dy_i}{P_i(T_i)}(\frac{1}{(e^{rT_i}-1)^2} + \frac{1}{e^{rT_i}})] + \frac{V(T_i)}{P_i(T_i)} - \frac{S_i'(T_i)}{e^{-rT_i}} \quad \text{for } i = 1, 2 \quad \text{(E.18)}$$

Now we have that $\frac{1}{e^{rT_i}} = e^{-rT_i}$ and $\frac{e^{rT_i}}{(e^{rT_i}-1)} = \frac{\frac{e^{rT_i}}{e^{rT_i}}}{(\frac{e^{rT_i}}{e^{rT_i}} - \frac{1}{e^{rT_i}})} = \frac{1}{1-e^{-rT_i}}$. By using these definitions in (E.18) we get that:

$$\frac{P_i'(T_i)}{P_i(T_i)} = r[\frac{1}{(1-e^{-rT_i})} - \frac{(\int_0^{T_i} e^{-ry_i}V(y_i)dy_i)}{P_i(T_i)(1-e^{-rT_i})}] + \frac{V(T_i)}{P_i(T_i)} - \frac{S_i'(T_i)}{e^{-rT_i}} \quad \text{for } i = 1, 2 \quad \text{(E.19)}$$

From (E.19) an increase in the marginal subsidy implies that the right-hand side decrease. Thus, the left-hand side of (E.19) must also decrease and since $P_i''(T_i) < 0$ this implies that the rotation period must increase.

E. 2. Correct revelation of the type of the forest owner.

From the main text we have that:

$$MIC(T_2) = (P_1'(T_2) - rP_1(T_2) + L_1'(T_2) - rL_1(T_2)) - (P_2'(T_2) - rP_2(T_2) + L_2'(T_2) - rL_2(T_2))e^{-rT_2})) \quad \text{(E.20)}$$

(E.20) may be rewritten as:

$$MIC(T_2) = -r(P_1(T_2) + L_1(T_2) - (P_2(T_2) + L_2(T_2)))e^{-rT_2} + (P_1'(T_2) + L_1'(T_2) - (P_2'(T_2) + L_2'(T_2))e^{-rT_2} \quad \text{(E.21)}$$

By differentiating (E.22) with respect to T_2 we get that:

$$MIC'(T_2) = -rMIC(T_2) - r(P_1'(T_2) + L_1'(T_2) - (P_2'(T_2) + L_2'(T_2))e^{-rT_2} + (P_1''(T_2) + L_1''(T_2) - (P_2''(T_2) + L_2''(T_2))e^{-rT_2} \quad \text{(E.22)}$$

Now from the assumptions in section 2.1 we have that $MIC(T_2) > 0,\ P_1'(T_2) > 0$ and $P_2'(T_2) > 0$. Based on the definition of the total and marginal future timber profit it also seems reasonable to assume that $L_1'(T_2) > 0$ and $L_2'(T_2) > 0$. Furthermore, from the dynamic single-crossing property we have that $(P_1'(T_2) + L_1'(T_2))e^{-rT_2} > (P_2'(T_2) + L_2'(T_2))e^{-rT_2}$. All these facts imply that the first two terms in (E.22) are negative. Concerning the third term, we have that $P_1''(T_2) < 0$ and $P_2''(T_2) < 0$ and based on the definitions that $L_1''(T_2) < 0$ and $L_2''(T_2) < 0$. Now we can impose a second-order dynamic single crossing property capturing that if $P_1'(T_2) + L_1'(T_2) > P_2'(T_2) + L_2'(T_2)$, then the numerical value of $P_1''(T_2) + L_1''(T_2)$ is higher than the numerical value of $P_2''(T_2) + L_2''(T_2)$. This implies that the third term in (E.22) is negative sign. Thus, in total, we obtain that $MIC'(T_2) < 0$.

Appendix F. Comparative statics.

F.1. Type 1.

From the main text we have that:

$$MCSA(T_1) + MFSA(T_1) + MCTP_1(T_1) + MFTP_1(T_1) + \\ \varepsilon(MCPA(T_1) + MFPA(T_1) + MCTP_1(T_1) + MFTP_1(T_1)) = 0 \quad \text{(F.1)}$$

From (F.1) we can derive the following second-order condition for a maximum:

$$C = \frac{\partial MCSA(T_1)}{\partial T_1} + \frac{\partial MFSA(T_1)}{\partial T_1} + \frac{\partial MCTP_1(T_1)}{\partial T_1} + \frac{\partial MFTP_1(T_1)}{\partial T_1} + \\ \varepsilon(\frac{\partial MCPA(T_1)}{\partial T_1} + \frac{\partial MFPA(T_1)}{\partial T_1} + \frac{\partial MCTP_1(T_1)}{\partial T_1} + \frac{\partial MFTP_1(T_1)}{\partial T_1}) < 0 \quad \text{(F.2)}$$

