

Online Appendix

Credibility and Backlash

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A Equilibrium selection

Farrell and Gibbons (1989, 1220) demonstrate that whenever a separating equilibrium exists in a cheap talk game with one sender and two receivers, the pooling equilibrium fails the criterion of *neologism-proofness* as long as the receivers' mappings from beliefs to actions satisfy a type of consistency with one another (*coherence*). The idea behind neologism-proofness is that the sender and receivers have access to a rich language with common and literal meaning. Essentially, the pooling equilibrium is selected against because of the idea that the sender would be able to make a speech like “I really am of type 1, and you should believe me because only a type 1 sender would have an incentive to convince you so” (Farrell 1993). While coherence is defined in a setting in which receivers have binary actions, its purpose is to ensure that the sender prefers separation. This holds presently:

Lemma A.1 (Politician preference for equilibrium). *When the separating equilibrium exists, the politician prefers it to the pooling equilibrium.*

Under pooling, both groups may grant support when it is hard enough to repurpose and when their prior belief that the politician is aligned is sufficiently great. But the inability to identify friends and enemies leaves this a speculative exercise, reducing the total amount that the politician receives in aggregate as well as the amount that the politician can use to achieve preferred objectives. For this reason, each politician type does better when she can credibly identify herself to both groups.¹ Given this, I reach the following result:

Proposition A.1 (Equilibrium selection). *When the separating equilibrium exists, the pooling equilibrium fails neologism-proofness.*

1. This is distinct from a main result of this paper, which is that separation is not necessarily better for a group. This stems from an asymmetry: for the politician, separation assures her of finding an ally. But for a group, separation might only find its opponent an ally. The inclusion of multiple politicians with independently-drawn types would not change this, as it is not clear that they would interact in any way, and whether any given politician separated would be independent of that same question for any other politician.

Consequently, I select the separating equilibrium when it exists.²

2. Alternatively, Harrington (1992, 265–7) adapts the equilibrium refinement of *announcement-proofness* (Matthews, Okuno-Fujiwara, and Postlewaite 1991) to a setting with multiple senders and receivers. It is straightforward to demonstrate that this refinement also selects the separating equilibrium presently.

B Endogenous capacity extension

I endogenize each ψ_I , allowing group I to choose to increase it from an initial value. Building on the results of the baseline model, I demonstrate that a weaker group may decline a free increase in capacity.

Preliminaries

In Stage 2, the baseline model plays out as before. In Stage 1, each $I \in \{A, B\}$ starts with an initial level of capacity $\underline{\psi}_I$. At no exogenous cost, I may later choose to increase ψ_I up to a maximum of $\bar{\psi}_I$ (but may not decrease it).

Sequence of moves

The sequence of moves is as in the baseline model, except preceding them is the following:

Stage 1

1. A selects its capacity $\psi_A \in [\underline{\psi}_A, \bar{\psi}_A]$.
2. B selects its capacity $\psi_B \in [\underline{\psi}_B, \bar{\psi}_B]$.

Subsequent moves shall collectively comprise Stage 2.

Utility functions

In Stage 2, P , A , and B shall have the same utility functions as before. In Stage 1, A and B shall have the following utility functions (P 's Stage 1 utility is inconsequential):

$$U_A^1(x) = -x,$$

$$U_B^1(x) = x.$$

Assumptions

The following assumption concerns the initial capacity of the groups:

Assumption B.1 (Initial group capacity). $\underline{\psi}_A < (1 - \phi)\underline{\psi}_B$.

Corresponding to the case of interest, this simply states that A starts off with lower capacity compared to B , such that only the pooling equilibrium is admitted.

Next, I assume the following:

Assumption B.2 (Intermediate initial capacity for B). $(1 - \phi)\bar{\psi}_A < \underline{\psi}_B < \frac{1}{1 - \phi}\bar{\psi}_A$.

The first part of this, $(1 - \phi)\bar{\psi}_A < \underline{\psi}_B$, simply states that no matter B 's choice of investment, A cannot induce pooling by becoming sufficiently higher-capacity than B . The second part of this, $\underline{\psi}_B < \frac{1}{1 - \phi}\bar{\psi}_A$, ensures non-triviality; it would otherwise be impossible for any strategy profile to lead to separation in equilibrium.

Finally, I assume the following:

Assumption B.3 (High potential capacity for B). $\frac{1}{1 - \phi}\bar{\psi}_A < \bar{\psi}_B$.

This simply states that no matter how much A invests, B can always induce pooling with sufficient investment. Results are similar without this assumption, but it greatly simplifies the analysis while corresponding substantively to the case of interest.

Summary

The exogenous parameters are $\underline{\psi}_A$, $\bar{\psi}_A$, $\underline{\psi}_B$, $\bar{\psi}_B$, p , ϕ , ψ_A , and ψ_B . The endogenous choices are ψ_A , ψ_B , m , s_A , s_B , and x . The random variable is σ . As a sequential game of imperfect information, the natural equilibrium concept is perfect Bayesian equilibrium (PBE). I continue to apply the equilibrium selection criterion described previously.

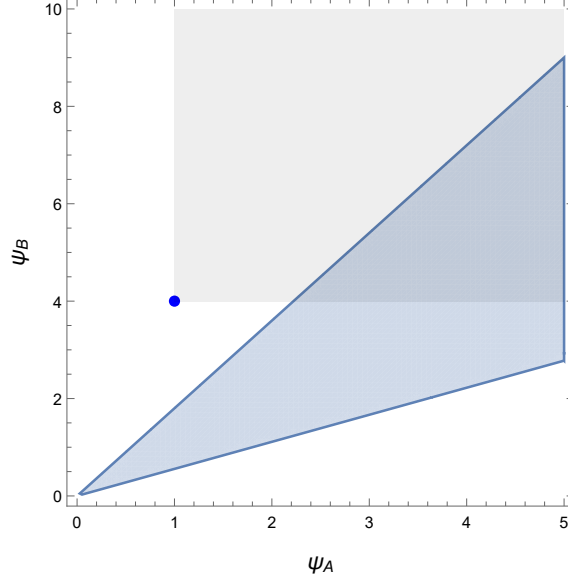


Figure B.1: An example fitting the assumptions. In particular, $\underline{\psi}_A = 1$, $\underline{\psi}_B = 4$, $\bar{\psi}_A = 5$, $\bar{\psi}_B = 10$, and $\phi = 4/9$. As before, the cone is the region in which separation occurs. The dot shows initial capacity, and the rectangle shows the set of points to which players may move capacity.

Discussion

I comment briefly on the assumptions. First, consider the order of moves. Allowing A to move first corresponds to the backlash dynamics that I explore. The question is, in anticipation of a higher-capacity group's strategic response, how does a lower-capacity group make decisions about building its capacity? The assumed order of moves fits this question.

