Online Appendix
Asset Market Equilibrium and Family Firm Cost of Capital: Implications for Corporate Finance

A. Appendix

A.1 Appendix 1

Let us denote **H** as $\mathbf{H} = [\boldsymbol{\Sigma}_{\boldsymbol{\theta}} + \boldsymbol{\Sigma}_{\mathbf{V}}]\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1}$. In the literature on asymmetric information, $\mathbf{H}^{-1}$ is the classic measure of the informativeness of the noisy information **I** since it can easily be shown that $\mathbf{H}^{-1}$ is equal to the percentage reduction in the uncertainty of $\boldsymbol{\theta}$ as beliefs are updated from the ex-ante to the ex-post probability distribution. Using all these, we get:

$$\begin{aligned}\mathbf{E}(\boldsymbol{\theta} \mid \mathbf{I}) &= \mathbf{E}(\boldsymbol{\theta}) + \boldsymbol{\Sigma}_{\boldsymbol{\theta}\mathbf{I}}\boldsymbol{\Sigma}_{\mathbf{I}}^{-1}[\mathbf{I} - \mathbf{E}(\mathbf{I})] \\ &= \bar{\boldsymbol{\theta}} + \boldsymbol{\Sigma}_{\boldsymbol{\theta}}[\boldsymbol{\Sigma}_{\boldsymbol{\theta}} + \boldsymbol{\Sigma}_{\mathbf{V}}]^{-1}[(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}) + \mathbf{Z}(\mathbf{x} - \bar{\mathbf{x}})]\end{aligned} \tag{22a}$$

$$\begin{aligned}\boldsymbol{\Sigma}_{\boldsymbol{\theta}|\mathbf{I}} &= \boldsymbol{\Sigma}_{\boldsymbol{\theta}} - \boldsymbol{\Sigma}_{\boldsymbol{\theta}\mathbf{I}}\boldsymbol{\Sigma}_{\mathbf{I}}^{-1}\boldsymbol{\Sigma}_{\mathbf{I}\boldsymbol{\theta}} \\ &= \boldsymbol{\Sigma}_{\boldsymbol{\theta}}\boldsymbol{\alpha}_1\boldsymbol{\Sigma}_{\mathbf{I}}^{-1}[\boldsymbol{\Sigma}_{\mathbf{I}}\boldsymbol{\alpha}_1^{-1} - \boldsymbol{\alpha}_1\boldsymbol{\Sigma}_{\boldsymbol{\theta}}] \\ &= \boldsymbol{\Sigma}_{\boldsymbol{\theta}}\boldsymbol{\alpha}_1(\boldsymbol{\alpha}_1[\boldsymbol{\Sigma}_{\boldsymbol{\theta}} + \boldsymbol{\Sigma}_{\mathbf{V}}]\boldsymbol{\alpha}_1)^{-1}[(\boldsymbol{\alpha}_1[\boldsymbol{\Sigma}_{\boldsymbol{\theta}} + \boldsymbol{\Sigma}_{\mathbf{V}}]\boldsymbol{\alpha}_1)\boldsymbol{\alpha}_1^{-1} - \boldsymbol{\alpha}_1\boldsymbol{\Sigma}_{\boldsymbol{\theta}}] \\ &= \boldsymbol{\Sigma}_{\boldsymbol{\theta}}[\boldsymbol{\Sigma}_{\boldsymbol{\theta}} + \boldsymbol{\Sigma}_{\mathbf{V}}]^{-1}\boldsymbol{\Sigma}_{\mathbf{V}}\end{aligned} \tag{22b}$$

and we can rewrite $\mathbf{E_2(R)}$ and $\boldsymbol{\Sigma}_2$ as

$$\mathbf{E}_2(\mathbf{R}) = \bar{\boldsymbol{\theta}} + \mathbf{H}^{-1}[(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}) + \mathbf{Z}(\mathbf{x} - \bar{\mathbf{x}})] - \mathbf{I} \tag{23}$$

and

$$\boldsymbol{\Sigma}_2 = \mathbf{H}^{-1}[\mathbf{H} + \boldsymbol{\Sigma}_{\mathbf{V}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}]\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} \tag{24}$$

The market equilibrium clearing condition is:

$$\overline{W_1}(\mathbf{X_1} - \mathbf{Y_1}) + \overline{W_2}(\mathbf{X_2} - \mathbf{Y_2}) = \overline{W_m}\mathbf{x}$$

Substituting for λ, $\mathbf{Y_i}$, and $\mathbf{X_i}$ (from equation (8)) into the market equilibrium clearing condition results in

$$\begin{aligned}&\lambda\left(\tfrac{1}{A}\right)\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}[\boldsymbol{\theta} - \boldsymbol{\alpha}_1\boldsymbol{\theta} - \boldsymbol{\alpha}_2\mathbf{x} - \boldsymbol{\alpha}_3 - R_f\mathbf{1} + \mathbf{B}] + (1-\lambda)\left(\tfrac{1}{A}\right)\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}[\mathbf{H} + \boldsymbol{\Sigma}_{\mathbf{V}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}]^{-1} \times \\ &[\mathbf{H}\bar{\boldsymbol{\theta}} + (\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}) + \mathbf{Z}(\mathbf{x} - \bar{\mathbf{x}}) - \mathbf{H}\boldsymbol{\alpha}_1\boldsymbol{\theta} - \mathbf{H}\boldsymbol{\alpha}_2\mathbf{x} - \mathbf{H}\boldsymbol{\alpha}_3 - \mathbf{H}R_f\mathbf{1}] = \mathbf{x} + \lambda\mathbf{Y}\end{aligned} \tag{23}$$

