Online Appendix for
"The Public Meeting Paradox: How NIMBY-Dominated Public Meetings Can Enable New Housing"

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May 2024

For Online Publication

A Proofs and Additional Formal Results

Assumption A.1. The players' preferences are such that no neighbor sues over the developer's break-even project, but all neighbors sue over the developer's ideal project. Formally, $\hat{x}_i(\overline{c}) < \hat{x}_i(\underline{c}) < \hat{x}_D$ and $\underline{x} < x_1 < x_2 < \hat{x}_D$, where $x_1 = \tilde{x}(\overline{c})$ and $x_2 = \tilde{x}(\underline{c})$.

Lemma A.1. $|x_0 - \hat{x}_i(c_i)| < |\tilde{x}(c_i) - \hat{x}_i(c_i)|$.

Proof of Lemma A.1. The lemma states that reversion policy x_0 is closer to the ideal point $\hat{x}_i(c_i)$ then is the lawsuit threshold $\tilde{x}(c_i)$. Assume not, then by the symmetry of the utility function $v_i(x_0, c_i) < v_i(\tilde{x}(c_i), c_i)$ and the neighbor would strictly prefer not to sue for proposal $\tilde{x}(c_i)$ a contradiction of the definition of $\tilde{x}(c_i)$. □

Lemma A.2. $\tilde{x}(\overline{c}) < \tilde{x}(\underline{c})$.

Proof of Lemma A.2. Let $\hat{x}_i(\overline{c}) < \hat{x}_i(\underline{c})$ (by the assumptions on the neighbors' ideal points) and $\tilde{x}(c_i) > \hat{x}_i(c_i)$ for all c_i (by the definition of $\tilde{x}(c_i)$ in Lemma 1). We consider two main cases.

Case 1. $\hat{x}_i(\overline{c}) < \hat{x}_i(\underline{c}) < x_0$ **or** $x_0 < \hat{x}_i(\overline{c}) < \hat{x}_i(\underline{c})$**.** Define $\tilde{x}_0(c_i) = \max\{x_0, 2\hat{x}_i(c_i) - x_0\}$ and $\tilde{x}(c_i)$ as defined in Lemma 1. Since v_i is strictly concave and $\kappa_P/(\tilde{\rho}_i(1,n) - \tilde{\rho}_i(0,n))$ is constant in

x, then it follows that $|\tilde{x}(\overline{c})-\tilde{x}_0(\overline{c})| < |\tilde{x}(\underline{c})-\tilde{x}_0(\underline{c})|$. Moreover, $\tilde{x}_0(\overline{c}) \leq \tilde{x}_0(\underline{c})$ since $\hat{x}_i(\overline{c}) < \hat{x}_i(\underline{c})$, and then:

$$\tilde{x}(\overline{c}) = \tilde{x}_0(\overline{c}) + |\tilde{x}(\overline{c}) - \tilde{x}_0(\overline{c})| < \tilde{x}_0(\underline{c}) + |\tilde{x}(\underline{c}) - \tilde{x}_0(\underline{c})| = \tilde{x}(\underline{c})$$

Case 2. $\hat{x}_i(\overline{c}) < x_0 < \hat{x}_i(\underline{c})$**.** Consider two subcases:

- **Case 2a:** $\hat{x}_i(\overline{c}) < \tilde{x}(\overline{c}) < x_0 < \hat{x}_i(\underline{c}) < \tilde{x}(\underline{c})$**.** This is not possible since by the definition of $\tilde{x}(\cdot)$, $x_0 \leq \tilde{x}(c_i)$ for all c_i.
- **Case 2b:** $\hat{x}_i(\overline{c}) < x_0 < \hat{x}_i(\underline{c}) < \tilde{x}(\underline{c}) < \tilde{x}(\overline{c})$**.** In this case, the relative distance between the weak opponent's ideal size ($\hat{x}_i(\underline{c})$) and the lawsuit threshold ($\tilde{x}(\underline{c})$) and the default size (x_0) is smaller than for the strong opponent. This combined with the fact that $\underline{c} < \overline{c}$ implies that the utility gain from a lawsuit is strictly smaller for the weak opponent. Since the strong opponent is indifferent between a lawsuit at $\tilde{x}(\overline{c})$, this implies the weak opponent strictly prefers not to sue, a contradiction of the definition of $\tilde{x}(\underline{c})$.