Next, the assumption that increasing capacity is free only strengthens the results. Strikingly, we shall see that A may still decline to do so.

Finally, consider the utility functions. In Stage 2, A and B incur a cost of supporting P . Yet in Stage 1, A and B are unconcerned with these future costs. This can be justified substantively. One can imagine the groups in Stage 1 as representing different actors compared to those in Stage 2. Donors or activists making decisions about how to build their organizations may care about policy but not about the effort that bureaucrats in the future will have to exert. Alternatively, the costs of granting support can capture a notion

of constraint at the moment that it is granted rather than a source of negative utility to an institutional designer. While this assumption simplifies the analysis, it also allows us to continue to focus on the substantively interesting question of how policy actually moves.

Analysis

Stage 2 plays out as before. In Stage 1, there are three cases. Under pooling, when $p < \frac{1-\phi}{2-\phi}$, only B supports (*Case 1*), when $\frac{1-\phi}{2-\phi} < p < \frac{1}{2-\phi}$, both support (*Case 2*), and when $\frac{1}{2-\phi} < p$, only A supports (*Case 3*).

A key observation is that once A has made a choice of ψ_A , only two things can be optimal for B : choose ψ_B just small enough such that a separating equilibrium continues to be possible, or choose ψ_B as large as possible. In Cases 1 and 2, which option B prefers is a function of ψ_A (while in Case 3, B grants zero support under pooling, so that its only consideration in selecting ψ_B is which equilibrium it wishes to induce; we shall see that this is not a function of ψ_A). For a small value of ψ_A , B would need to forgo a large potential increase in ψ_B to maintain separation. As ψ_A increases, though, this sacrifice diminishes, and setting $\psi_B = \frac{1}{1-\phi}\psi_A$ (the largest value of ψ_A compatible with separation) becomes relatively more attractive. This is summarized in the following lemma:

Lemma B.1 (*B's best response*). *Suppose that Case 1 or 2 holds. There exists a threshold value of ψ_A , call it $\tilde{\psi}_A$, such that $\psi_A \leq \tilde{\psi}_A$ implies that B will induce pooling by setting $\psi_B = \bar{\psi}_B$, while $\psi_A > \tilde{\psi}_A$ implies that B will induce separation by setting $\psi_B = \frac{1}{1-\phi}\psi_A$.*

Suppose instead that Case 3 holds. Then B either always prefers pooling or always prefers separation irrespective of ψ_A . If B always prefers pooling, it sets $\psi_B = \bar{\psi}_B$. If B always prefers separation, it sets $\psi_B = \frac{1}{1-\phi}\psi_A$.

See Figure B.2 for an illustration of this result.³ Effectively, when ψ_A is chosen to be small,

3. To guarantee equilibrium existence, and because A can move ψ_A rightward or leftward from $\tilde{\psi}_A$ by any

B would need to set ψ_B much smaller than $\bar{\psi}_B$ to allow for separation, i.e. $\frac{1}{1-\phi}\psi_A$ is small. In such case, B does better to increase capacity as much as possible and give up on separation. Yet when ψ_A becomes larger, setting $\psi_B = \frac{1}{1-\phi}\psi_A$ becomes relatively more attractive, such that B eventually prefers to sacrifice some capacity to allow separation to happen.

Given B 's best response, we shall see that A 's optimum can be one of two things. First, A may seek to avoid separation by setting $\psi_A = \tilde{\psi}_A$. That is to say, A chooses the largest ψ_A compatible with pooling. Second, A may select ψ_A as large as possible, with either separation or pooling resulting depending on B 's best reply.

To help characterize equilibrium outcomes, I define cutoff values of p . Letting

$$T_p \equiv \frac{\bar{\psi}_A - \sqrt{t\bar{\psi}_A \left(\bar{\psi}_A - 4(1-\phi)^2 \underline{\psi}_B \right)}}{2(1-\phi)^2(2-\phi)\underline{\psi}_B} + \frac{1-\phi}{2-\phi},$$

I shall say that p is *low* when $p < \frac{1}{2-\phi}$, *intermediate* when $\frac{1}{2-\phi} < p < \min \left\{ T_p, \frac{1-\phi(1-\phi)}{2-\phi(3-\phi)} \right\}$, *high* when $T_p < p < \frac{1-\phi(1-\phi)}{2-\phi(3-\phi)}$, and *very high* when $\frac{1-\phi(1-\phi)}{2-\phi(3-\phi)} < p$. These regions are illustrated in Figure B.3. We are now ready for the following result:

Proposition B.1. *When p is low, A sets $\psi_A = \max\{\tilde{\psi}_A, (1-\phi)\underline{\psi}_B\}$, B sets $\psi_B = \bar{\psi}_B$, and pooling occurs. When p is intermediate, A sets $\psi_A = (1-\phi)\underline{\psi}_B$, B sets $\psi_B = \bar{\psi}_B$, and pooling occurs. When p is high, A sets $\psi_A = \bar{\psi}_A$, B sets $\psi_B = \frac{1}{1-\phi}\bar{\psi}_A$, and separation occurs. Finally, when p is very high, A sets $\psi_A = \bar{\psi}_A$, B sets $\psi_B = \bar{\psi}_B$, and pooling occurs.*

When p is low, A holds back on increasing ψ_A too far because it fears the consequences of separation. This is because p is simply too small, such that when friends and enemies can be identified, this more often benefits the higher-capacity B .

Next, when p is intermediate, B always wants to separate: it grants zero support under

 $\epsilon > 0$, I assume that A can break B 's indifference whichever way A prefers when $\psi_A = \tilde{\psi}_A$.

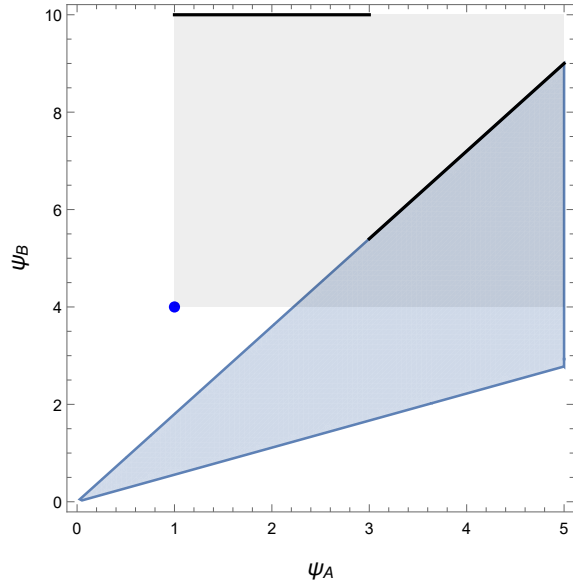


Figure B.2: Maintaining the parametric assumptions of Figure B.1 and fixing $p = \frac{207}{700}$ (so Case 1 holds), the black line is B 's optimal choice of ψ_B given ψ_A . The discontinuity is at $\tilde{\psi}_A = 3$.

