Online Appendix to Pricing Transition Risk with a Jump-Diffusion Credit Risk Model: Evidences from the CDS market

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The Online Appendix is organized as follows. The derivation of the default probability within the jump-diffusion model is presented in Section 1, which is followed by the proof of the complete pricing formulas, in Section 2. The numerical method to compute the inverse Laplace transform is introduced in Section 3. The CDS pricing formula for the no-jump credit risk model is mentioned in Section 4. The sensitivity analysis of the theoretical credit spread generated by the jump-diffusion model is discussed in Section 5. In Section 6, a comprehensive representation of the results of the investigation is proposed. Finally, the descriptive statistics of the dataset is reported in Section 7. References to Propositions are intended to the main paper.

1 The Default Probability

In this section we outline the derivation of the default probability for the model described in Section 3 of the paper, which constitutes a fundamental building block in the derivation of the semi-closed formulas defined in Propositions 1 and 2. Employing the notation $x = \ln(V)$, the default probability in $(t, T]$ is given by:

$$D(x, t; T) = \mathbb{E}\left[\mathbb{1}_{\{T \geq t_d \geq t\}} \middle| \mathcal{F}_t \right] \tag{1}$$

Following the approach of Kou and Wang [2003], which is consistent within our model[1], we can define the Laplace transform with parameter $\omega > 0$ of the default probability (1), that is $H(x, t; \omega) =$

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[1] In Kou and Wang [2003], the Laplace transform of the first time passage model has been derived for a jump-diffusion model with a downward and upward component. For our model, which is included in the mentioned version as it has only one jump component, the Laplace transform (reported in (2)) can be derived in a similar manner. For the sake of brevity, we do not report the proof.

$\mathcal{L}\left(D\left(x,t;T\right)\right)(\omega)$, as:

$$H\left(x,t;\omega\right)=\frac{1}{\omega}\mathbb{E}\left[\left.e^{-\omega(t_d-t)}\right|\mathcal{F}_t\right] \tag{2}$$

where, defining $\hat{x}:=\ln\left(V_t/V_{def}\right)$, we have:

$$\mathbb{E}\left[\left.e^{-\omega(t_d-t)}\right|\mathcal{F}_t\right]=\left[\frac{\gamma(\beta-\eta)}{\eta(\beta-\gamma)}e^{-\beta\hat{x}}+\frac{\beta(\eta-\gamma)}{\eta(\beta-\gamma)}e^{-\gamma\hat{x}}\right] \tag{3}$$

The values of β and γ are the two positive roots of the polynomial $\mathrm{G}\left(q\right)=\omega$, where:

$$\mathrm{G}\left(q\right)=\frac{1}{2}\sigma^2q^2-\psi q+\lambda\left(\frac{\eta}{\eta-q}-1\right) \tag{4}$$

In equation (4), ψ is the drift of the log-process of the firm, consistent with the risk-adjusted measure Q, defined as:

$$\psi=r-\frac{1}{2}\sigma^2-\lambda\left(\frac{\eta}{\eta+1}-1\right) \tag{5}$$

2 Technical Results

Proof of Proposition 1. Consider the bond price $B\left(x,t;T\right)$ defined in (5) of the paper, and denote by $F\left(x,t;\omega\right)=\mathcal{L}\left(B\left(x,t;T\right)\right)(\omega)$ its Laplace transform. Then,

$$F\left(x,t;\omega\right)=\int_t^{\infty}e^{-\omega(T-t)}\mathbb{E}\left[e^{-r(T-t)}\mathbb{1}_{\{t_d>T\}}+\Upsilon e^{-r(t_d-t)}\mathbb{1}_{\{T\geq t_d\geq t\}}+\left.\int_t^T be^{-r(z-t)}\mathbb{1}_{\{t_d>z\}}dz\right|\mathcal{F}_t\right]dT \tag{6}$$

By Fubini's theorem and straightforward considerations, formula (6) can be rewritten as follows

$$\begin{aligned}F\left(x,t;\omega\right) &= \int_t^{\infty}e^{-(\omega+r)(T-t)}\mathbb{E}\left[\left.\mathbb{1}_{\{t_d>T\}}\right|\mathcal{F}_t\right]dT+\int_t^{\infty}\Upsilon e^{-\omega(T-t)}\mathbb{E}\left[\left.e^{-r(t_d-t)}\mathbb{1}_{\{T\geq t_d\geq t\}}\right|\mathcal{F}_t\right]dT\\ &\quad+\int_t^{\infty}e^{-\omega(T-t)}\mathbb{E}\left[\left.\int_t^T be^{-r(z-t)}\mathbb{1}_{\{t_d>z\}}dz\right|\mathcal{F}_t\right]dT\end{aligned} \tag{7}$$

The following are the computations to obtain the final Laplace transform, which will be splitted in order to compute the three addendum of (7) separately. The *first addendum* is defined as:

$$\begin{aligned}\int_t^{\infty}e^{-(\omega+r)(T-t)}\mathbb{E}\left[\left.\mathbb{1}_{\{t_d>T\}}\right|\mathcal{F}_t\right]dT &= \mathbb{E}\left[\left.\int_t^{t_d}e^{-(\omega+r)(T-t)}dT\right|\mathcal{F}_t\right]\\ &= \frac{1}{\omega+r}\left[1-\mathbb{E}\left[\left.e^{-(r+\omega)(t_d-t)}\right|\mathcal{F}_t\right]\right]\end{aligned} \tag{8}$$

The *second addendum* is defined as:

$$\begin{aligned}\Upsilon\int_0^{\infty}e^{-\omega(T-t)}\mathbb{E}\left[\left.e^{-r(t_d-t)}\mathbb{1}_{\{T\geq t_d\geq 0\}}\right|\mathcal{F}_t\right]dT &= \Upsilon\mathbb{E}\left[\left.\int_{t_d}^{\infty}e^{-\omega(T-t)-r(t_d-t)}dT\right|\mathcal{F}_t\right]\\ &= \frac{\Upsilon}{\omega}\mathbb{E}\left[\left.e^{-(r+\omega)(t_d-t)}\right|\mathcal{F}_t\right]\end{aligned} \tag{9}$$

Before providing the final form of *third addendum*, we note that:

$$\begin{aligned}\mathbb{E}\left[\left.\int_t^T be^{-r(z-t)}\mathbb{1}_{\{t_d>z\}}dz\right|\mathcal{F}_t\right] &= b\mathbb{E}\left[\left.\int_t^{T\wedge t_d}e^{-r(z-t)}dz\right|\mathcal{F}_t\right]\\ &= \frac{b}{r}\mathbb{E}\left[\left.1-e^{-r((T\wedge t_d)-t)}\right|\mathcal{F}_t\right]\\ &= \frac{b}{r}\mathbb{E}\left[\left.1-e^{-r(t_d-t)}\mathbb{1}_{\{t_d<T\}}\right|\mathcal{F}_t\right]-\frac{b}{r}e^{-r(T-t)}\mathbb{E}\left[\left.\mathbb{1}_{\{t_d>T\}}\right|\mathcal{F}_t\right]\end{aligned} \tag{10}$$

where $\wedge$ is the min operator. Employing (10), the *third addendum* will be:

$$\int_t^{\infty} e^{-\omega(T-t)}\mathbb{E}\left[\int_t^T be^{-r(z-t)}\mathbb{1}_{\{t_d>z\}}dz\middle|\mathcal{F}_t\right]dT = \int_t^{\infty} e^{-\omega(T-t)}\frac{b}{r}\mathbb{E}\left[1-e^{-r(t_d-t)}\mathbb{1}_{\{t_d<T\}}\middle|\mathcal{F}_t\right]dT - \int_t^{\infty} e^{-\omega(T-t)}\frac{b}{r}e^{-r(T-t)}\mathbb{E}\left[\mathbb{1}_{\{t_d>T\}}\middle|\mathcal{F}_t\right]dT \tag{11}$$