By the pigeonhole principle, this leaves the only remaining, viable ordering for Case 2 as: $x_0 < \tilde{x}(\overline{c}) < \tilde{x}(\underline{c})$. We have thus shown that $\tilde{x}(\overline{c}) < \tilde{x}(\underline{c})$. □

A.1 Deriving the Developer's Belief at a Representative Meeting

Let $a_i(c_i)$ be the strategy used by a neighbor i of type $c_i \in \{\underline{c}, \overline{c}\}$. Since neighbors are behavioral and thus behave independently and have the same incentives, then we can compactly write the weak opponents' strategy as $\underline{a}$ and the strong opponents' strategy as $\overline{a}$. While we assume players use pure strategies in the main text, here we will allow $\underline{a}, \overline{a} \in (0,1)$. Let a_W and a_S be the total number of weak and strong type attendees at the meeting, respectively. Similarly, let m_W and m_S be the total number of weak and strong type attendees *observed* at the meeting, respectively. The total number of attendees is $a = a_W + a_S$ and the total number of *observed* attendees is $m = m_W + m_S$.

We will characterize the developer's belief at a meeting in several steps. First, what is the probability that there are $0 \leq z \leq a$ weak type neighbors *in the meeting*, i.e. Pr$(a_W = z)$? This can be formally represented by a binomial distribution that characterizes the probability of z "successes" (i.e., weak opponents) out of a independent trials. One wrinkle is that the probability of "success" and "failure" depend in part on the weak opponents' strategies and in part on the prevalence of weak opponents in the neighborhood. Formally,

$$\Pr(a_W = z) = \binom{a}{z}(\underline{a}\omega)^z(1-\underline{a}\omega)^{a-z}$$

For example, in an unrepresentative meeting equilibrium in which $\underline{a} = 0$ (and letting $0^0 = 1$), this collapses to

$$\Pr(a_W = z) = \binom{a}{z} 0^z 1^{a-z} = \begin{cases} 1 & \text{if } z = 0 \\ 0 & \text{if } z > 0 \end{cases}$$

Next, given an arbitrary number of z weak opponents in attendance at a meeting of size a, what is the probability that all m observed attendees are weak opponents (i.e. $\Pr(m = m_L | a_W = z)$)? Given that m attendees are independently sampled without replacement, the probability that *only* weak opponents are sampled can be written as follows (with some abuse of notation):

$$\frac{z}{a} \times ... \times \frac{z-(m-1)}{a-(m-1)}$$

This is more precisely written as

$$\Pr(m = m_W | a_W = z) = \prod_{y=0}^{m-1} \frac{z-y}{a-y}$$

For example, suppose that there are 4 weak opponents in attendance in a meeting attended by 10 neighbors total and only 2 weak opponents are observed by the developer. What is the probability of this occurring?

$$\Pr(m = m_W = 2 | a_W = 4) = \frac{4}{10} \times \frac{3}{9} = \frac{6}{45} \approx 0.13$$

To see where this comes from, note that the first draw yields a weak opponent with probability 4/10 since 4 of the 10 attendees are weak opponents. The second draw yields a weak type with probability 3/9 since 3 of the *remaining* 9 attendees are weak opponents.

Recall that what the developer cares about is whether there is a strong opponent that could sue her. So, a key factor in her decision making will be her belief that there are any strong opponents who could sue her—i.e., the probability there are strong opponents at the meeting—given that she observes m_W and m_S. Of course, if $m_S > 0$, then she knows for sure that there is at least one strong opponent who would sue her, so she makes her decision anticipating the possibility of such a suit.