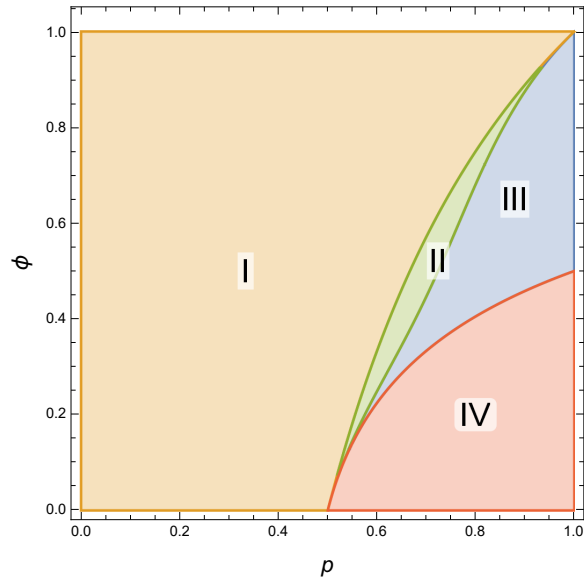


Figure B.3: In regions I, II, III, and IV, p is low, intermediate, high, and very high, respectively. Very high p coincides with separation strongly favoring A . In this example, $\bar{\psi}_A = \underline{\psi}_B = 8$.

pooling, while p is tilted enough in A 's favor that it grants positive support. If separation were instead to occur, the higher-capacity B would identify and support more friends than A would like, relative to A 's benefit of identifying its own friends.

Next, when p is high, B still always wants to separate. What has changed is A 's calculation. Now, p has become sufficiently large such that A 's benefit of identifying its friends improves relative to the cost of B being able to identify its friends. While B still does better under separation, it has become relatively attractive to A compared to the alternative of keeping ψ_A so small that for B it is infeasible to induce separation.

Finally, when p is very high, separation strongly favors A in the sense defined above. Large p and small ϕ means that most politicians are likely to be A 's friend. Yet without the ability to identify friends or grant support that can only be used for agreeable purposes, there is a high potential for A 's support to be repurposed. Therefore, B always wants to induce pooling, so both players increase their capacity as far as possible.

A comparative static implication we thus see is that increasing p sufficiently may make it larger than T_p , implying that A comes to prefer separation. That is to say, when A is more likely to identify a friend, it becomes more valuable for it to try to do so. Of course, increasing p too much may therefore lead B to induce pooling. I additionally find the following:

Proposition B.2 (Comparative statics). *The measure of ϕ in which separation occurs is increasing in $\bar{\psi}_A$ and decreasing in $\underline{\psi}_B$.*

These comparative statics essentially reflect a change in various forms of relative capacity of A compared to B . When A 's maximum potential capacity decreases, separation becomes less desirable to A . And when B 's initial capacity is greater, this gives A room to increase its capacity more while still not triggering separation, making pooling relatively attractive. In summary, then, increasing B 's relative current and potential capacity leads A to be increasingly wary of choosing to increase its own capacity to the maximum that is feasible.

C Group selection of complementarity extension

I extend the baseline model to examine how groups might endogenously choose complementarity ϕ . I therefore relax the assumption that there is a common value of ϕ and instead allow it to be specific to each player, i.e. ϕ_I is the fraction of I 's support that cannot be repurposed, with $I \in \{A, B\}$. Additionally, selection of each ϕ_I occurs simultaneously before the baseline model plays out.

Formal definition

Preliminaries are as in the baseline model, except an endogenously chosen fraction ϕ_I of the support granted by group $I \in \{A, B\}$, must either be used to move policy in the specified direction or disposed. The sequence of moves is as before, except preceding them is the following:

Stage 1

1. Each group $I \in \{A, B\}$ simultaneously selects $\phi_I \in [\underline{\phi}_I, \bar{\phi}_I]$.

Subsequent moves shall collectively comprise Stage 2. Utility functions are as in the endogenous capacity extension. Assumption ?? is maintained. Next, to analyze a non-trivial case, I assume the following:

Assumption C.1 (Non-triviality). $\underline{\phi}_B \leq 1 - \frac{\psi_A}{\psi_B} < \bar{\phi}_B$.

This ensures that B (who we shall see holds the keys to separation) actually has a choice of inducing pooling or separation.

Summary

The exogenous parameters are p , $\underline{\phi}_A$, $\underline{\phi}_B$, $\bar{\phi}_A$, $\bar{\phi}_B$, ψ_A , and ψ_B . The endogenous choices are ϕ_A , ϕ_B , m , s_A , s_B , and x . The random variable is σ . As a sequential game of imperfect information, the natural equilibrium concept is perfect Bayesian equilibrium (PBE). I focus exclusively on pure-strategy PBE.

Analysis

In Stage 2, from an analysis that is analogous to that in the baseline model, we have

$$s_A^*(\mu; \phi_A) = \max \{ (-(1 - \phi_A) + \mu(2 - \phi_A))\psi_A, 0 \},$$
$$s_B^*(\mu; \phi_B) = \max \{ (1 - \mu(2 - \phi_B))\psi_B, 0 \}.$$

Then the conditions required by a separating equilibrium are as follows:

$$(1) \quad (1 - \phi_B)\psi_B \leq \psi_A,$$
$$(2) \quad (1 - \phi_A)\psi_A \leq \psi_B.$$

Because $\psi_A \leq \psi_B$, it is immediate that Condition 2 is always satisfied. That is to say, A 's choice of ϕ_A never determines whether the separating equilibrium is possible. We therefore see that it is always a weakly dominant strategy for A to select ϕ_A as large as possible. Whether we are in the separating or pooling equilibrium is in B 's hands, with separation occurring whenever ϕ_B is selected to satisfy Condition 1.⁴ Analogous to A 's choice, then, selecting $\phi_B = 1 - \frac{\psi_A}{\psi_B}$ weakly dominates any $\phi_B < 1 - \frac{\psi_A}{\psi_B}$. That is to say, if pooling is going to happen, better that ϕ_B be as large as possible. This is summarized in the following

4. To ensure that an equilibrium exists, I assume that on the boundary at which the separating equilibrium comes into existence, the pooling equilibrium is still played.