Then, computing separately the two parts of the right-hand side of (11), we have:

$$\begin{aligned}\int_t^{\infty} e^{-\omega(T-t)}\left(\frac{b}{r}\mathbb{E}\left[1-e^{-r(t_d-t)}\mathbb{1}_{\{t_d<T\}}\middle|\mathcal{F}_t\right]\right)dT &= \frac{b}{r}\int_t^{\infty} e^{-\omega(T-t)}dT-\frac{b}{r}\mathbb{E}\left[\int_{t_d}^{\infty} e^{-r(t_d-t)-\omega(T-t)}dT\middle|\mathcal{F}_t\right]\\ &= \frac{b}{r\omega}-\frac{b}{r\omega}\mathbb{E}\left[e^{-(r+\omega)(t_d-t)}\middle|\mathcal{F}_t\right]\end{aligned} \tag{12}$$

and

$$\begin{aligned}-\int_0^{\infty} e^{-(\omega+r)(T-t)}\frac{b}{r}\mathbb{E}\left[\mathbb{1}_{\{t_d>T\}}\middle|\mathcal{F}_t\right]dT &= -\frac{b}{r}\mathbb{E}\left[\int_0^{t_d} e^{-(\omega+r)(T-t)}dT\middle|\mathcal{F}_t\right]\\ &= \frac{b}{r\left(r+\omega\right)}\mathbb{E}\left[e^{-(r+\omega)(t_d-t)}\middle|\mathcal{F}_t\right]-\frac{b}{r\left(r+\omega\right)}\end{aligned} \tag{13}$$

Finally, the *third addendum* is given by:

$$\int_0^{\infty} e^{-\omega(T-t)}\mathbb{E}\left[\int_t^T be^{-r(z-t)}\mathbb{1}_{\{t_d>z\}}dz\middle|\mathcal{F}_t\right]dT = \frac{b}{r\omega}-\frac{b}{r(r+\omega)}-\left(\frac{b}{r\omega}-\frac{b}{r(r+\omega)}\right)\mathbb{E}\left[e^{-(r+\omega)(t_d-t)}\middle|\mathcal{F}_t\right] \tag{14}$$

Collecting all the terms derived ((9),(8),(14)) and recalling formula (3), we obtain the Laplace transform of the bond price. ■

Proof of Proposition 2. Denote by Π the CDS spread which is payed continuously. Then, by definition and based on the modeling framework, the Premium Leg is defined as:

$$\text{PremiumLeg}\left(x,t;T\right) = \mathbb{E}\left[\Pi\int_t^T e^{-r(z-t)}\mathbb{1}_{\{t_d\geq z\}}dz\middle|\mathcal{F}_t\right] \tag{15}$$

The Laplace transform of the Premium Leg, denoted by $F_{PremiumLeg}\left(x,t;\omega\right)$, can be computed as already shown in (14), obtaining:

$$F_{PremiumLeg}\left(x,t;\omega\right) = \mathbb{E}\left[e^{-(r+\omega)(t_d-t)}\middle|\mathcal{F}_t\right]\left(\frac{\Pi}{r\left(r+\omega\right)}-\frac{\Pi}{r\omega}\right)+\frac{\Pi}{r\omega}-\frac{\Pi}{r\left(r+\omega\right)} \tag{16}$$

Moreover, the Protection Leg, denoting with $1-\Upsilon$ the loss-given default, is given by:

$$\text{ProtectionLeg}(x,t;T) = \mathbb{E}\left[e^{-r(t_d-t)}\left(1-\Upsilon\right)\mathbb{1}_{\{T\geq t_d\geq t\}}\middle|\mathcal{F}_t\right] \tag{17}$$

Similar as before, the Laplace transform of the Protection Leg, denoted by $F_{ProtectionLeg}\left(x,t;\omega\right)$, can be derived as in (9), obtaining:

$$F_{ProtectionLeg}\left(x,t;\omega\right) = \mathbb{E}\left[e^{-(r+\omega)(t_d-t)}\middle|\mathcal{F}_t\right]\frac{\left(1-\Upsilon\right)}{\omega} \tag{18}$$

Collecting the terms derived ((17), (18)) and recalling (3), the semi-analytical expression of the CDS spread in (11) of the paper follows by choosing Π that makes the Premium Leg equals to the Protection Leg. ■

3 Numerical Inversion of the Laplace transform

The semi-closed formulas in Propositions 1 and 2 for the pricing of defaultable coupon bonds and CDSs, respectively, require the numerical inversion of the Laplace transform. There are several methods we can use to approximate the solution. In this study, we adopt the so-called Gaver-Stehfest algorithm, see, e.g., Stehfest [1970] and Gaver-Jr [1966]. Assume that $F(x, t, \omega)$ is the Laplace transform of $\Phi(x, t, T)$. According to the Gaver-Stehfest algorithm, the inverse Laplace transform is approximated as follows:

$$\Phi(x, t, T) \approx \frac{\ln(2)}{T-t} \sum_{k=1}^{2M} \alpha_k^M F\left(x, t, \frac{k \ln(2)}{T-t}\right) \tag{19}$$

where

$$\alpha_k^M = \frac{(-1)^{M+k}}{M!} \sum_{j=\lfloor (k+1)/2 \rfloor}^{\min(k,M)} j^{M+1} \binom{M}{j} \binom{2j}{j} \binom{j}{(k-j)} \tag{20}$$

and $\lfloor \diamond \rfloor$ stands for the integer part of $\diamond$.

The accuracy of the algorithm depends on the parameter M, which has to be chosen correctly in order to avoid rounding errors. We observed via heuristic analysis that it is recommended to choose values of M between 6 and 8, as confirmed in the literature, see, e.g., Ballestra et al. [2017]. Indeed, for small values of M, a proper level of accuracy is not always obtained, whereas if the value of M increases, the stability of the algorithm declines as rounding errors can undermine the sum.

In order to validate all the formulas and, therefore, the Gaver-Stehfest algorithm, we study the accuracy of the method by comparing the solutions with an alternative approach. In particular, we computed the survival probability with a finite difference method (FDM). The numerical scheme has been implemented using a sufficiently thin grid, which ensures maximum precision. By fixing $M = 8$, in all the experiments conducted, we observed that the Gaver-Stehfest algorithm achieves a satisfactory level of accuracy of 10^{-4} in comparison with the FDM method.

Moreover, other numerical algorithms to compute the inverse of the Laplace transform have been tested. Specifically, we have also implemented the Bromwich Integral, see, e.g., Cathcart and El-Jahel [2006]. Although it was capable to obtain a proper level of accuracy, the Gaver-Stehfest algorithm was preferred for its computational performance, as it is twice as fast as the Bromwich Integral.

4 The Credit-Risk Model Without Jump

In this section we summarize credit-risk structural model without jump and report the pricing formula for a CDS. In this simplified framework, the value of a firm is driven by a Geometric Brownian Motion, defined as:

$$\frac{dS_t}{S_t} = r dt + \sigma_s dW_t^Q \tag{21}$$

where S_t denotes the firm's asset value, r is the risk-free interest rate and σ_s is the volatility parameter and W_t is a standard Weiner process under the risk neutral measure Q. As in the model defined in Section 3 , the firm goes bankrupt when breaches from above a constant default barrier S_{def}, and it coincides with the first-time passage:

$$t_d = \inf \{t > 0 : S_t \leq S_{def}\}, \quad t \in (0, T] \tag{22}$$

Following Brigo et al. [2013] and emplying the notation $z = \ln(S)$, the survival probability $\mathbb{P}^{surv}(z, t; T)$ computed in t is defined as:

$$\mathbb{P}^{surv}(z, t; T) = N\left(\frac{\ln \frac{S_t}{S_{def}} + m(T-t)}{\sigma\sqrt{T-t}}\right) - \left(\frac{S_{def}}{S_t}\right)^{\frac{2m}{\sigma^2}} N\left(\frac{\ln \frac{S_{def}}{S_t} + m(T-t)}{\sigma\sqrt{T-t}}\right) \tag{23}$$

where $m = r - \frac{1}{2}\sigma_s^2$ and T indicates the maturity.