However, if she only observes weak opponents so that $m_W = m$ and $m_S = 0$, she has an inferential problem. There are two scenarios. First, it is possible that there are only weak opponents in attendance, in which case she would only need to compromise with them (a "partial compromise"). Second, it is possible there are strong opponents in attendance, but she just didn't see them, in which case she would want to compromise with the (unobserved) strong opponents to prevent a lawsuit (a "full compromise"). We use the previous two probabilities to formally characterize the developer's belief about the probability that all attendees are actually weak opponents when she only observes weak opponents, i.e. $\Pr(a_W = a|m = m_W)$? We do so using Bayes' rule,

$$\Pr(a_W = a|m_W = m) = \frac{\Pr(m_W = m|a_W = a)\Pr(a_W = a)}{\sum_{z=0}^{a}\Pr(m_W = m|a_W = z)\Pr(a_W = z)}$$

We can immediately make one simplification. Since $\Pr(m_W = m|a_W = z) = 0$ for all $z < m$, we can simplify to:

$$\Pr(a_W = a|m_W = m) = \frac{\Pr(m_W = m|a_W = a)\Pr(a_W = a)}{\sum_{z=m}^{a}\Pr(m_W = m|a_W = z)\Pr(a_W = z)}$$

Next, note that the numerator (and one term in the denominator) is:

$$\Pr(m_W = m|a_W = a)\Pr(a_W = a) = \left[\prod_{y=0}^{m-1}\frac{a-y}{a-y}\right]\left[\binom{a}{a}(\underline{a}\omega)^a(1-\underline{a}\omega)^{a-a}\right] = (\underline{a}\omega)^a$$

Finally, we can write the developer's belief as follows:

$$\Pr(a_W = a|m_W = m) = \frac{(\underline{a}\omega)^a}{(\underline{a}\omega)^a + \sum_{z=m}^{a-1}\Pr(m_W = m|a_W = z)\Pr(a_W = z)}$$

A.1.1 Developer Beliefs at a Representative Meeting with Pure Strategies

To develop intuition, let's consider how this belief looks for a representative meeting equilibrium where all neighbors use pure strategies (implying $\underline{a} = 1$) and in a neighborhood of size 3 (i.e., $n = a = 3$) with a meeting with channel 2 (i.e., $m = 2$).

$$\begin{aligned}
\Pr(a_W = 3|m_W = 2) &= \frac{\omega^3}{\omega^3 + \sum_{z=2}^{2}\left[\prod_{y=0}^{1}\frac{z-y}{3-y}\right]\left[\binom{3}{z}\omega^z(1-\omega)^{3-z}\right]} \\
&= \frac{\omega^3}{\omega^3 + \left[\prod_{y=0}^{1}\frac{2-y}{3-y}\right]\left[\binom{3}{2}\omega^2(1-\omega)^1\right]} \\
&= \frac{\omega^3}{\omega^3 + \left[\frac{2}{3}\times\frac{1}{2}\right]\left[3\omega^2(1-\omega)^1\right]} \\
&= \frac{\omega^3}{\omega^3 + \omega^2(1-\omega)} \\
&= \omega
\end{aligned}$$

So, conditional on observing 2 weak opponents at a representative meeting, she believes all attendees are weak opponents with probability ω and she believes there is at least one strong opponent with probability $1 - \omega$.

We can generalize this to a neighborhood of an arbitrary size n and a meeting channel of arbitrary size m:

$$\Pr(a_W = n | m_W = m) = \frac{\omega^n}{\omega^n + \sum_{z=m}^{n-1} \left[\prod_{y=0}^{m-1} \frac{z-y}{n-y}\right] \left[\binom{n}{z}\omega^z(1-\omega)^{n-z}\right]}$$

Note that the product inside the summation in the denominator is a ratio of falling factorials:

$$\left[\prod_{y=0}^{m-1} \frac{z-y}{n-y}\right] = \frac{(z)_m}{(n)_m} = \frac{z!(n-m)!}{n!(z-m)!}$$

We can use this fact to simply each term in the summation:

$$\Pr(a_W = n | m_W = m) = \frac{\omega^n}{\omega^n + \sum_{z=m}^{n-1} \binom{n-m}{z-m}\omega^z(1-\omega)^{n-z}}$$

The summation yields a tidy expression:

$$\Pr(a_W = n | m_W = m) = \frac{\omega^n}{\omega^n + (\omega^m - \omega^n)} = \omega^{n-m}$$

Then, in a representative meeting with n attendees, after having observed m weak opponents, the developer believes that there are only weak opponents in the meeting with probability ω^{n-m}. She believes there is at least one strong opponent with probability $1 - \omega^{n-m}$.