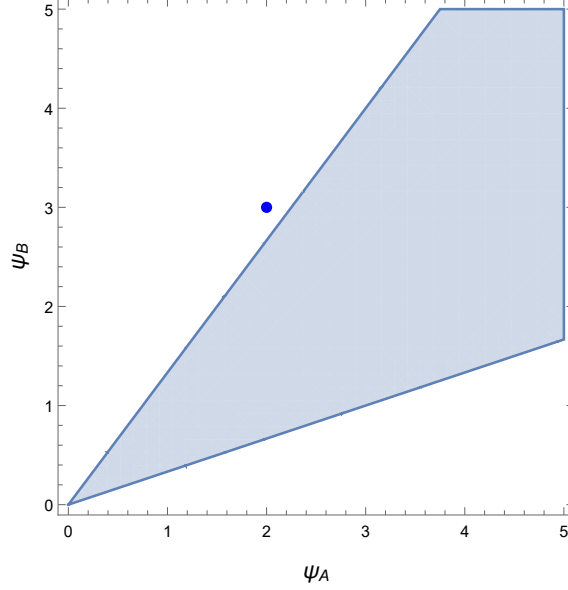


Figure C.1: An example in which $\psi_A = 2$, $\psi_B = 3$, $\phi_A = 1/4$, $\phi_B = 2/3$, and pooling occurs. Because B can move the upper boundary of the cone, $\psi_B > \psi_A$ implies that B is in control of whether separation is possible.

lemma:

Lemma C.1 (Player strategies). *It is a weakly dominant strategy for A to set $\phi_A = \bar{\phi}_A$. For B , setting $\phi_B = 1 - \frac{\psi_A}{\psi_B}$ weakly dominates setting ϕ_B smaller.*

However, B also realizes that ϕ_B even larger may bring about separation, at which point the specific choice of ϕ_B otherwise does not matter. Therefore, in determining the equilibrium, I consider B 's two candidates for optimal play. First, B can select the largest ϕ_B that is still compatible with pooling. Second, B can select anything larger than that to induce the separating equilibrium. Define

$$T'_p \equiv \frac{\psi_A(1 - (2 - \bar{\phi}_A)\bar{\phi}_A) + \psi_B}{\psi_A(2 - \bar{\phi}_A)^2}.$$

We are now ready for the main result of this analysis:

Proposition C.1 (Equilibrium outcomes). *When $p \leq T'_p$, there exists a PBE in which A sets $\phi_A = \bar{\phi}_A$, B sets $\phi_B = \bar{\phi}_B$, and separation occurs. When $p \geq T'_p$, there exists a PBE in which A sets $\phi_A = \bar{\phi}_A$, B sets $\phi_B = 1 - \frac{\psi_A}{\psi_B}$, pooling occurs, and $s_B^* = 0$.*

A small value of p , then, means that B prefers separation. That is, when P is not overwhelmingly likely to be aligned with A , it benefits B 's policy goals more for both players to be able to identify their friends and enemies. And in keeping with B having more capacity than A , notice that $T'_p \geq 1/2$, so even if P is somewhat more likely to be aligned with A , it may still benefit B to separate. When p is large, it is remarkable that B can induce pooling by setting ϕ_B sufficiently small but then does not end up having to grant any support at all. The mere presence of its superior, nonspecific resources proves tempting enough to opposition politicians such as to destroy any possibility for a separating equilibrium, thus preventing A from being able to identify its friends and enemies.

I now look at comparative statics on T'_p . An increase in T'_p means separation becomes more desirable for B , while a decrease means that pooling becomes more desirable:

Proposition C.2 (Comparative statics). *T'_p increases in $\bar{\phi}_A$ and ψ_B and decreases in ψ_A .*

Intuitively, as B 's capacity increases more relative to A , separation comes to benefit B more. Finally, as $\bar{\phi}_A$ increases, A is able to do increasingly well under pooling, eventually inducing B to want to bring about separation.

D Politician selection of complementarity extension

Suppose that before the baseline model plays out, the politician can determine the value of ϕ , with a value admitting separation feasible. To rule out a trivial and implausible case, assume that the choice of ϕ is observable. We then have a multi-stage signaling game, to which I apply the *never dissuaded once convinced* refinement (Osborne and Rubenstein 1990, 96–8). I find that one politician type must strictly prefer separation.⁵ That politician type may select a corresponding value of ϕ . Then the other type can either select a different value of ϕ , separating immediately, or the same value of ϕ , only deferring separation until later. Thus, separation always occurs. However, there may still be a role for increasing a weaker group’s capacity in enabling backlash: the minimum value of ϕ admitting separation in the baseline model is a decreasing function of ψ_A when $\psi_A < \psi_B$.

Formal definition

Preliminaries are as in the baseline model. The sequence of moves is as follows:

Stage 1

1. P ’s type $\sigma \in \{-1, 1\}$ is drawn and revealed to P . With probability $p \in (0, 1)$, $\sigma = -1$ and P agrees with A . Otherwise, P agrees with B .
2. P selects $\phi \in [\underline{\phi}, \bar{\phi}]$.

Moves 2-5 from the baseline model shall comprise Stage 2. Utility functions are as in the baseline model. Assumption ?? is maintained. Next, I assume the following:

Assumption D.1. $\underline{\phi} \leq 1 - \frac{\psi_A}{\psi_B} < \bar{\phi}$.

5. This is distinct from the result of Lemma A.1, which held ϕ fixed.

This ensures that P has a choice of values of ϕ that, given pooling in Stage 1, may correspond either to pooling or separation in Stage 2.

Summary

The exogenous parameters are p , $\underline{\phi}$, $\bar{\phi}$, ψ_A , and ψ_B . The endogenous choices are ϕ , m , s_A , s_B , and x . The random variable is σ . As a sequential game of imperfect information, the natural equilibrium concept is perfect Bayesian equilibrium (PBE). I focus exclusively on pure-strategy PBE. I further apply the *never dissuaded once convinced* refinement (Osborne and Rubenstein 1990, 96–8): once a group assigns probability one to any type, it does not engage in any further updating regardless of P 's subsequent actions.⁶

Analysis

Analysis of Stage 2 is analogous to that in the baseline model. In the overall game, pooling may only occur if both types of P would select the same value of ϕ and that value implies pooling in the baseline model.