4.1 Pricing a CDS in the No-Jump Framework

Having derived the survival probability $\mathbb{P}^{surv}(z, t; T)$ defined in (23), we follow the approach proposed in Fang et al. [2010] to derive the formula to price a CDS spread, denoted by Λ. It follows that:

$$\Lambda(z, t; T) = \frac{(1 - \Upsilon)\left(\int_t^T e^{-r(T-t)} d\mathbb{P}^{def}(z, t; s)\right)}{\int_t^T e^{-r(s-t)} \mathbb{P}^{surv}(z, t; s) ds} \tag{24}$$

Integrating by parts and defining the default probability as $\mathbb{P}^{def}(z, t; T) = 1 - \mathbb{P}^{surv}(z, t; T)$, we obtain the final form of the pricing formula:

$$\Lambda(z, t; T) = (1 - \Upsilon)\left(\frac{1 - e^{-r(T-t)} \mathbb{P}^{surv}(z, t; T)}{\int_t^T e^{-r(s-t)} \mathbb{P}^{surv}(z, t; s) ds} - r\right) \tag{25}$$

The integral at the denominator of formula (25) can be approximated using a classical quadrature scheme method, see, e.g., Fang et al. [2010].

5 Theoretical Term-Structure of the Credit Spread

In the modeling framework outlined in Section 3, the parameters λ and η drive the transition risk whereby, the former models the arrival rate of more stringent green regulations, and the latter describes the magnitude of downward jumps on the firm's value. In fact, as underlined in Agliardi and Agliardi [2021], the parameter η describes the impact of stricter green policies on the value of a firm, in the manner whereby the higher is η, the greener is the firm.

In this section we conduct a sensitivity analysis on the parameter η, generating several credit spreads. First, in Figure 1, we report the theoretical term-structure of a defaultable (green) bond and a CDS spread, generated by the model pricing formulas for different level of greenness. We observe an increment of the price of the bond as η increases (greenness is higher). A lower bond price, corresponding to a lower η value, guarantees a higher return to the investor, compensating for the higher exposure to transition risk.

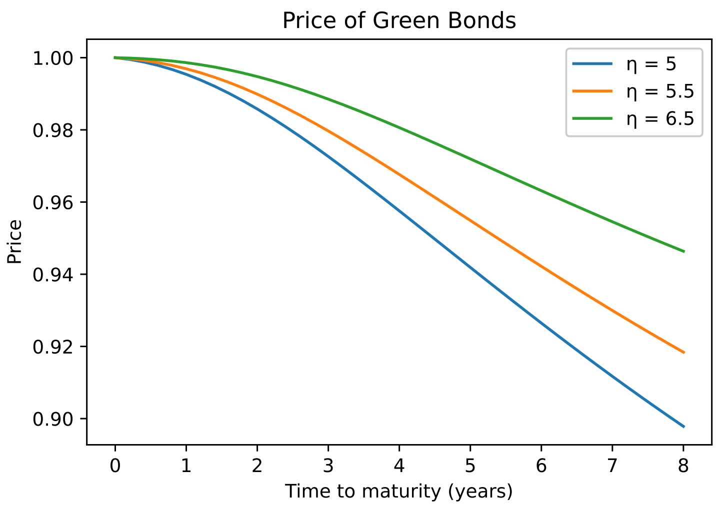

Figure 1: $V_t = 4$, $V_{def} = 1$, $\Upsilon = 0.6$, b = 0, r = 0.02 , $\sigma = 0.2$, $\lambda = 0.4$.

Similarly, in Figure 2 we present the CDS spreads generated by the model. We can observe that, as η increases, the CDS spread decreases. This indicates that the CDS spread of green companies is lower than that of brown companies.

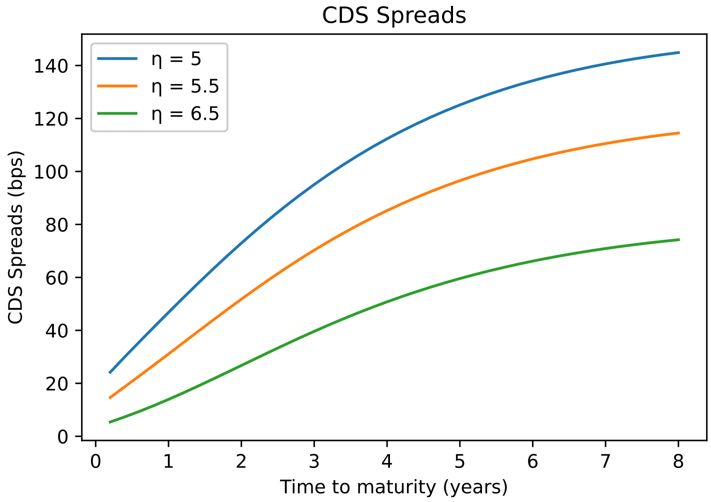

Figure 2: $V_t = 4$, $V_{def} = 1$, $\Upsilon = 0.6$, r $= 0.02$, $\sigma = 0.2$, $\lambda = 0.4$.

In the last Figure 3 we derive the green spread generated by the model, computing the yields of a pure green bond and of a bond issued by a firm with a given shade of greenness. Specifically, a pure green bond is a bond issued by a firm that is not affected by transition risk, so that, its value follows a diffusion process without jumps. Assuming that the bond considered neither pays coupons nor refunds the holder with the recovery rate in the default event, the green spread is given by:

$$\Psi(x,t;T,\eta) = \frac{-\log(B(x,t;T,\eta)/(e^{-r(T-t)}\mathbb{P}^{surv}(x,t;T))}{T-t} \tag{26}$$

where $\mathbb{P}^{surv}(x,t;T)$ is defined in (23) and B is the bond price derived by our model, under the same assumptions as the pure green bond. The theoretical term-structure of the green spreads for different shade of greenness is reported in Figure 3.

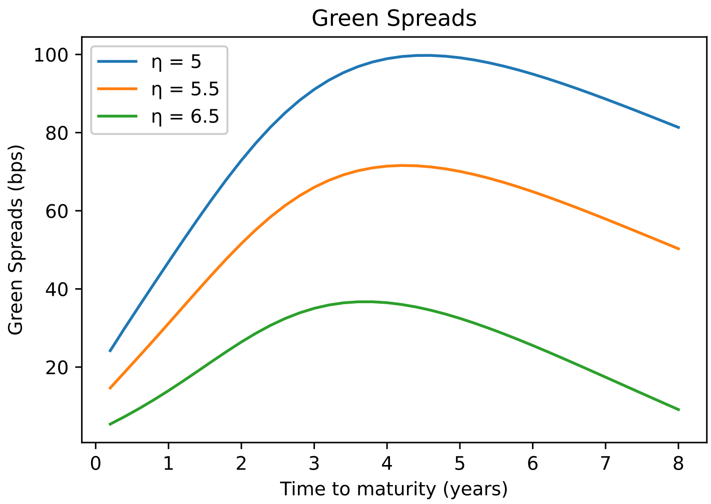

Figure 3: $V_t = 4$, $V_{def} = 1$, $\Upsilon = 0$, b $= 0$, r $= 0.02$, $\sigma = 0.2$, $\lambda = 0.4$.