The developer's "inferential problem" is the fact that neither of these quantities is zero or one. Her inferential problem becomes "worse" when $\omega^{n-m} \to \frac{1}{2}$.

A.2 Weak Opponents' Incentives in the Unrepresentative Meeting Equilibrium

We consider the strategic calculations of a neighbor i who is a weak opponent. Let $\tilde{h}_i$ be his belief that at least one strong opponent will attend the meeting. He gets the following payoff from not attending an unrepresentative meeting:

$$\underbrace{(1-\tilde{h}_i)v_i(x_M,\underline{c})}_{\substack{\text{All other neighbors}\\ \text{are weak opponents}\\ \text{(no attendance)}}} + \underbrace{\tilde{h}_i v_i(x_1,\underline{c})}_{\substack{\text{At least one other neighbor}\\ \text{is a strong opponent}}}$$

However, what happens if he deviates and attends the meeting? Since observing weak opponents at the meeting is off the equilibrium path, perfect Bayesian equilibrium does not pin down the developer's belief in that situation. By Assumption 2, if she observes any strong opponents, she (correctly) believes there is a strong opponent at the meeting who is eligible to sue. And, if she observes all weak opponents, she believes there are no strong opponents in the meeting.

There are two scenarios in which the developer observes only weak opponents. First, all neighbors could be weak opponents, and without the deviation, there would be no meeting attendance. The weak opponent i believes this happens with probability $(1-\tilde{h}_i)$. Second, the neighborhood could have a mix of weak and strong opponents (probability $\tilde{h}_i$), but with the deviation, only weak opponents are randomly chosen to be observed. In this situation, suppose that a deviating weak opponent i believes only weak opponents are chosen with probability $\tilde{\phi}$.

Recall that each neighbor i believes there will be a successful lawsuit with probability $\tilde{\rho}_i(s_i,n)$. Moreover, since each neighbor's cost is private information, a neighbor i only learns the types of the m attendees whose types are revealed.[1] Let $x_2' \equiv \min\{x_2, x_M\}$ Then, the expected payoff to a weak opponent i for deviating and attending the meeting is:

$$\begin{aligned}(1-&\tilde{h}_i)v_i(x_2,\underline{c})\\ &+\tilde{h}_i\tilde{\phi}\big[(1-\tilde{\rho}_i(0,n))v_i(x_2',\underline{c})+\tilde{\rho}_i(0,n)v_i(x_0,\underline{c})\big]\\ &+\tilde{h}_i(1-\tilde{\phi})v_i(x_1,\underline{c})\\ &-k\end{aligned}$$

From Assumption 2, it follows that $\tilde{\phi}=0$. Then the condition reduces to

$$(1-\tilde{h}_i)v_i(x_2,\underline{c})+\tilde{h}_i v_i(x_1,\underline{c})-k$$

Then there is no incentive to deviate if

$$(1-\tilde{h}_i)v_i(x_M,\underline{c})+\tilde{h}_i v_i(x_1,\underline{c}) \geq (1-\tilde{h}_i)v_i(x_2,\underline{c})+\tilde{h}_i v_i(x_1,\underline{c})-k$$

1. It is possible that a given neighbor i ends up knowing the types of $m+1$ neighbors if she is not among the m whose types are publicly revealed. This won't dramatically change our analysis, and we thus ignore it to make our results more parsimonious.

This reduces to

$$v_i(x_M, \underline{c}) \geq v_i(x_2, \underline{c}) - \frac{k}{1 - \tilde{h}_i} \tag{A.1}$$

Since $v_i(\cdot, \underline{c})$ is decreasing for $x_M > \hat{x}_i(\underline{c})$ and $x_2 > \hat{x}_i(\underline{c})$, then there exists a threshold $x_M^{\text{ur}} > x_2$ where (A.1) binds. Then for all $x_M \leq x_M^{\text{ur}}$, a weak opponent has no incentive to deviate.