Recalling Assumption 1, by Proposition 1 a value of ϕ implies pooling in the baseline model if and only if $(1 - \phi)\psi_B \leq \psi_A$. This can be rearranged as $\phi \leq 1 - \frac{\psi_A}{\psi_B}$. Considering this along with expressions for optimal support s_A^* and s_B^* in the baseline model, the following cases yield (presently setting aside the possibility of separation in Stage 1):

6. In the present setting, this appears to be a more reasonable refinement than that of Vincent (1998). Suppose instead that groups continue to update after the selection of ϕ . Consider the case in which the prior probability of a politician aligned with group I is small, and the capacity of I is low. If the politician aligned with I does appear, she would be able to select a small value of ϕ to ensure that she receives support from group J under a subsequent pooling equilibrium, which may exceed the support that she would receive from I under a subsequent separating equilibrium (this does not contradict Lemma A.1, which relied on ϕ being fixed). But then it would have been sensible for both groups to continue to rely on the politician's selection of ϕ small to infer her type, given that only the politician aligned with I has an incentive to do so.

$$\begin{aligned}
p \leq 1/2 : & \begin{cases} \phi \leq \min \left\{ 1 - \frac{p}{1-p}, 1 - \frac{\psi_A}{\psi_B} \right\} & \text{Pooling; } 0 = s_A^* < s_B^* \\ 1 - \frac{p}{1-p} < \phi \leq 1 - \frac{\psi_A}{\psi_B} & \text{Pooling; } 0 < s_A^*, s_B^* \\ 1 - \frac{\psi_A}{\psi_B} < \phi & \text{Separation} \end{cases} , \\
p \geq 1/2 : & \begin{cases} \phi \leq \min \left\{ 2 - \frac{1}{p}, 1 - \frac{\psi_A}{\psi_B} \right\} & \text{Pooling; } 0 = s_B^* < s_A^* \\ 2 - \frac{1}{p} < \phi \leq 1 - \frac{\psi_A}{\psi_B} & \text{Pooling; } 0 < s_A^*, s_B^* \\ 1 - \frac{\psi_A}{\psi_B} < \phi & \text{Separation} \end{cases} .
\end{aligned}$$

I reach the following result:

Proposition D.1. *One type of P strictly prefers to select a value of ϕ that implies separation in Stage 2, such that separation in the overall game is guaranteed.*

E Formal proofs

Proof of Proposition 1. Denote P of type $\sigma = -1$ as P_L and P of type $\sigma = 1$ as P_R . A separating equilibrium takes the following form:

- Strategy for P_L : set $m = L$.
- Strategy for P_R : set $m = R$.
- Strategy for $I \in \{A, B\}$: grant $s_I^*(1)$ if $m = L$ and grant $s_I^*(0)$ otherwise.
- Beliefs: $\mu_L = 1$ and $\mu_R = 0$.

Holding fixed the behavior of groups, I check when both politician types have no incentive to deviate. The utility to P_L from setting $m = L$ is ψ_A while the utility to P_L from misrepresenting and setting $m = R$ is $(1 - \phi)\psi_B$. Then the utility of being truthful exceeds that of misrepresenting when $\frac{\psi_A}{\psi_B} \geq 1 - \phi$. Next, the utility to P_R from setting $m = R$ is ψ_B while the utility to P_R from misrepresenting and setting $m = L$ is $(1 - \phi)\psi_A$. Then the utility of being truthful exceeds that of misrepresenting when $\frac{\psi_B}{\psi_A} \geq 1 - \phi$ or equivalently, $\frac{\psi_A}{\psi_B} \leq \frac{1}{1 - \phi}$. Taken together, this is $1 - \phi \leq \frac{\psi_A}{\psi_B} \leq \frac{1}{1 - \phi}$. Given this, beliefs are consistent. Finally, $s_I^* : I \in \{A, B\}$ was already constructed to be optimal. \square

Proof of Proposition 2. Let $\mathbb{E}^{\tilde{\mathbb{S}}}[x|\psi_A, \psi_B]$ denote expected policy under a perfect information baseline and $\mathbb{E}^{\tilde{\mathbb{P}}}[x|\psi_A, \psi_B]$ denote expected policy when P is banned from communicating. Notice of course that $\mathbb{E}^{\tilde{\mathbb{P}}}[x|\psi_A, \psi_B] = \mathbb{E}^{\mathbb{P}}[x|\psi_A, \psi_B]$, and $\mathbb{E}^{\tilde{\mathbb{S}}}[x|\psi_A, \psi_B] = \mathbb{E}^{\mathbb{S}}[x|\psi_A, \psi_B]$ when the separating equilibrium is supportable.

Expected policy under the perfect information benchmark is as follows:

$$\mathbb{E}^{\tilde{\mathbb{S}}}[x|\psi_A, \psi_B] = p(-\psi_A) + (1 - p)\psi_B.$$

Expected policy under the no-communication benchmark is as follows:

$$\mathbb{E}^{\tilde{\mathbb{P}}}[x|\psi_A, \psi_B] = \begin{cases} (1 - p(2 - \phi))^2 \psi_B & p \leq \frac{1-\phi}{2-\phi} \\ -(1 - p(2 - \phi) - \phi)^2 \psi_A + (1 - p(2 - \phi))^2 \psi_B & \frac{1-\phi}{2-\phi} \leq p \leq \frac{1}{2-\phi} \\ -(1 - p(2 - \phi) - \phi)^2 \psi_A & \frac{1}{2-\phi} \leq p \end{cases}.$$

First observe that $\frac{\partial}{\partial \psi_A} \mathbb{E}^{\tilde{\mathbb{S}}}[x|\psi_A, \psi_B] = -p$. Next,

$$\frac{\partial}{\partial \psi_A} \mathbb{E}^{\tilde{\mathbb{P}}}[x|\psi_A, \psi_B] = \begin{cases} 0 & p \leq \frac{1-\phi}{2-\phi} \\ -(1 - p(2 - \phi) - \phi)^2 & \text{o/w} \end{cases},$$

so the proposition follows. \square

Proof of Proposition 3. We must ask the conditions under which $\mathbb{E}^{\tilde{\mathbb{S}}}[x|(1 - \phi)\psi_B, \psi_B] > \mathbb{E}^{\tilde{\mathbb{P}}}[x|(1 - \phi)\psi_B, \psi_B]$. There are of course three cases: $p < \frac{1-\phi}{2-\phi}$, $\frac{1-\phi}{2-\phi} \leq p < \frac{1}{2-\phi}$, and $\frac{1}{2-\phi} \leq p$. In Cases 1 and 2, reduction of the system of inequalities consisting of the initial hypothesis and case (and basic initial assumptions of the model) demonstrates that the former always holds. In Case 3, the same process demonstrates that the initial hypothesis holds if and only if $p < \frac{1-\phi(1-\phi)}{2-\phi(3-\phi)}$. Because Cases 1 and 2 always imply $p < \frac{1-\phi(1-\phi)}{2-\phi(3-\phi)}$, the proposition follows. \square

Proof of Proposition 4. The condition

$$\lim_{\psi_A \uparrow (1-\phi)\psi_B} \mathbb{E}[x] < \lim_{\psi_A \uparrow \frac{1}{1-\phi}\psi_B} \mathbb{E}[x]$$

is equivalent to

$$\mathbb{E}^{\mathbb{P}}[x|(1-\phi)\psi_B, \psi_B] < \mathbb{E}^{\mathbb{S}}\left[x\left|\frac{1}{1-\phi}\psi_B, \psi_B\right.\right].$$