6 Panel Quantile Regressions Tables

		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
6 m	Return	−4.1068***	−2.3116***	−1.3085***	−0.8791***	−0.6093***	−0.8423***	−1.4223***	−2.4705***	−5.1219***
		(0.4655)	(0.1055)	(0.0400)	(0.0203)	(0.0125)	(0.0164)	(0.031)	(0.078)	(0.4157)
	Δ Vol	−3.3603***	−0.7522***	−0.2759***	−0.1066***	−0.0086***	0.0728***	0.2226***	0.5595***	2.7067***
		(0.0262)	(0.0061)	(0.0024)	(0.0013)	(0.0008)	(0.0010)	(0.0019)	(0.0044)	(0.0221)
	Δ MRI	0.3477***	0.3116***	0.2888***	0.2737***	0.2700***	0.2789***	0.3211***	0.3947***	0.5663***
		(0.0178)	(0.0030)	(0.0010)	(0.0004)	(0.0002)	(0.0003)	(0.0006)	(0.0017)	(0.0115)
	Δ TR	0.0943***	0.0760***	0.0702***	0.0663***	0.0644***	0.0717***	0.0815***	0.0925***	0.1421***
		(0.0134)	(0.0028)	(0.0010)	(0.0004)	(0.0002)	(0.0003)	(0.0007)	(0.0019)	(0.0105)
1 y	Return	−6.3824***	−3.1756***	−1.9581***	−1.3818***	−0.9189***	−1.3005***	−2.1068***	−3.5178***	−7.0436***
		(0.5008)	(0.1389)	(0.0588)	(0.0330)	(0.0184)	(0.0254)	(0.0465)	(0.1001)	(0.4515)
	Δ Vol	−3.8476***	−0.9763***	−0.4102***	−0.1749***	−0.0139***	0.1111***	0.3207***	0.7686***	2.9986***
		(0.0278)	(0.0081)	(0.0036)	(0.0021)	(0.0011)	(0.0016)	(0.0028)	(0.0055)	(0.0231)
	Δ MRI	0.4105***	0.3998***	0.3862***	0.3745***	0.3674***	0.3767***	0.4117***	0.4782***	0.6534***
		(0.0152)	(0.0033)	(0.0012)	(0.0006)	(0.0003)	(0.0004)	(0.0008)	(0.0019)	(0.0105)
	Δ TR	0.0922***	0.0923***	0.0924***	0.0897***	0.0850***	0.0937***	0.1094***	0.1210***	0.1417***
		(0.0126)	(0.0032)	(0.0012)	(0.0006)	(0.0003)	(0.0004)	(0.0008)	(0.0020)	(0.0101)
2 y	Return	−7.6878***	−3.9118***	−2.7359***	−2.009***	−1.4553***	−2.0721***	−2.9864***	−4.7830***	−9.2548***
		(0.6422)	(0.1765)	(0.0978)	(0.0598)	(0.0374)	(0.0509)	(0.0810)	(0.1555)	(0.5562)
	Δ Vol	−4.1547***	−1.2675***	−0.6649***	−0.2695***	−0.0113***	0.2052***	0.5546***	1.1956***	3.3557***
		(0.0340)	(0.0101)	(0.0059)	(0.0037)	(0.0023)	(0.0031)	(0.0047)	(0.0085)	(0.0272)
	Δ MRI	0.4236***	0.3769***	0.3753***	0.3645***	0.3590***	0.3673***	0.4055***	0.4567***	0.5719***
		(0.0134)	(0.0029)	(0.0014)	(0.0008)	(0.0004)	(0.0006)	(0.0010)	(0.0022)	(0.0093)
	Δ TR	0.1451***	0.1486***	0.1443***	0.1409***	0.1403***	0.1521***	0.1656***	0.1790***	0.1969***
		(0.0135)	(0.0033)	(0.0016)	0.000964	0.000583	0.00079	0.001298	0.002672	0.010565
3 y	Return	−8.5255***	−4.6270***	−3.2277***	−2.5058***	−1.9138***	−2.6231***	−3.8275***	−5.9435***	−11.1866***
		(0.7384)	(0.2370)	(0.1324)	(0.0844)	(0.0539)	(0.0696)	(0.1132)	(0.2085)	(0.6588)
	Δ Vol	−4.3768***	−1.5770***	−0.8194***	−0.3748***	−0.0381***	0.2488***	0.7247***	1.4830***	3.7591***
		(0.0381)	(0.0133)	(0.0079)	(0.0052)	(0.0033)	(0.0042)	(0.0066)	(0.0112)	(0.0310)
	Δ MRI	0.3993***	0.3560***	0.3489***	0.3461***	0.3462***	0.3526***	0.3789***	0.4192***	0.5136***
		(0.0110)	(0.0029)	(0.0014)	(0.0008)	(0.0005)	(0.0006)	(0.0011)	(0.0022)	(0.0079)
	Δ TR	0.1607***	0.1583***	0.1554***	0.1521***	0.1493***	0.1583***	0.1666***	0.1790***	0.1986***
		(0.0126)	(0.0036)	(0.0018)	(0.0011)	(0.0006)	(0.0008)	(0.0014)	(0.0028)	(0.0097)
4 y	Return	−9.7164***	−5.0982***	−3.6035***	−2.7001***	−2.0107***	−2.7411***	−3.8935***	−6.4579***	−12.8842***
		(0.8356)	(0.2724)	(0.1548)	(0.1011)	(0.0653)	(0.0845)	(0.1288)	(0.2316)	(0.7601)
	Δ Vol	−4.6445***	−1.7249***	−0.9787***	−0.4528***	−0.0575***	0.2619***	0.7880***	1.6494***	4.3665***
		(0.0424)	(0.0153)	(0.0093)	(0.0063)	(0.0041)	(0.0052)	(0.0075)	(0.0124)	(0.0358)
	Δ MRI	0.4402***	0.4031***	0.4047***	0.4008***	0.4016***	0.4090***	0.4380***	0.4710***	0.5695***
		(0.0119)	(0.0032)	(0.0016)	(0.0009)	(0.0005)	(0.0007)	(0.0012)	(0.0023)	(0.0087)
	Δ TR	0.1908***	0.1864***	0.1816***	0.1788***	0.1771***	0.1873***	0.2002***	0.2213***	0.2362***
		(0.0139)	(0.0040)	(0.0020)	(0.0012)	(0.0008)	(0.0010)	(0.0016)	(0.0031)	(0.0112)
5 y	Return	−11.4430***	−5.8459***	−4.2022***	−3.0990***	−2.4444***	−3.2634***	−4.5913***	−7.0986***	−14.0178***
		(0.9349)	(0.2954)	(0.1726)	(0.1101)	(0.0720)	(0.0908)	(0.1408)	(0.2307)	(0.7192)
	Δ Vol	−4.9478***	−1.9719***	−1.0746***	−0.4912***	−0.0566***	0.2644***	0.8293***	1.7603***	4.5607***
		(0.0469)	(0.0166)	(0.0103)	(0.0068)	(0.0045)	(0.0056)	0.008272	(0.01237)	(0.03361)
	Δ MRI	0.4271***	0.3940***	0.4001***	0.4037***	0.4105***	0.4188***	0.4473***	0.4818***	0.5825***
		(0.0129)	(0.0033)	(0.0017)	(0.0010)	(0.0006)	(0.0007)	(0.0012)	(0.0022)	(0.0080)
	Δ TR	0.2106***	0.2092***	0.1978***	0.1898***	0.1839***	0.1949***	0.2127***	0.2343***	0.2505***
		(0.0150)	(0.0042)	(0.0022)	(0.0013)	(0.0008)	(0.0010)	(0.0017)	(0.0030)	(0.0103)
7 y	Return	−13.7272***	−6.8163***	−4.6931***	−3.4492***	−2.7840***	−3.5714***	−4.9224***	−7.7603***	−15.7068***
		(1.0001)	(0.3354)	(0.1851)	(0.1176)	(0.0831)	(0.1011)	(0.1497)	(0.2427)	(0.8547)
	Δ Vol	−5.8357***	−2.2695***	−1.1539***	−0.5138***	−0.0590***	0.3022***	0.9254***	2.0163***	5.0416***
		(0.0502)	(0.0188)	(0.0111)	(0.0073)	(0.0052)	(0.0062)	(0.0087)	(0.0130)	(0.0396)
	Δ MRI	0.3728***	0.3363***	0.3422***	0.3419***	0.3476***	0.3536***	0.3827***	0.4242***	0.5267***
		(0.0129)	0.003616	0.001758	0.001023	0.000689	0.000832	0.001282	0.002266	(0.0090)
	Δ TR	0.2736***	0.2676***	0.2531***	0.2533***	0.2480***	0.2623***	0.2848***	0.3086***	0.3351***
		(0.0157)	(0.0047)	(0.0023)	(0.0014)	(0.0009)	(0.0011)	(0.0018)	(0.0031)	(0.0121)
10 y	Return	−14.6564***	−7.4115***	−5.1175***	−3.7870***	−2.9852***	−3.5927***	−5.1934***	−8.5166***	−16.3607***
		(0.9858)	(0.3367)	(0.1842)	(0.1188)	(0.0819)	(0.0943)	(0.1479)	(0.2567)	(0.7713)
	Δ Vol	−5.7388***	−2.3635***	−1.1795***	−0.5289***	−0.0846***	0.2555***	0.8601***	2.1040***	5.3533***
		(0.0492)	(0.0189)	(0.0110)	(0.0074)	(0.0051)	(0.0058)	(0.0086)	(0.0137)	(0.0352)
	Δ MRI	0.3569***	0.3195***	0.3240***	0.3179***	0.3184***	0.3283***	0.3515***	0.3881***	0.4769***
		(0.0124)	(0.0035)	(0.0017)	(0.0010)	(0.0006)	(0.0007)	(0.0012)	(0.0023)	(0.0080)
	Δ TR	0.258281***	0.260511***	0.239783***	0.239518***	0.238234***	0.250074***	0.273796***	0.306784***	0.349737***
		(0.0152)	(0.0046)	(0.0023)	(0.0014)	(0.0009)	(0.0010)	(0.0017)	(0.0032)	(0.0107)
20 y	Return	−14.7697***	−7.1142***	−4.6905***	−3.2396***	−2.6567***	−3.0845***	−4.5915***	−7.8075***	−16.1003***
		(0.9495)	(0.2864)	(0.1589)	(0.1026)	(0.0728)	(0.0831)	(0.1223)	(0.2288)	(0.7432)
	Δ Vol	−5.7421***	−2.2686***	−1.0401***	−0.4137***	−0.0417***	0.2859***	0.8837***	1.9942***	5.1096***
		(0.0478)	(0.0162)	(0.0096)	(0.0064)	(0.0046)	(0.0051)	(0.0072)	(0.0121)	(0.0345)
	Δ MRI	0.2214***	0.2061***	0.2078***	0.2045***	0.2028***	0.2092***	0.2313***	0.2630***	0.3280***
		(0.0107)	(0.0027)	(0.0013)	(0.0007)	(0.0005)	(0.0006)	(0.0009)	(0.0019)	(0.0074)
	Δ TR	0.2256***	0.2127***	0.1953***	0.1874***	0.1894***	0.2012***	0.2276***	0.2568***	0.2922***
		(0.0135)	(0.0037)	(0.0018)	(0.0011)	(0.0007)	(0.0009)	(0.0013)	(0.0027)	(0.0100)
30 y	Return	−14.5589***	−7.2858***	−4.8345***	−3.2554***	−2.571***	−3.0110***	−4.4499***	−7.6243***	−15.3198***
		(0.9252)	(0.2788)	(0.1450)	(0.0936)	(0.0655)	(0.0735)	(0.1109)	(0.2160)	(0.7042)
	Δ Vol	−5.9629***	−2.2174***	−0.9481***	−0.3754***	−0.0258***	0.2872***	0.8230***	1.9192***	5.3181***
		(0.0469)	(0.0158)	(0.0088)	(0.0059)	(0.0042)	(0.0046)	(0.0065)	(0.0115)	(0.0332)
	Δ MRI	0.1936***	0.1713***	0.1679***	0.1601***	0.1596***	0.1634***	0.1785***	0.2107***	0.2743***
		(0.0099)	(0.0025)	(0.0011)	(0.0006)	(0.0004)	(0.0005)	(0.0008)	(0.0018)	(0.0070)
	Δ TR	0.1893***	0.1715***	0.1559***	0.1541***	0.1524***	0.1661***	0.1855***	0.2130***	0.2449***
		(0.0124)	(0.0034)	(0.0016)	(0.0010)	(0.0006)	(0.0007)	(0.0012)	(0.0025)	(0.0091)

