

Online Appendix
“Liability for Homelessness”
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A Parametric results

In this section, we present versions of the formal propositions under the assumption that $F(e_t) = 1/e_t$.

Proposition A.1 (Policy implementation under electoral accountability). *Under the electoral accountability regime, equilibrium policy implementation is as follows:*

$$r_t^* = \begin{cases} 0 & \rho < \frac{\eta\theta^A}{\delta} \mid \theta^A < \theta^V, \\ 1 & \rho \geq \frac{\eta\theta^A}{\delta} \ \& \ \theta^A \geq \theta^V. \end{cases}$$

Proposition A.2 (Voter utility under electoral accountability). *Under the electoral accountability regime, equilibrium voter utility is as follows:*

$$U_t^{V*} = \begin{cases} 0 & \rho < \frac{\eta\theta^A}{\delta} \mid \theta^A < \theta^V, \\ \frac{1-\theta^V}{1-\delta} & \rho \geq \frac{\eta\theta^A}{\delta} \ \& \ \theta^A \geq \theta^V. \end{cases}$$

Proposition A.3 (Policy implementation under private liability). *If the policy would be implemented in the electoral accountability regime, it is always implemented in the private liability regime. Specifically, under private liability, we have the following:*

$$r_t^* = \begin{cases} 0 & \sqrt{\frac{\theta^R}{\eta}} < \theta^V \ \& \ \theta^A < \theta^V, \\ 1 & \sqrt{\frac{\theta^R}{\eta}} \geq \theta^V \mid \theta^A \geq \theta^V. \end{cases}$$

Proposition A.4 (Voter utility under private liability). *Under the private liability regime, equilibrium voter utility is weakly greater than under the electoral accountability regime. Specifically, it is as follows:*

$$U_t^{V*} = \begin{cases} 0 & \sqrt{\frac{\theta^R}{\eta}} < \theta^V \ \& \ \theta^A < \theta^V, \\ \frac{1-\theta^A}{1-\delta} & \sqrt{\frac{\theta^R}{\eta}} < \theta^V \ \& \ \theta^A \geq \theta^V, \\ \frac{(1-\theta^V)\sqrt{\frac{\eta}{\theta^R}}}{1-\delta} & \sqrt{\frac{\theta^R}{\eta}} \geq \theta^V. \end{cases}$$

Proposition A.5 (Comparison of utility for the rich). *Under the private liability regime, equilibrium utility for the rich is greater than under the electoral accountability regime precisely under the following condition:*

$$\eta < \min \left\{ \frac{\delta \rho}{\theta^A}, \frac{\theta^R}{4\theta^{A^2}} \right\} \ \& \ \theta^A \geq \theta^V.$$

B Formal proofs

Proof of Proposition 1. In text. □

Proof of Proposition 2. If $\rho < \frac{\eta F^{-1}\left(\frac{1}{\theta^A}\right)}{\delta}$ or $\theta^A < \theta^V$, then Proposition 1 states that policy implementation does not occur. Then clearly the voter earns zero utility. If instead both $\rho \geq \frac{\eta F^{-1}\left(\frac{1}{\theta^A}\right)}{\delta}$ and $\theta^A \geq \theta^V$, Proposition 1 states that policy implementation occurs. Because the voter cannot directly observe effort e_t , the politician chooses the minimum that induces the administrator to implement the policy, which is $\underline{e} = F^{-1}\left(\frac{1}{\theta^A}\right)$. Then turning to the voter's utility function, the voter earns $1 - 1/\theta^A$ in each stage in perpetuity. As future payoffs are discounted by δ , the voter's net present value of this stream of payoffs is $\frac{1 - \frac{1}{\theta^A}}{1 - \delta}$. □

Proof of Proposition 3. **If the policy is implemented in the electoral accountability regime, it is implemented in the private liability regime.** Assume that policy is implemented under the electoral accountability regime. Then the voter must have preferred to demand policy implementation given that it occurs with $e_t = \underline{e}$. Then under the private liability regime, the voter can choose to adjudicate disputes in favor of the homeless, leading the rich to forestall a dispute by inducing policy implementation with $e_t \geq \underline{e}$. As the voter's utility is increasing in e_t , the voter must then prefer to choose an adjudication rule that leads to policy implementation under private liability.