There are three cases to consider: $p < \frac{1-\phi}{2-\phi}$, $\frac{1-\phi}{2-\phi} \leq p < \frac{1}{2-\phi}$, and $\frac{1}{2-\phi} \leq p$. In Cases 2 and 3, reduction of the system of inequalities consisting of the initial hypothesis and case (and basic initial assumptions of the model) demonstrates that the former never holds. In Case 1, the same process demonstrates that the initial hypothesis holds if and only if $p < \frac{1-2\phi}{2-\phi(3-\phi)}$. This is precisely the definition of separation strongly favoring B . Because each step in the chain of logical relationships was biconditional, the proposition follows. \square

Proof of Lemma A.1. Denote P of type $\sigma = -1$ as P_L and P of type $\sigma = 1$ as P_R . Let superscript \mathbb{S} denote separation and superscript \mathbb{P} denote pooling. Expected utilities from separation are $\mathbb{E}U_{P_L}^{\mathbb{S}} = \psi_A$ and $\mathbb{E}U_{P_R}^{\mathbb{S}} = \psi_B$. Expected utilities from pooling are

$$\begin{aligned}\mathbb{E}U_{P_L}^{\mathbb{P}} &= s_A^*(p) + (1-\phi)s_B^*(p) \\ &= \max\left\{\left(- (1-\phi) + p(2-\phi)\right)\psi_A, 0\right\} + (1-\phi)\left\{\left(1-p(2-\phi)\right)\psi_B, 0\right\}, \\ \mathbb{E}U_{P_R}^{\mathbb{P}} &= (1-\phi)s_A^*(p) + s_B^*(p) \\ &= (1-\phi)\max\left\{\left(- (1-\phi) + p(2-\phi)\right)\psi_A, 0\right\} + \max\left\{\left(1-p(2-\phi)\right)\psi_B, 0\right\}.\end{aligned}$$

Recall the initial assumptions that $\psi_A \leq \psi_B$, $0 \leq t < 1$, and $0 < p < 1$. Next, by Proposition 1 and the hypothesis that separation is possible, it follows that $\psi_B \leq \frac{1}{1-\phi}\psi_A$. There are six possible cases: the Cartesian product of types of P with contribution behavior under pooling ($p \leq \frac{1-\phi}{2-\phi}$ and only receiver B contributes, $\frac{1-\phi}{2-\phi} < p < \frac{1}{2-\phi}$ and both receivers contribute, and $\frac{1}{2-\phi} \leq p$ and only receiver A contributes). In each case, application of the assumptions along with $\psi_B \leq \frac{1}{1-\phi}\psi_A$ implies that $\mathbb{E}U_{P_I}^{\mathbb{S}} > \mathbb{E}U_{P_I}^{\mathbb{P}}$, with I the corresponding type of P . \square

Proof of Proposition A.1. Follows from Proposition 4 of Farrell and Gibbons (1989) taken together with the present Lemma A.1 (which substitutes for their Proposition 2, allowing application of the logic of Proposition 4 to the present case of continuous actions). \square

Proof of Lemma B.1. Notice first that within pooling or separation, only the largest ψ_B compatible with said equilibrium can be optimal.

In any Case, if B cannot induce separation (i.e. $\psi_A < (1 - \phi)\underline{\psi}_B$), it is clear that setting $\psi_B = \bar{\psi}_B$ is optimal. Suppose instead that $\psi_A \geq (1 - \phi)\underline{\psi}_B$. Then B 's expected utility from separation (setting $\psi_B = \frac{1}{1-\phi}\psi_A$) is $\mathbb{E}U_B^S = \frac{((1-p(2-\phi))\psi_A}{1-\phi}$.

Suppose that Case 1 holds. B 's expected utility from pooling (setting $\psi_B = \bar{\psi}_B$) is $\mathbb{E}U_B^P = (1-p(2-\phi))^2\bar{\psi}_B$. Then $\mathbb{E}U_B^S \geq \mathbb{E}U_B^P$ implies (and is implied by) $\psi_A \geq (1-p(2-\phi))(1-\phi)\bar{\psi}_B$. Because $\psi_A < (1-\phi)\underline{\psi}_B$ makes separation infeasible for B so that setting $\psi_B = \bar{\psi}_B$ must be optimal, it therefore follows that

$$\tilde{\psi}_A = \max\{(1-p(2-\phi))(1-\phi)\bar{\psi}_B, (1-\phi)\underline{\psi}_B\}.$$

Now suppose that Case 2 holds. B 's expected utility from pooling (setting $\psi_B = \bar{\psi}_B$) is

$$\mathbb{E}U_B^P = -(1-p(2-\phi)-\phi)^2\psi_A + (1-p(2-\phi))^2\bar{\psi}_B.$$

Then $\mathbb{E}U_B^S \geq \mathbb{E}U_B^P$ implies (and is implied by)

$$(3) \quad \psi_A \geq \frac{(1-p(2-\phi))^2(1-\phi)}{(1-p)(2-\phi)(1-p(2-\phi)(1-\phi)-t(1-\phi))}\bar{\psi}_B,$$

so analogously to Case 1 it follows that

$$\tilde{\psi}_A = \max\left\{\frac{(1-p(2-\phi))^2(1-\phi)}{(1-p)(2-\phi)(1-p(2-\phi)(1-\phi)-t(1-\phi))}\bar{\psi}_B, (1-\phi)\underline{\psi}_B\right\}.$$

Suppose that Case 3 holds. B 's expected utility from pooling (setting $\psi_B = \bar{\psi}_B$) is

$$\mathbb{E}U_B^{\mathbb{P}} = -(1 - p(2 - \phi) - \phi)^2 \psi_A.$$

Then $\mathbb{E}U_B^{\mathbb{S}} \geq \mathbb{E}U_B^{\mathbb{P}}$ is equivalent to $p \leq \frac{1-\phi(1-\phi)}{2-\phi(3-\phi)}$, a condition unrelated to ψ_A . \square

Proof of Proposition B.1. Given what we know from Lemma B.1 about B 's choice of ψ_B , A 's expected utility from separation in any Case is $\mathbb{E}U_A^{\mathbb{S}} = \frac{(p(2-\phi)-1)\psi_A}{1-\phi}$. Then $\frac{d\mathbb{E}U_A^{\mathbb{S}}}{d\psi_A} = \frac{p(2-\phi)-1}{1-\phi}$, so it follows that $\frac{d\mathbb{E}U_A^{\mathbb{S}}}{d\psi_A} < 0$ in Cases 1 and 2, and $\frac{d\mathbb{E}U_A^{\mathbb{S}}}{d\psi_A} > 0$ in Case 3. Therefore, I conclude that if A were to induce separation, in Cases 1 and 2, A would set $\psi_A = \tilde{\psi}_A$ (if B were ever so averse to separation such that $\tilde{\psi}_A > \bar{\psi}_A$, then A simply cannot induce separation and sets $\psi_A = \bar{\psi}_A$). In Case 3, A would set $\psi_A = \bar{\psi}_A$.