$^{***}p < 0.001$

Table 1: This table presents the results of the panel regression for the *jump-diffusion* model, where the dependent variable is the first-difference model implied CDS spread ($\Delta \text{CDS}_{i,t}^{model}$) computed between two consecutive days. The columns contains the estimates for the quantiles $\tau \in \{0.1, ..., 0.9\}$. Standard errors are reported between brackets ().

		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
6 m	Return	-0.9614^{**}	$-0.0350^{\diamond}$	-0.0001^{**}	$-0.00002^{\diamond}$	$0.000002^{\diamond}$	$0.000003^{\diamond}$	$0.000004^{\diamond}$	$-0.00001^{\diamond}$	$0.0009^{\diamond}$
		(0.2930)	(0.0175)	(0.00007)	(0.00001)	(0.000007)	(0.000007)	(0.000007)	(0.0001)	(0.0477)
	Vol	-2.4894^{***}	0.0539^{***}	-0.00003^{***}	-0.00002^{***}	-0.000005^{***}	0.000003^{***}	0.00002^{***}	0.00063^{***}	0.0611^{**}
		(0.0268)	(0.0017)	(0.000007)	(0.000001)	(0.000001)	(0.000001)	(0.000001)	(0.00001)	(0.0051)
	TR	-0.0377^{***}	-0.0103^{***}	-0.00001^{***}	-0.000001^{***}	0.0^{***}	0.0^{***}	0.0^{***}	-0.000004^{***}	-0.0003^{***}
		(0.0007)	(0.00005)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0001)
1 y	Return	$-0.5000^{\diamond}$	$-0.0292^{\diamond}$	-0.366554^{***}	$-0.0002^{\diamond}$	$-0.0393^{\diamond}$	-1.0287^{***}	-1.4334^{***}	-1.7115^{***}	-2.3132^{*}
		(2.0729)	(0.2365)	(0.0797)	(0.0464)	(0.0426)	(0.2276)	(0.2666)	(0.2746)	(1.1636)
	Vol	-9.0090^{***}	-1.3912^{***}	-0.2472^{***}	$-0.00005^{\diamond}$	$0.001489^{\diamond}$	0.4965^{***}	1.5059^{***}	2.6191^{***}	5.774416^{***}
		(0.2043)	(0.0230)	(0.0079)	(0.0046)	(0.0042)	(0.0230)	(0.0270)	(0.0277)	(0.1163)
	TR	0.039776^{***}	0.00185^{**}	0.001208^{***}	$0.0^{\diamond}$	$0.0001^{\diamond}$	$0.0014^{\diamond}$	-0.000176^{***}	0.007189^{***}	0.009777^{**}
		(0.0053)	(0.0005)	(0.0002)	(0.0001)	(0.0001)	(0.0005)	(0.0006)	(0.0007)	(0.0029)
2 y	Return	$1.8717^{\diamond}$	$1.2692^{\diamond}$	$0.3357^{\diamond}$	$-0.0186^{\diamond}$	$0.0374^{\diamond}$	$0.01779^{\diamond}$	$-0.0514^{\diamond}$	$-0.3220^{\diamond}$	$-1.5636^{\diamond}$
		(2.5714)	(0.7613)	(0.2972)	(0.1339)	(0.1088)	(0.1296)	(0.4283)	(0.3963)	(0.8213)
	Vol	-13.6256^{***}	-4.5514^{***}	-1.2938^{***}	-0.3106^{***}	-0.1369^{***}	-0.1881^{***}	1.00632^{***}	2.1511^{***}	6.9873^{***}
		(0.3022)	(0.0914)	(0.0342)	(0.0152)	(0.0124)	(0.0148)	(0.0490)	(0.0464)	(0.1066)
	TR	0.0605^{***}	0.02805^{***}	0.01205^{***}	0.0069^{***}	0.0054^{***}	0.0098^{***}	0.0092^{***}	0.0093^{***}	-0.0161^{***}
		(0.0056)	(0.0016)	(0.0006)	(0.0002)	(0.0002)	(0.0002)	(0.0008)	0.0008)	(0.0018)
3 y	Return	4.8017^{*}	4.4491^{***}	2.6799^{***}	2.7078^{***}	2.4222^{***}	1.3505^{***}	0.8565^{***}	1.3805^{***}	$0.4613^{\diamond}$
		(2.5142)	(0.6963)	(0.4859)	(0.4142)	(0.3829)	(0.2543)	(0.2263)	(0.3366)	(0.8128)
	Vol	-9.6000^{***}	-3.9788^{***}	-2.1711^{***}	-1.6477^{***}	-0.9967^{***}	0.5893^{***}	1.0509^{***}	2.0409^{***}	6.8891^{***}
		(0.3248)	(0.0874)	(0.0604)	(0.0519)	(0.0483)	(0.0324)	(0.0289)	(0.0440)	(0.1105)
	TR	-0.004047	-0.0173^{***}	-0.0184^{***}	-0.0133^{***}	-0.0128^{***}	-0.0203^{***}	-0.0149^{***}	-0.0173^{***}	-0.0373^{***}
		(0.0043)	(0.0011)	(0.0008)	(0.0006)	(0.0006)	(0.0004)	(0.0003)	(0.0005)	(0.0014)
4 y	Return	$0.7850^{\diamond}$	0.497041^{*}	0.185274^{*}	0.002744^{*}	$-0.00017^{\diamond}$	$0.00981^{\diamond}$	$-0.548479^{\diamond}$	$-0.539229^{\diamond}$	$-1.559522^{\diamond}$
		(0.7707)	(0.2488)	(0.0938)	(0.0802)	(0.0848)	(0.1146)	(0.2766)	(0.3805)	(0.7517)
	Vol	-4.5027^{***}	-1.5483^{***}	-0.6131^{***}	-0.1837^{***}	-0.0926^{***}	-0.0235^{*}	0.7201^{***}	2.2797^{***}	6.4748^{***}
		(0.1170)	(0.0358)	(0.0134)	(0.0113)	(0.0117)	(0.0156)	(0.0373)	(0.0484)	(0.0940)
	TR	0.01441^{***}	0.0064^{***}	0.0037^{***}	0.0013^{***}	0.0009^{***}	0.0026^{***}	0.0017^{***}	-0.0017^{***}	-0.0087^{***}
		(0.0011)	(0.0003)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0003)	(0.0004)	(0.0009)
5 y	Return	5.21413^{*}	0.975125^{*}	$-0.224915^{\diamond}$	$-0.261743^{\diamond}$	-0.657334^{**}	-0.78973^{*}	-1.167283^{***}	-1.543307^{***}	-2.259644^{**}
		(2.5557)	(0.5411)	(0.3899)	(0.2167)	(0.2479)	(0.3236)	(0.3037)	(0.4103)	(0.8594)
	Vol	-31.4475^{***}	-7.1585^{***}	-1.7761^{***}	-0.2479^{***}	$0.0440^{\diamond}$	0.7375^{***}	0.9706^{***}	3.2140^{***}	6.7653^{***}
		(0.4325)	(0.0827)	(0.0573)	(0.0316)	(0.0356)	(0.0461)	(0.0414)	(0.0529)	(0.1084)
	TR	0.1200^{***}	0.0265^{***}	0.00714^{***}	0.00378^{***}	0.0050^{***}	0.0059^{***}	0.0126^{***}	0.0107^{***}	0.0189^{***}
		(0.0035)	(0.0006)	(0.0004)	(0.0002)	0.0002)	(0.0003)	(0.0003)	(0.0004)	(0.0008)
7 y	Return	8.2547^{**}	$0.7634^{\diamond}$	-2.0723^{*}	$-0.9384^{\diamond}$	-4.1131^{***}	-4.6397^{***}	-3.8456^{***}	-5.7417^{***}	-7.2098^{***}
		(3.0663)	(1.6973)	(1.1137)	(0.6278)	0.8715)	(0.9780)	(1.0697)	(1.3881)	(1.6044)
	Vol	-58.1959^{***}	-17.2456^{***}	-5.5184^{***}	0.8484^{***}	3.3317^{***}	7.1436^{***}	8.1941^{***}	9.7285^{***}	14.4300^{***}
		(0.5367)	(0.2884)	(0.1812)	(0.0991)	(0.1339)	(0.1448)	(0.1490)	(0.1799)	(0.2011)
	TR	0.1481^{***}	0.0473^{***}	0.0093^{***}	-0.0028^{***}	$0.0011^{\diamond}$	$-0.0008^{\diamond}$	0.0136^{***}	0.0295^{***}	0.0421^{***}
		(0.0031)	(0.0017)	(0.0010)	(0.0005)	(0.0008)	(0.0008)	(0.0009)	(0.0011)	(0.00123)
10 y	Return	$0.7107^{\diamond}$	$-2.3099^{\diamond}$	-4.1078^{*}	-2.7753^{***}	-2.1724	-2.8258^{***}	-2.7153^{*}	-4.5127^{**}	$-1.7540^{\diamond}$
		(5.3367)	(2.8837)	(2.3708)	(1.1838)	(0.6202)	(0.8541)	(1.2523)	(1.6100)	(2.3109)
	Vol	-66.5803^{***}	-13.4016^{***}	-3.8290^{***}	2.3298^{***}	1.8154^{***}	6.5870^{***}	8.3139^{***}	11.7102^{***}	1.4856^{***}
		(0.7702)	(0.4435)	(0.3890)	(0.1920)	(0.0986)	(0.1311)	(0.1874)	(0.2304)	(0.3735)
	TR	0.0400^{***}	-0.0717^{***}	-0.0417^{***}	-0.0244^{***}	-0.0054^{***}	-0.0116^{***}	0.0076^{***}	0.0221^{***}	0.1431^{***}
		(0.0040)	(0.0023)	(0.0020)	(0.0010)	(0.0005)	(0.0006)	(0.0009)	(0.0012)	(0.0019)
20 y	Return	28.8820^{**}	$2.5063^{\diamond}$	$0.2324^{\diamond}$	$-0.8500^{\diamond}$	$0.0291^{\diamond}$	$0.4854^{\diamond}$	$2.1068^{\diamond}$	$1.0833^{\diamond}$	$1.4380^{\diamond}$
		(9.1559)	(4.4486)	(2.8916)	(1.7687)	(1.6311)	(1.4683)	(1.3019)	(1.3198)	(1.3672)
	Vol	-81.2161^{***}	-17.2253^{***}	4.0578^{***}	8.2122^{***}	9.6627^{***}	8.2464^{***}	8.3767^{***}	10.3641^{***}	8.2334^{***}
		(1.2334)	(0.6497)	(0.4547)	(0.2860)	(0.2607)	(0.2357)	(0.2096)	(0.2162)	(0.2384)
	TR	0.04070^{***}	-0.0776^{***}	-0.0810^{***}	-0.0602^{***}	-0.0508^{***}	-0.0363^{***}	-0.0274^{***}	-0.0225^{***}	0.0099^{***}
		(0.0062)	(0.0032)	(0.0022)	(0.0014)	(0.0012)	(0.0011)	(0.0010)	(0.0010)	(0.0011)
30 y	Return	42.7162^{***}	14.7453^{**}	7.3467^{*}	$1.7350^{\diamond}$	$-0.6425^{\diamond}$	$-0.0578^{\diamond}$	$0.5922^{\diamond}$	$0.2169^{\diamond}$	1.3834^{**}
		(12.9178)	(5.4523)	(2.9647)	(2.7206)	(2.5582)	(1.8428)	(0.5233)	(0.4528)	(0.6551)
	Vol	-128.7750^{***}	-31.0030^{***}	-6.6050^{***}	9.0796^{***}	14.4791^{***}	9.5935^{***}	2.7888^{***}	0.3753^{***}	5.1133^{***}
		(1.7104)	(0.7795)	(0.4573)	(0.4358)	(0.4087)	(0.2861)	(0.0795)	(0.0744)	(0.1183)
	TR	$0.0032^{\diamond}$	-0.1559^{***}	-0.1590^{***}	-0.1758^{***}	-0.1549^{***}	-0.0823^{***}	-0.0195^{***}	-0.0021^{***}	-0.0204^{***}
		(0.0084)	(0.0037)	(0.0021)	(0.0020)	(0.0019)	(0.0013)	(0.0003)	(0.0003)	(0.0005)

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.1$, $^{\diamond}$ $0.1 < p < 1$

Table 2: This table presents the results of the panel regression for the *jump-diffusion* model, where the dependent variable is the error $\mathcal{E}_{i,t}$ between model and market CDS. The columns contains the estimates for the quantiles $\tau \in \{0.1, ..., 0.9\}$. Standard errors are reported between brackets ().