Derivation of conditions for policy implementation. Given that policy is implemented whenever the voter sets an adjudication rule in favor of the homeless, we ask when the voter prefers to induce policy implementation. This occurs whenever the choice of e_t weakly exceeds the minimum value that makes the voter indifferent between implementation and not, namely $F^{-1}\left(\frac{1}{\theta^V}\right)$. There are two ways for this to occur. First, the administrator (who observes e_t and must be made at least indifferent) is weakly more demanding than the voter, so $e_t = F^{-1}\left(\frac{1}{\theta^A}\right) \geq F^{-1}\left(\frac{1}{\theta^V}\right)$ and thus $\theta^A \geq \theta^V$ as $F(\cdot)$ is a decreasing

function. Second, if facing the prospect of a dispute being decided in favor of the homeless, the rich prefer to exert weakly more effort than would make the voter indifferent, so $e_t = e_t^{R^\circ} = F'^{-1}\left(-\frac{\eta}{\theta^R}\right) \geq F^{-1}\left(\frac{1}{\theta^V}\right)$. \square

Proof of Proposition 4. If $F'^{-1}\left(-\frac{\eta}{\theta^R}\right) < F^{-1}\left(\frac{1}{\theta^V}\right)$ and $\theta^A < \theta^V$, then Proposition 3 states that policy implementation does not occur. Then clearly the voter earns zero utility.

If $F'^{-1}\left(-\frac{\eta}{\theta^R}\right) < F^{-1}\left(\frac{1}{\theta^V}\right)$ and $\theta^A \geq \theta^V$, then Proposition 3 states that policy implementation occurs. The fact that $F'^{-1}\left(-\frac{\eta}{\theta^R}\right) < F^{-1}\left(\frac{1}{\theta^V}\right)$ is equivalent to $e_t^{R^\circ} < \underline{e}$, such that $e_t^{R^*} = \underline{e}$. Then as with electoral accountability, the voter earns $1 - 1/\theta^A$ in each stage in perpetuity, so the voter's net present value of this stream of payoffs is $\frac{1 - \frac{1}{\theta^A}}{1 - \delta}$.

If $F'^{-1}\left(-\frac{\eta}{\theta^R}\right) \geq F^{-1}\left(\frac{1}{\theta^V}\right)$, then Proposition 3 states that policy implementation occurs. The fact that $F'^{-1}\left(-\frac{\eta}{\theta^R}\right) \geq F^{-1}\left(\frac{1}{\theta^V}\right)$ is equivalent to $e_t^{R^\circ} \geq \underline{e}$, such that $e_t^{R^*} = e_t^{R^\circ} = F'^{-1}\left(-\frac{\eta}{\theta^R}\right)$. Then turning to the voter's utility function, the voter earns $1 - \theta^V F\left(F'^{-1}\left(-\frac{\eta}{\theta^R}\right)\right)$ in each stage in perpetuity. As future payoffs are discounted by δ , the voter's net present value of this stream of payoffs is $\frac{1 - \theta^V F\left(F'^{-1}\left(-\frac{\eta}{\theta^R}\right)\right)}{1 - \delta}$. \square

Proof of Proposition 5. If the policy is implemented under neither regime, utility is zero in both cases. If the policy is implemented under both regimes, voter utility is higher under private liability because it is increasing in e_t , and we know that $e_t^{R^*} \geq \underline{e}$ (i.e., the equilibrium choice of e_t is weakly greater under private liability). If the policy is implemented only under private liability, it must have provided utility weakly greater than zero under private liability, as the voter could instead have guaranteed that implementation did not occur by choosing to adjudicate disputes in favor of the rich; compare to utility under electoral accountability, which is zero as the policy is not implemented. Finally, Proposition 3 states that we never have policy implementation under electoral accountability but not under private liability. \square