Note also that whenever A desires pooling, A sets ψ_A as large as is compatible with this.

Suppose that Case 1 or 2 holds. Suppose first that $\tilde{\psi}_A \geq (1 - \phi)\underline{\psi}_B$. Then at $\psi_A = \tilde{\psi}_A$, A can induce either pooling or separation. But recall that $\tilde{\psi}_A$ is defined as the value of ψ_A such that B is indifferent between pooling and separation, and because the game in Stage 1 is constant-sum, this implies A 's indifference between pooling and separation (of course B would not prefer separation with ψ_A even greater). I conclude that A sets $\psi_A = \tilde{\psi}_A$ and can assume that when indifferent, A induces pooling.⁷ Suppose instead that $\tilde{\psi}_A < (1 - \phi)\underline{\psi}_B$. Because it was just demonstrated that A 's Stage 1 utility under separation is strictly decreasing in ψ_A , this implies that, since at $\tilde{\psi}_A$ A is indifferent between pooling and separation, at $(1 - \phi)\underline{\psi}_B$ A must strictly prefer pooling. Then A sets $\psi_A = \tilde{\psi}_A$ and induces pooling.

Suppose that Case 3 holds. Suppose that B prefers pooling, i.e. $p \geq \frac{1-\phi(1-\phi)}{2-\phi(3-\phi)}$. Then B always sets $\psi_B = \bar{\psi}_B$ regardless of ψ_A , so A sets $\psi_A = \bar{\psi}_A$. Suppose instead that B

7. A lexicographic preference relation for A by which A first maximizes what is presently given as its Stage 1 utility function and next minimizes its Stage 2 cost of granting support would yield this as the optimum.

always prefers separation, i.e. $p \leq \frac{1-\phi(1-\phi)}{2-\phi(3-\phi)}$. Then A can either induce pooling by setting $\psi_A = (1-\phi)\underline{\psi}_B$ or induce separation by setting $\psi_A = \bar{\psi}_A$. A 's utility from pooling is

$$\mathbb{E}U_A^{\mathbb{P}}((1-\phi)\underline{\psi}_B) = (1-p(2-\phi)-\phi)^2(1-\phi)\underline{\psi}_B,$$

while its utility from separation is $\mathbb{E}U_A^{\mathbb{S}}(\bar{\psi}_A) = \frac{\bar{\psi}_A(p(2-\phi)-1)}{1-\phi}$. Then $\mathbb{E}U_A^{\mathbb{S}} \geq \mathbb{E}U_A^{\mathbb{P}}$ implies (and is implied by)

$$(4) \quad \bar{\psi}_A \geq \frac{(1-p(2-\phi)-\phi)^2(1-\phi)^2}{p(2-\phi)-1}\underline{\psi}_B.$$

Then clearly A induces separation by setting $\psi_A = \bar{\psi}_A$ if this condition holds and induces pooling by setting $\psi_A = (1-\phi)\underline{\psi}_B$ otherwise. Recalling that we are in Case 3 and B always prefers separation, the condition can be rearranged as

$$(5) \quad p \geq \frac{\bar{\psi}_A - \sqrt{t\bar{\psi}_A(\bar{\psi}_A - 4(1-\phi)^2\underline{\psi}_B)}}{2(1-\phi)^2(2-\phi)\underline{\psi}_B} + \frac{1-\phi}{2-\phi} (= T_p).$$

Examining the right-hand side of Condition 4, observe that whenever $\phi > 0$, it follows that

$$\lim_{p \downarrow \frac{1}{2-\phi}} \frac{(1-p(2-\phi)-\phi)^2(1-\phi)^2}{p(2-\phi)-1}\underline{\psi}_B = \infty.$$

implying that approaching the boundary of Case 3 from within the case, Condition 4 is never satisfied. Next, if $\phi = 0$, to be in Case 3 we must have $p \geq 1/2$. Given this, B is indifferent to separation rather than strictly dispreferring it (implying that A is indifferent) only when $p = 1/2$. These observations imply that the right-hand side of Condition 5 must be greater than or equal to $\frac{1}{2-\phi}$. The proposition follows. \square

Proof of Proposition B.2. The Condition 4 LHS increases in $\bar{\psi}_A$ and RHS increases in $\underline{\psi}_B$. \square

Proof of Lemma C.1. As discussed, A 's choice of ϕ_A cannot determine whether pooling or separation occurs. If pooling occurs, A 's Stage 1 expected utility is

$$\mathbb{E}U_A^{\mathbb{P}} = ((1-p)\phi_A - (1-2p))s_A^*(p; \phi_A) - (p\phi_B - (2p-1))s_B^*(p; \phi_B).$$

Suppose $p \leq 1/2$ and $\phi_A < \frac{1-2p}{1+p}$. Then $\frac{\partial \mathbb{E}U_A^{\mathbb{P}}}{\partial \phi_A} = 0$. Suppose instead that either $\phi_A > \frac{1-2p}{1+p}$ or $p \geq 1/2$ (or both). We have $\frac{\partial \mathbb{E}U_A^{\mathbb{P}}}{\partial \phi_A} = 2(1-p)((1-p)\phi_A - (1-2p))\psi_A > 0$. Then given that pooling occurs, $\phi_A = \bar{\phi}_A$ is always optimal. Given that separation occurs, A 's expected utility is not a function of ϕ_A and similarly, $\phi_A = \bar{\phi}_A$ is always optimal. A symmetric argument applies to B , except any $\phi_B > 1 - \frac{\psi_A}{\psi_B}$ leads to separation in Stage 2. \square

Proof of Proposition C.1. Analysis of the Stage 2 subgame is as before. Next, Lemma C.1 tells us 1. $\phi_A = \bar{\phi}_A$ is always optimal for A and 2. given that B chooses to induce pooling, the largest such value of ϕ_B is selected, namely $1 - \frac{\psi_A}{\psi_B}$. We are left to determine which of two candidates is optimal for B : pooling with $\phi_B = 1 - \frac{\psi_A}{\psi_B}$ or separation with $\phi_B = \bar{\phi}_B$.

Utility to B from separation is $\mathbb{E}U_B^{\mathbb{S}} = -p\psi_A + (1-p)\psi_B$. To determine utility to B from pooling, allow two cases: $p \leq 1/2$ and $p > 1/2$. Suppose first that $p \leq 1/2$. Then utility from pooling is

$$\mathbb{E}U_B^{\mathbb{P}} = \frac{(-p\psi_A + (1-p)\psi_B)^2}{\psi_B} - c_A^*(p; \bar{\phi}_A)((1-p)\bar{\phi}_A - (1-2p)).$$

Given the assumed constraints on possible parameter values, $\mathbb{E}U_B^{\mathbb{S}} \geq \mathbb{E}U_B^{\mathbb{P}}$ must follow.