We assume that the second-order conditions in (F.2) is fulfilled implying that $C < 0$. By totally differentiating (F.1) with respect to T_1 and ε we get that:

$$CdT_1 + Fd\varepsilon = 0 \quad \text{(F.3)}$$

In (F.3) we have that:

$$F = MCPA(T_1) + MFPA(T_1) + MCTP_1(T_1) + MFTP_1(T_1) < 0 \quad \text{(F.4)}$$

From the main text we have that $F < 0$. By reorganizing (F.3) we obtain that:

$$\frac{dT_1}{d\varepsilon} = -\frac{F}{C} < 0 \quad \text{(F.5)}$$

From above we have that $C < 0$ and that $F < 0$. Thus, from (F.5) we obtain that $\frac{dT_1}{d\varepsilon} < 0$.

F.2. Type 2.

From the main text we have that:

$$
\begin{aligned}
&MCSA(T_2) + MFSA(T_2) + MCTP_2(T_2) + MFTP_2(T_2) + \\
&\varepsilon(MCPA(T_2) + MFPA(T_2) + MCTP_2(T_2) + MFTP_2(T_2)) - \\
&\frac{\pi_1}{\pi_2}\varepsilon MIC(T_2) = 0
\end{aligned} \tag{F.6}
$$

From (F.6) we arrive at the following second-order condition:

$$
\begin{aligned}
&E = \frac{\partial MCSA(T_2)}{\partial T_2} + \frac{\partial MFSA(T_2)}{\partial T_2} + \frac{\partial MCTP_2(T_2)}{\partial T_2} + \frac{\partial MFTP_2(T_2)}{\partial T_2} + \\
&\varepsilon(\frac{\partial MCPA(T_2)}{\partial T_2} + \frac{\partial MFPA(T_2)}{\partial T_2} + \frac{\partial MCTP_2(T_2)}{\partial T_2} + \frac{\partial MFTP_2(T_2)}{\partial T_{12}}) - \\
&\frac{\pi_1}{\pi_2}\varepsilon \frac{\partial MIC(T_2)}{\partial T_2} < 0
\end{aligned} \tag{F.7}
$$

We assume that the second-order condition in (F.7) is fulfilled so $E < 0$. Now we can totally differentiate (F.6) with respect to T_2, ε, π_1 and π_2:

$$
EdT_2 + Gd\varepsilon + Hd\pi_1 + Jd\pi_2 = 0 \tag{F.8}
$$

In (F.8) we have that:

$$
\begin{aligned}
&G = MCPA(T_2) + MFPA(T_2) + MCTP_2(T_2) + \\
&MFTP_2(T_2) - \frac{\pi_1}{\pi_2} MIC(T_2) < 0
\end{aligned} \tag{F.9}
$$

$$
H = -\frac{1}{\pi_2}\varepsilon MIC(T_2) < 0 \tag{F.10}
$$

$$
J = \frac{\pi_1}{(\pi_2)^2}\varepsilon MIC(T_2) > 0 \tag{F.11}
$$

From the main text we have that G < 0, H < 0 and J > 0as indicated in (F.9) – (F.11). To derive a comparative static result for ε we will use $d\pi_1 = d\pi_2 = 0$ in (F.8) and reorganize to get:

$$
\frac{dT_2}{d\varepsilon} = -\frac{G}{E} < 0 \tag{F.12}
$$

From above we have that $E < 0$ and that $G < 0$. Thus, from (F.12) we obtain that $\frac{dT_2}{d\varepsilon}$. Concerning a comparative static result for π_1 we will use $d\varepsilon = d\pi_2 = 0$ in (F.8) and rewrite to obtain that:

$$\frac{dT_2}{d\pi_1} = -\frac{H}{E} < 0 \tag{F.13}$$

From above we have that $E < 0$ and that $H < 0$. Thus, as indicated in (F.13) we obtain that $\frac{dT_2}{d\pi_1} < 0$. Finally, by setting $d\varepsilon = d\pi_1 = 0$ in (A.8) and reorganizing we obtain that:

$$\frac{dT_2}{d\pi_2} = -\frac{J}{E} > 0 \tag{F.14}$$

From above we have that $E < 0$ and that $J > 0$. Thus, as indicated in (F.14) we obtain that $\frac{dT_2}{d\pi_2} > 0$.