		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
6 m	Return	3.1055◇	4.0041◇	2.3562◇	1.8887◇	1.1614◇	0.3790◇	1.1465◇	−0.0997◇	0.0223◇
		(6.1933)	(4.6106)	(2.1093)	(1.7640)	(1.1208)	(0.8851)	(0.6865)	(0.6457)	(0.5893)
	Δ Vol	−252.2250***	−70.5118***	−43.6450***	−31.3166***	−21.7309***	−16.6341***	−12.8272***	−8.6226***	−4.9525***
		(0.3538)	(0.2844)	(0.1323)	(0.1128)	(0.0727)	(0.0583)	(0.0457)	(0.0434)	(0.0393)
	Δ MRI	−0.6949***	−0.4702**	−0.344013***	−0.3260***	−0.2174***	−0.2156***	−0.1851***	−0.1981***	−0.1701***
		(0.1973)	(0.1444)	(0.0566)	(0.0418)	(0.0235)	(0.0165)	(0.0112)	(0.0089)	(0.0071)
	Δ TR	−0.1276◇	−0.2353*	−0.1569**	−0.1882***	−0.2093***	−0.1997***	−0.1477***	−0.1259***	−0.0801***
		(0.1416)	(0.1075)	(0.0484)	(0.0405)	(0.0256)	(0.0198)	(0.0149)	(0.0134)	(0.0116)
1 y	Return	8.9587◇	5.5347◇	4.2768◇	3.9028*	2.9270*	1.5977◇	0.3075◇	−0.2208◇	0.3661◇
		(7.1282)	(4.7106)	(2.6958)	(1.9723)	(1.2941)	(1.0615)	(0.7826)	(0.7758)	(0.8562)
	Δ Vol	−205.6918***	−71.8713***	−46.1858***	−32.9265***	−22.9435***	−16.9662***	−12.3693***	−9.4609***	−4.5862***
		(0.404351	(0.285805	(0.1685)	(0.1256)	(0.0839)	(0.0701)	(0.0526)	(0.0528)	(0.0589)
	Δ MRI	−0.322135*	−0.218918*	−0.2002***	−0.2232***	−0.1914***	−0.1078***	−0.0959***	0.0168◇	0.0630***
		(0.1783)	(0.1148)	(0.0575)	(0.0378)	(0.0225)	(0.0173)	(0.0125)	(0.0134)	(0.0182)
	Δ TR	−0.0744◇	−0.1773*	−0.16506***	−0.2017***	−0.1870***	−0.2004***	−0.1742***	−0.1649***	−0.0951***
		(0.1435)	(0.0932)	(0.0523)	(0.0384)	(0.0246)	(0.0193)	(0.0134)	(0.0126)	(0.0128)
2 y	Return	0.812715◇	3.2147◇	3.1538◇	0.3426◇	0.2168◇	−0.1906◇	−0.6844◇	−1.0564◇	−0.7422◇
		(3.1085)	(2.5786)	(1.9360)	(1.5240)	(1.1930)	(1.0468)	(1.0290)	(1.1760)	(1.2994)
	Δ Vol	−57.6960***	−41.7341***	−32.1719***	−24.8106***	−20.1381***	−15.6891***	−11.2799***	−6.7300***	0.6516***
		(0.1838)	(0.1609)	(0.1218)	(0.0962)	(0.0759)	(0.0667)	(0.0654)	(0.0745)	(0.0791)
	Δ MRI	−0.0796◇	−0.0295◇	0.0227◇	0.0476*	0.0518***	0.0406***	0.0568***	0.0658***	0.1228***
		(0.0487)	(0.0397)	(0.0268)	(0.01997)	(0.0153)	(0.0137)	(0.0142)	(0.0179)	(0.0239)
	Δ TR	−0.0644◇	−0.0926*	−0.1478***	−0.2204***	−0.1896***	−0.1909***	−0.1881***	−0.1286***	−0.0858***
		(0.0540)	(0.0439)	(0.0322)	(0.0246)	(0.0185)	(0.0157)	(0.0148)	(0.0168)	(0.0187)
3 y	Return	−2.4544*	−1.3447◇	−0.9795◇	−1.9624*	−1.3671◇	−1.2944◇	−0.00002◇	−0.000003◇	−0.1707◇
		(1.3012)	(1.2575)	(0.9699)	(0.9791)	(1.0420)	(1.0314)	(0.4189)	(0.3595)	(1.8849)
	Δ Vol	−26.1262***	−25.94676***	−23.5189***	−18.7001***	−15.4271***	−8.240209◇	−0.00001◇	0.000001◇	10.6677***
		(0.0754)	(0.0777)	(0.0603)	(0.0617)	(0.0656)	(0.0644)	(0.0257)	(0.0215)	(0.1029)
	Δ MRI	−0.0317***	0.0011◇	0.0262***	0.014433*	0.0285***	0.0241***	0.0***	0.0◇	0.0015◇
		(0.0104)	(0.0100)	(0.0082)	(0.0086)	(0.0099)	(0.0105)	(0.0043)	(0.0040)	(0.0215)
	Δ TR	−0.173613***	−0.1577***	−0.1734***	−0.1658***	−0.1652***	−0.1343***	−0.000001◇	0.0◇	−0.008198◇
		(0.0211)	(0.0176)	(0.01336)	(0.0130)	(0.0132)	(0.0126)	(0.0048)	(0.0042)	(0.0219)
4 y	Return	−2.3510*	−2.8125*	−1.9876*	−1.9116*	−2.1913**	−0.00005◇	−4.1577***	−4.0595*	−3.9568*
		(1.3822)	(1.2432)	(1.1208)	(0.9088)	(0.7883)	(0.3988)	(1.2386)	(1.7229)	(2.0509)
	Δ Vol	−17.8775***	−19.0492***	−14.2031***	−9.311***	−5.2140***	−0.00001◇	2.9852***	14.8115***	35.7578***
		(0.0860)	(0.0780)	(0.0708)	(0.0575)	(0.0495)	(0.0247)	(0.074861)	(0.1001)	(0.1060)
	Δ MRI	0.0206*	0.012609◇	0.0477***	0.0254***	0.0415***	0.000002◇	0.0720***	0.0459*	0.0704**
		(0.0087)	(0.0088)	(0.0086)	(0.0076)	(0.0071)	(0.0039)	(0.0131)	(0.0196)	(0.0251)
	Δ TR	−0.1175***	−0.1096***	−0.1080***	−0.1012***	−0.0986***	−0.000003◇	−0.0791***	−0.0345◇	−0.02479◇