Suppose instead that $p > 1/2$. Then utility from pooling is

$$\mathbb{E}U_B^{\mathbb{P}} = \frac{(-p\psi_A + (1-p)\psi_B)c_B^*(p; 1 - \frac{\psi_A}{\psi_B})}{\psi_B} - \psi_A((1-p)\bar{\phi}_A - (1-2p))^2.$$

Then $\mathbb{E}U_B^{\mathbb{S}} \geq \mathbb{E}U_B^{\mathbb{P}}$ implies (and is implied by) $p \leq T'_p$.

Suppose that $p \geq T'_p$ and B induces pooling. To see that $c_B^* = 0$, observe that $c_B^*(p; 1 - \frac{\psi_A}{\psi_B}) > 0$ implies $p < \frac{\psi_B}{\psi_A + \psi_B}$, which contradicts $p \geq T'_p$.

Finally, observing that $T'_p > 1/2$, I find that T'_p is always the threshold dividing the region of p in which the specified separating equilibrium exists from that in which the specified pooling equilibrium exists. \square

Proof of Proposition C.2. We have $\frac{\partial T'_p}{\partial \psi_A} = -\frac{\psi_B}{\psi_A^2(2-\bar{\phi}_A)^2} < 0$, $\frac{\partial T'_p}{\partial \psi_B} = \frac{1}{\psi_A(2-\bar{\phi}_A)^2} > 0$, and $\frac{\partial T'_p}{\partial \phi_A} = \frac{2(\psi_B - (1-\bar{\phi}_A)\psi_A)}{\psi_A(\bar{\phi}_A - 2)^3} > 0$. \square

Proof of Proposition D.1. Case 1: $p \leq 1/2$. Suppose that P_B selects $\phi \leq \min\left\{1 - \frac{p}{1-p}, 1 - \frac{\psi_A}{\psi_B}\right\}$.

Expected utility to P_B is

$$\mathbb{E}U_{P_B} = (\phi p + (1 - 2p))\psi_B,$$

which is maximized at $\phi = \min\left\{1 - \frac{p}{1-p}, 1 - \frac{\psi_A}{\psi_B}\right\}$. Suppose next that P_B selects $\phi \in \left(1 - \frac{p}{1-p}, 1 - \frac{\psi_A}{\psi_B}\right]$. Expected utility to P_B is

$$(6) \quad \mathbb{E}U_{P_B} = (1 - \phi)((1 - p)\phi + 2p - 1)\psi_A + (\phi p + (1 - 2p))\psi_B.$$

This has a critical point at

$$(7) \quad \phi^* = \frac{(2 - 3p)\psi_A + p\psi_B}{2(1 - p)\psi_A}.$$

The second derivative test demonstrates that this is globally concave in ϕ . But notice that $\phi^* > 1 - \frac{\psi_A}{\psi_B}$, such that if any value of $\phi \in \left[0, 1 - \frac{\psi_A}{\psi_B}\right]$ were optimal, it must be $1 - \frac{\psi_A}{\psi_B}$. Then we are left to compare expected utility from pooling at $1 - \frac{\psi_A}{\psi_B}$ to that from separation. The former is Expression 6 setting $\phi = 1 - \frac{\psi_A}{\psi_B}$. The latter is simply ψ_B . The latter is strictly greater than the former, such that separation is always strictly preferred.

Case 2: $p \geq 1/2$. Suppose that P_B selects $\phi \leq \min \left\{ 2 - \frac{1}{p}, 1 - \frac{\psi_A}{\psi_B} \right\}$. Expected utility to P_B is

$$\mathbb{E}U_{P_B} = (1 - \phi)((1 - p)\phi + 2p - 1)\psi_A.$$

This has a critical point at

$$\phi^* = \frac{2 - 3p}{2(1 - p)}.$$

The second derivative test demonstrates that this is globally concave in ϕ , but note for later that $\phi^* < 2 - \frac{1}{p}$ coincides with $p > 2 - \sqrt{2}$.

Suppose next that P_B selects $\phi \in \left(2 - \frac{1}{p}, 1 - \frac{\psi_A}{\psi_B} \right]$. Expected utility to P_B is as in Expression 6, and so the critical point is as in Expression 7. But as before, $\phi^* > 1 - \frac{\psi_A}{\psi_B}$.

We are left to perform three expected utility comparisons: separation vs. 1. pooling at $\phi = \frac{2-3p}{2(1-p)}$ (when $p > 2 - \sqrt{2}$), 2. pooling at $\phi = 2 - \frac{1}{p}$ (when $p \leq 2 - \sqrt{2}$), and 3. pooling at $1 - \frac{\psi_A}{\psi_B}$. The third comparison was already performed in Case 1, demonstrating that separation is strictly preferred. Performing the second comparison also shows that separation is strictly preferred. The first comparison implies that pooling is preferred if and only if $\psi_B \leq \frac{p^2}{4(1-p)}\psi_A$.

The final step, then, is to determine what P_A prefers to do when $\psi_B \leq \frac{p^2}{4(1-p)}\psi_A$ and $p > 2 - \sqrt{2}$. Suppose that P_A selects $\phi \leq \min \left\{ 2 - \frac{1}{p}, 1 - \frac{\psi_A}{\psi_B} \right\}$. Expected utility to P_A is

$$\mathbb{E}U_{P_A} = ((1 - p)\phi + 2p - 1)\psi_A,$$

which is clearly increasing in ϕ , implying a maximum at the upper corner. Suppose next

that P_A selects $\phi \in \left(2 - \frac{1}{p}, 1 - \frac{\psi_A}{\psi_B}\right]$. Expected utility to P_A is

$$(8) \quad \mathbb{E}U_{P_A} = (p - (1 - \phi)(1 - p))\psi_A + (1 - \phi)(1 - p(2 - \phi))\psi_B.$$

This has a critical point at

$$(9) \quad \phi^* = \frac{(1 - p)\psi_A + (3p - 1)\psi_B}{2p\psi_B}.$$

The second derivative test demonstrates that this is globally concave in ϕ . But notice that $\phi^* > 1 - \frac{\psi_A}{\psi_B}$, such that if any value of $\phi \in \left[0, 1 - \frac{\psi_A}{\psi_B}\right]$ were optimal, it must be $1 - \frac{\psi_A}{\psi_B}$. Then we are left to compare expected utility from pooling at $1 - \frac{\psi_A}{\psi_B}$ to that from separation. The former is Expression 8 setting $\phi = 1 - \frac{\psi_A}{\psi_B}$. The latter is simply ψ_A . The latter is strictly greater than the former, such that separation is always strictly preferred.

Then one type of P always strictly prefers separation. This can always be induced by selecting ϕ sufficiently large, such that failure to do so is informative in itself. \square

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