Proof of Proposition 6. First observe that policy must have been implemented under the

electoral accountability regime, else the rich would already attain their maximum possible utility. Then by Proposition 1, we must have

$$(1) \quad \begin{aligned} \rho &\geq \frac{\eta F^{-1}\left(\frac{1}{\theta^A}\right)}{\delta} \ \& \ \theta^A \geq \theta^V \iff \\ \eta &\leq \frac{\delta \rho}{F^{-1}\left(\frac{1}{\theta^A}\right)} \ \& \ \theta^A \geq \theta^V. \end{aligned}$$

Then by Proposition 3, we must also have policy implementation under private liability. Recognize also that if the rich's effort $e_t = \underline{e}$, it would not be possible for their utility to be higher under private liability. Given this, the rich's utility is higher under private liability compared to electoral accountability when

$$(2) \quad \begin{aligned} -\eta e_t - 1 - \theta^R F\left(F'^{-1}\left(-\frac{\eta}{\theta^R}\right)\right) &\geq -1 - \frac{\theta^R}{\theta^A} \iff \\ \eta &\leq \frac{\theta^R \left(\frac{1}{\theta^A} - F\left(F'^{-1}\left(-\frac{\eta}{\theta^R}\right)\right)\right)}{F'^{-1}\left(-\frac{\eta}{\theta^R}\right)}. \end{aligned}$$

As both Conditions 1 and 2 must be satisfied, the result follows. □

C Formal details of uncertainty extension

Let the voter receive a noisy signal $s_t \in \{0, 1\}$ of policy implementation. With probability $q > 1/2$, the signal correctly reveals whether policy implementation occurred and $s_t = r_t$. With probability $1 - q$, the signal is incorrect and $s_t \neq r_t$. The threshold for the voter to prefer to demand policy implementation is the same as before, as the administrator continues to observe the politician's choice of e_t . However, the politician's incentive to comply with the demand is mitigated, as sacking now sometimes follows implementation and retention sometimes follows no implementation. The politician's continuation value of holding office in stage $t + 1$ now satisfies

$$\begin{aligned} v_O^P &= \rho - \eta \underline{e} + \delta (q v_O^P + (1 - q) 0) \\ &= \rho - \eta F^{-1}\left(\frac{1}{\theta^A}\right) + \delta q v_O^P, \end{aligned}$$

implying that

$$v_O^P = \frac{\rho - \eta F^{-1}\left(\frac{1}{\theta^A}\right)}{1 - \delta q}.$$

which is less than the quantity in the main text if it was positive. The politician is now willing to implement policy as long as

$$\begin{aligned} \rho - \eta \underline{e} + \delta (q v_O^P + (1 - q) 0) &\geq \rho - \eta 0 + \delta (q 0 + (1 - q) v_O^P) \iff \\ (3) \quad \rho &\geq \frac{(1 - \delta (1 - q)) \eta F^{-1}\left(\frac{1}{\theta^A}\right)}{\delta (2q - 1)}. \end{aligned}$$

(Of course, when $q = 1$, we recover the original condition presented in the main text).

Noticing that

$$\frac{\partial}{\partial q} \left[\frac{(1 - \delta(1 - q))\eta F^{-1}\left(\frac{1}{\theta^A}\right)}{\delta(2q - 1)} \right] = -\frac{(2 - \delta)\eta F^{-1}\left(\frac{1}{\theta^A}\right)}{\delta(2q - 1)^2} < 0,$$

we see that as $q \downarrow 1/2$, the right-hand side of Condition 3 increases, making it stricter and thus harder to satisfy. Thus, electoral accountability performs worse than before. By comparison, private liability performs the same: because the homeless's information about policy implementation is assumed to be verifiable, the voter always learns whether it occurred, and play proceeds as above.