This market equilibrium condition must hold as an identity (i.e., for any realization of **x**) which implies the following three equations:

$$\lambda\left(\tfrac{1}{A}\right)\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}(\mathbf{1} - \boldsymbol{\alpha}_1) + (1-\lambda)\left(\tfrac{1}{A}\right)\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}[\mathbf{H} + \boldsymbol{\Sigma}_{\mathbf{V}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}]^{-1}[\mathbf{1} - \mathbf{H}\boldsymbol{\alpha}_1] = \mathbf{0} \tag{a1}$$

$$-\lambda\left(\tfrac{1}{A}\right)\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{\alpha}_2 + (1-\lambda)\left(\tfrac{1}{A}\right)\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}[\mathbf{H} + \boldsymbol{\Sigma}_{\mathbf{V}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}]^{-1}[\mathbf{Z} - \mathbf{H}\boldsymbol{\alpha}_2] = \mathbf{1} \tag{a2}$$

$$-\lambda\left(\frac{1}{A}\right)\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}\left[\boldsymbol{\alpha}_3 + R_f\mathbf{1} - \mathbf{B}\right] + (1-\lambda)\left(\frac{1}{A}\right)\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}\left[\mathbf{H} + \boldsymbol{\Sigma}_{\mathbf{V}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}\right]^{-1} \times \left[\mathbf{H}\bar{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}} - \mathbf{Z}\bar{\mathbf{x}} - \mathbf{H}\boldsymbol{\alpha}_3 - \mathbf{H}\mathbf{R}_{\mathbf{f}}\mathbf{1}\right] = \lambda\mathbf{Y} \qquad \text{(a3)}$$

Solving equations (a1) to (a3) simultaneously[24] results in the information equilibrium expressions for $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$, and $\boldsymbol{\alpha}_3$.

A.2 Appendix 2

Recall equation (16):

$$E[\mathbf{R}] = R_f\mathbf{1} + ACov(\mathbf{R}, R_m + I_m) + ACov(\mathbf{R}, \mathbf{R}+\mathbf{I})\lambda\mathbf{Y} - Cov(\mathbf{R}, \mathbf{R}+\mathbf{I})\lambda\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}\mathbf{B}$$

The matrix representation of this equation is:

$$\mathrm{E}\begin{bmatrix} R_{FF} \\ R_{NF} \end{bmatrix} = \begin{bmatrix} R_f \\ R_f \end{bmatrix} + A\begin{bmatrix} Cov(R_{FF}, R_m + I_m) \\ Cov(R_{NF}, R_m + I_m) \end{bmatrix} + \begin{bmatrix} Cov(R_{FF}, R_{FF} + I_{FF}) & Cov(R_{FF}, R_{NF} + I_{NF}) \\ Cov(R_{NF}, R_{FF} + I_{FF}) & Cov(R_{NF}, R_{NF} + I_{NF}) \end{bmatrix} \times \lambda \begin{bmatrix} Y \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} Cov(R_{FF}, R_{FF} + I_{FF}) & Cov(R_{FF}, R_{NF} + I_{NF}) \\ Cov(R_{NF}, R_{FF} + I_{FF}) & Cov(R_{FF}, R_{NF} + I_{NF}) \end{bmatrix} \times \lambda \begin{bmatrix} \sigma_{\varepsilon_{FF}}^2 & \sigma_{\varepsilon_{FF},\varepsilon_{NF}} \\ \sigma_{\varepsilon_{FF},\varepsilon_{NF}} & \sigma_{\varepsilon_{NF}}^2 \end{bmatrix}^{-1} \begin{bmatrix} B \\ 0 \end{bmatrix}$$

The third term in this equation can be rewritten as:

$$\begin{bmatrix} Cov(R_{FF}, R_{FF} + I_{FF}) & Cov(R_{FF}, R_{NF} + I_{NF}) \\ Cov(R_{NF}, R_{FF} + I_{FF}) & Cov(R_{FF}, R_{NF} + I_{NF}) \end{bmatrix} \begin{bmatrix} \sigma_{\varepsilon_{NF}}^2 \\ -\sigma_{\varepsilon_{FF},\varepsilon_{NF}} \end{bmatrix} \frac{\lambda B}{|\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}|}$$

Taking element by element of this matrix representation gives us the representation in equation (17) with

$$R_I + I_I = \lambda(R_{NF} + I_{NF})$$

A.3 Appendix 3

Recall the (scalar) elements of $\boldsymbol{\Sigma}_{\mathbf{W}}^{-1}$ as

$$\boldsymbol{\Sigma}_{\mathbf{W}}^{-1} = \frac{1}{|\boldsymbol{\Sigma}_{\mathbf{W}}|}\begin{bmatrix} Cov(R_m, R_m + I_m) & Cov(R_m, R_B + I_B) & Cov(R_m, R_I + I_I) \\ Cov(R_B, R_m + I_m) & Cov(R_p, R_B + I_B) & Cov(R_B, R_I + I_I) \\ Cov(R_I, R_m + I_m) & Cov(R_I, R_B + I_B) & Cov(R_I, R_I + I_I) \end{bmatrix}$$

Aggregating equation (18) across the family and nonfamily firms, weighting them first by their relative market values (traded and nontraded) and then finding portfolios that will generate the ***net*** returns on private wealth and interaction returns results in the three equations.

[24] The detailed proof is available from the authors upon request.