		(0.0192)	(0.0165)	(0.0145)	(0.0115)	(0.0097)	(0.0048)	(0.0149)	(0.0209)	(0.0262)
5 y	Return	−0.0067◇	−0.0301◇	−0.000006◇	−0.000003◇	−0.0034◇	−0.1061◇	−0.1736◇	−0.2643◇	−0.3338◇
		(0.6872)	(0.4328)	(0.1026)	(0.1153)	(0.1387)	(0.3506)	(0.3659)	(0.5416)	(0.8066)
	Δ Vol	−13.5941***	−2.1323***	−0.000008◇	0.000001◇	0.1432***	5.0340***	9.7155***	19.419***	33.1278***
		(0.0441)	(0.0281)	(0.0066)	(0.0073)	(0.0087)	(0.0218)	(0.0222)	(0.0313)	(0.0421)
	Δ MRI	0.0055◇	0.0012◇	0.0◇	0.0◇	0.0001◇	0.0019◇	0.0033◇	0.0088◇	0.0353◇
		(0.0063)	(0.0036)	(0.0008)	(0.000964	(0.0012)	(0.0032)	(0.0036)	(0.0061)	(0.0101)
	Δ TR	−0.005616◇	−0.0010◇	0.0◇	0.0◇	0.00007◇	0.0012◇	−0.0016◇	−0.0038◇	−0.0091◇
		(0.0086)	(0.0054)	(0.0012)	(0.0014)	(0.0016)	(0.0041)	(0.0043)	(0.0068)	(0.0107)
7 y	Return	0.03120◇	−0.1883◇	−0.00001◇	−1.5935◇	−0.5418◇	−0.5164◇	−0.1722◇	−0.7829◇	−0.4409◇
		(0.7621)	(0.2771)	(0.3604)	(0.7684)	(0.8093)	(0.7500)	(0.7768)	(0.8997)	(1.1200)
	Δ Vol	−6.9746***	−0.4479***	0.000001◇	2.7797***	6.6894***	10.2932***	13.7541***	17.1105***	23.5668***
		(0.0488)	(0.0178)	(0.0229)	(0.0487)	(0.0510)	(0.0464)	(0.0463)	(0.0503)	(0.0555)
	Δ MRI	0.0067◇	0.0089***	0.00001◇	0.0294***	0.0509***	0.0475***	0.0445***	0.0409***	0.03808***
		(0.0041)	(0.0020)	(0.0028)	(0.0061)	(0.0067)	(0.0066)	(0.0074)	(0.0097)	(0.0133)
	Δ TR	0.0018◇	0.0072*	−0.0◇	0.0434***	0.0379***	0.0446***	0.0645***	0.0463**	0.02992**
		(0.0073)	(0.0031)	(0.0042)	(0.0090)	(0.0095)	(0.0089)	(0.0096)	(0.0116)	(0.0157)
10 y	Return	2.1100◇	0.9802◇	0.00002◇	0.00001◇	0.6532◇	1.0444◇	1.0633◇	1.3617◇	0.5832◇
		(1.4818)	(0.9560)	(0.2908)	(0.3208)	(0.5810)	(0.9268)	(0.8033)	(1.0227)	(1.2039)
	Δ Vol	−16.7535***	−4.0113***	−0.00001◇	0.000001◇	1.1344***	6.2161***	11.1312***	14.2431***	13.9336***
		(0.0860)	(0.0593)	(0.0181)	(0.0203)	(0.0367)	(0.0588)	(0.0509)	(0.0644)	(0.0737)
	Δ MRI	0.0025◇	−0.00169◇	0.0◇	0.0◇	0.0095◇	0.0150*	−0.0061◇	0.00495◇	0.02904***
		(0.0118)	(0.0076)	(0.0023)	(0.0026)	(0.0047)	(0.0072)	(0.0057)	(0.0071)	(0.0087)
	Δ TR	0.1310***	0.0802***	0.000002◇	0.000001◇	0.0545***	0.0854***	0.1030***	0.1046***	0.0753***
		(0.0170)	(0.0112)	(0.0033)	(0.0036)	(0.0067)	(0.0111)	(0.0097)	(0.0125)	(0.0157)
20 y	Return	8.6324**	6.4267*	5.2899*	3.9391*	4.2590◇	3.0114◇	0.8525◇	0.9701◇	0.6682◇
		(4.7369)	(3.6232)	(2.6405)	(2.0422)	(2.4208)	(2.2034)	(1.8653)	(1.8132)	(1.7537)
	Δ Vol	−70.1374***	−42.8053***	−20.0820***	−2.6321***	7.5991***	18.7052***	24.8702***	22.2789***	18.8805***
		(0.2487)	(0.2090)	(0.1606)	(0.1279)	(0.1542)	(0.1422)	(0.1227)	(0.1221)	(0.1222)
	Δ MRI	−0.0836*	−0.0975**	−0.0860***	−0.0967***	−0.1124***	−0.0780***	−0.0797***	−0.0750***	−0.0811***
		(0.0430)	(0.0324)	(0.0217)	(0.0161)	(0.0181)	(0.0153)	(0.0123)	(0.0117)	(0.0097)
	Δ TR	0.0636◇	0.1007*	0.1095***	0.1082***	0.1238***	0.1548***	0.1754***	0.1421***	0.1095***
		(0.0554)	(0.0396)	(0.0283)	(0.0221)	(0.0263)	(0.0245)	(0.0211)	(0.0206)	(0.0184)
30 y	Return	21.4858**	7.7933*	11.2974**	8.8951**	6.132**	7.9725***	3.1319◇	0.0821◇	−2.0407◇
		(7.3755)	(4.5123)	(3.8768)	(2.5783)	(2.1219)	(2.3578)	(2.4674)	(1.9346)	(1.7070)
	Δ Vol	−98.2669***	−64.1962***	−36.5270***	−16.8234***	−2.6711***	7.5035***	17.3678***	19.5688***	21.5032***
		(0.3890)	(0.2621)	(0.2349)	(0.1616)	(0.1361)	(0.1537)	(0.1630)	(0.1315)	(0.1183)
	Δ MRI	−0.0493◇	−0.1026**	−0.1088***	−0.1384***	−0.1467***	−0.1468***	−0.1270***	−0.1345***	−0.1319***
		(0.0595)	(0.0364)	(0.0297)	(0.0191)	(0.0154)	(0.0164)	(0.0171)	(0.0138)	(0.0103)
	Δ TR	0.0582◇	0.0734◇	0.1108**	0.1449***	0.1321***	0.1523***	0.1311***	0.1001***	0.1000***
		(0.0808)	(0.0473)	(0.0403)	(0.0267)	(0.0222)	(0.0253)	(0.0271)	(0.0212)	(0.0165)