D Formal details of competing policy demands extension

Let there be a continuum of policies on the unit interval indexed by i and varying in θ^A and thus \underline{e} (now denoted \underline{e}^i) as well as θ^V (now denoted θ^{V^i}). For a given policy i in Stage t , the productivity of policy (the voter's benefit per unit of politician effort, denoted π^i) is

$$\pi^i = \frac{1 - \theta^{V^i}}{\underline{e}^i}.$$

Let the index i specifically denote the policies in descending order of π^i , with $G(i) \equiv \pi^i$. Additionally, assume that $G(i)$, θ^{V^i} , and \underline{e}^i are Lebesgue-integrable functions. For nontriviality, assume the following:

1. The politician's reelection incentive ρ is sufficiently small to preclude willingness to implement all policies, i.e., $\int_0^1 \underline{e}^i di > \rho$.
2. The voter prefers to demand implementation of a strictly positive measure of policies, corresponding to $G(0) > 0$.

Then the voter maximizes utility by demanding implementation of policies between zero and \bar{i} , where \bar{i} is the largest value of i satisfying the following two constraints:

$$\int_0^{\bar{i}} \underline{e}^i di \leq \rho,$$

$$G(\bar{i}) \geq 0.$$

That is, the politician must be willing to satisfy the demands, and the voter must be willing to make them. Given the assumptions, we must have $\bar{i} \in (0, 1)$.

We argue that homelessness policy is difficult for the politician and a low priority for the voter, implying a small value of π^i and thus a high index number i . Therefore, the policy is likely not to be demanded, in favor of competing demands that are more productive for the

voter. By comparison, the performance of the private liability regime does not suffer, as the size of the office-holding benefit (ρ) is not a constraint on which policies would be demanded. Additionally, as shown in the main text, it is possible that under the private liability regime, effort on homelessness policy would be greater than the bare minimum needed to induce implementation.

E Formal details of effort evasion extension

We consider the possibilities both of a collective action problem among the rich and the possibility that rich individuals might pay a cost to exit. We consider two rich individuals $R_i : i \in \{1, 2\}$, either of whom can choose to exert effort e_{ti} for collective benefit. To create a collective action problem, we now allow \underline{e} to be drawn independently according to the same probability distribution in each stage, indexing it by subscript t .¹ Constraining $e_{t1}, e_{t2} \geq 0$, we assume the following:

$$\Pr(\underline{e}_t < e_{t1} + e_{t2}) = 1 - \frac{1}{1 + e_{t1} + e_{t2}}.$$

We now make the following assumptions for tractability. First, the binding constraint on policy implementation is the administrator rather than the voter. Second, in the event of a failure to meet this threshold, the total surcharge imposed on all rich individuals is $\psi > 0$, with R_1 paying a proportion $s_1 \equiv s \in (1/2, 1]$ (the progressivity of the surcharge) and R_2 paying a proportion $s_2 \equiv 1 - s$. We also let $F(e_{t1} + e_{t2}) = \frac{1}{e_{t1} + e_{t2}}$.

Before the realization of \underline{e}_t , we also allow each R_i to pay a cost of exit $\kappa > 0$ (every stage)² and be assured of avoiding the cost of policy implementation or surcharge, thus allowing for the possibility of capital flight. Then expected utility from exiting in a given stage (denoted

1. If \underline{e} were fixed, there would be no collective action problem: there could be equilibria in which each individual contributes some positive amount, and these amounts sum to exactly \underline{e} ; then everyone faces a sharp cliff of surcharges when reducing effort by an arbitrarily small amount.

2. Whether exit is paid every stage or only once is immaterial; an equivalent cost of permanent exit is $\frac{\kappa}{1-\delta}$, with this being compared to the expected per-period costs of not exiting also divided by $1 - \delta$. Then players' strategic calculation is the same.

by subscript \mathbb{X}) is

$$\mathbb{E}U_t^{R_1}_{\mathbb{X}} = -\kappa.$$

By comparison, R_i has the following expected utility from not exiting in a given stage (denoted by subscript \mathbb{N}):

$$\begin{aligned} \mathbb{E}U_t^{R_1}_{\mathbb{N}}(e_i; e_j) &= \Pr(\underline{e}_t < e_{ti} + e_{tj}) \cdot \left(-1 - \theta^R \frac{1}{e_{ti} + e_{tj}}\right) + (1 - \Pr(\underline{e}_t < e_{ti} + e_{tj})) \cdot (-s_i \psi) \\ &= \left(1 - \frac{1}{1 + e_{ti} + e_{tj}}\right) \left(-1 - \theta^R \frac{1}{e_{ti} + e_{tj}}\right) + \left(\frac{1}{1 + e_{ti} + e_{tj}}\right) (-s_i \psi). \end{aligned}$$

Maximizing this with respect to e_{ti} implies the following best response function for each R^i :

$$BR_t^{R_i}(e_{tj}) = \max \left\{ \sqrt{\frac{\theta^R + s_i \psi - 1}{\eta}} - 1 - e_{tj}, 0 \right\}.$$

Given the assumption that $s_1 > 1/2 > s_2$, the only possible equilibrium effort levels with any positive effort (specifically, when $\theta^R + s \psi - 1 > \eta$) are

$$\begin{aligned} e_{t1}^* &= \sqrt{\frac{\theta^R + s \psi - 1}{\eta}} - 1, \\ e_{t2}^* &= 0. \end{aligned}$$

If R_1 does not exit, we see then that anticipating the effort of R_1 , the less-exposed R_2 declines to put in any effort at all. But R_1 is not fully exposed to the surcharge, with R_1 's effort decreasing as s decreases. Consequently, there may be a benefit to increasing s and thus designing the surcharge with greater progressivity.

However, this has not yet taken into account the possibility that R_1 may choose to exit (R_2 is less tempted to do so), which places an upper-bound on the values of s that

are compatible with strictly positive effort.³ Denoting this as \bar{s} and supposing then that $e_1^* > 0$, we see that \bar{s} is the minimum of 1 (due to feasibility) and the value of s that solves the following equation:⁴

$$\begin{aligned}
& \mathbb{E}U_t^{R_1}_{\mathbb{N}}(e_1^*; 0) = \mathbb{E}U_t^{R_1}_{\mathbb{X}} \iff \\
& -1 + \eta - 2\sqrt{\eta(\theta^R + s\psi - 1)} = -\kappa \iff \\
(4) \quad & \sqrt{\theta^R + s\psi - 1} = \frac{\kappa + \eta - 1}{2\sqrt{\eta}} \implies \\
& \bar{s} = \min \left\{ \frac{1}{4\psi} \left(\eta + 2(\kappa + 1 - 2\theta^R) + \frac{(\kappa - 1)^2}{\eta} \right), 1 \right\}.
\end{aligned}$$

When $\bar{s} < 1$, we have

$$\frac{\partial \bar{s}}{\partial \kappa} = \frac{\kappa + \eta - 1}{2\eta\psi}.$$

Given our initial supposition that $e_{t1}^* > 0$, then we know that $\sqrt{\theta^R + s\psi - 1} > 0$, implying that for Equation 4 to have had a solution, we must have had $\kappa + \eta - 1 > 0 \iff \kappa > 1 - \eta$.⁵ Then clearly $\frac{\partial \bar{s}}{\partial \kappa} > 0$. That is to say, the *lower* are the costs of exit, the *less* progressivity of the surcharge is possible, potentially increasing concerns about a collective action problem among the rich. The extent to which these strategic factors interact and trade off in the real world is an empirical question.

3. One might imagine that R_1 's exit itself would solve the collective action problem, with only R_2 left to bear the entire surcharge. This would imply that progressivity is less deleterious than initially thought, supporting our argument in the main text. However, as we are presently concerned with exploring its bounds, we assume that if R_1 exits, then R_2 would still only bear a proportion $s_2 = 1 - s$ of the surcharge, which could correspond to successive rich individuals simply having fewer resources to be extracted.

4. If the solution were less than $1/2$, then s could indeed be selected accordingly to retain R_1 while accepting that R_2 would exit; instead choosing, say, $s = 1/2$ would induce both to exit.

5. Otherwise, exit would be guaranteed regardless of the choice of s .