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.1$, $^{\diamond}$ $0.1 < p < 1$

Table 3: This table presents the results of the panel regression for the *diffusion* model, where the dependent variable is first-difference model implied CDS spread ($\Delta \mathrm{CDS}^{model}_{i,t}$) computed between two consecutive days. The columns contains the estimates for the quantiles $\tau \in \{0.1, ..., 0.9\}$. Standard errors are reported between brackets ().

		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	Return	−0.0054***	−0.0016***	−0.0009***	−0.0008***	−0.0006***	−0.0006***	−0.0008***	−0.0014***	−0.0072***
		(0.0004)	(0.00008)	(0.00004)	(0.00003)	(0.00002)	(0.00002)	(0.00003)	(0.00007)	(0.0006)
6 m	Vol	−0.0035***	−0.0015***	−0.0009***	−0.0006***	−0.0002***	−0.0001***	−0.00008***	−0.00002***	0.0014***
		(0.000038)	(0.000008)	(0.000004)	(0.000003)	(0.000003)	(0.000002)	(0.000003)	(0.000007)	(0.000058)
	TR	−0.000004***	−0.000007***	−0.000006***	−0.000002***	0.000001***	0.000004***	0.000012***	0.00002***	0.000031***
		(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
	Return	−1.6624***	−0.2780***	−0.0817***	−0.0295***	−0.0142***	−0.0127***	−0.0247***	−0.1075***	−1.0267***
		(0.2130)	(0.0135)	(0.0017)	(0.0003)	(0.0001)	(0.0001)	(0.0002)	(0.0019)	(0.0617)
1 y	Vol	−0.5075***	−0.0599***	−0.0124***	−0.0035***	−0.0012***	−0.0004***	0.0004***	0.0070***	0.13367***
		(0.0192)	(0.0013)	(0.0001)	(0.00003)	(0.00001)	(0.00001)	(0.00002)	(0.0001)	(0.0050)
	TR	0.0012*	0.0003***	0.00009***	0.00003***	0.00001***	0.00001***	0.00001***	0.00004***	0.00061***
		(0.0005)	(0.00003)	(0.000005)	(0.000001)	(0.0)	(0.0)	(0.000001)	(0.000005)	(0.0001)
	Return	−13.8329***	−4.9175***	−2.5092***	−1.4395***	−0.6667***	−0.7363***	−1.8540***	−4.6526***	−15.0207***
		(0.8864)	(0.1217)	(0.0396)	(0.01622)	(0.0059)	(0.0062)	(0.0197)	(0.0806)	(0.5781)
2 y	Vol	−3.3682***	−0.8664***	−0.3742***	−0.1533***	−0.0378***	−0.0106***	0.0750***	0.3574***	1.6894***
		(0.0935)	(0.0137)	(0.0045)	(0.0018)	(0.0006)	(0.0006)	(0.0020)	(0.0081)	(0.0544)
	TR	0.0042*	0.0027***	0.0018***	0.0009***	0.0003***	0.0003***	0.0004***	0.0009***	0.0058***
		(0.0017)	(0.0002)	(0.00008)	(0.00003)	(0.00001)	(0.00001)	(0.00003)	(0.0001)	(0.0010)
	Return	−26.2692***	−11.8761***	−7.0762***	−4.2664***	−1.7042***	−2.2444***	−6.3563***	−13.2778***	−33.4054***
		(1.2244)5	(0.242857	(0.1011)	(0.0464)	(0.0155)	(0.0193)	(0.0707)	(0.2090)	(1.0716)
3 y	Vol	−6.4163***	−1.9866***	−0.9455***	−0.4107***	−0.0745◇	0.0025***	0.2890***	0.9586***	3.7684***
		(0.1444)	(0.0300)	(0.0126)	(0.0058)	(0.0019)	(0.0023)	(0.0082)	(0.0231)	(0.1127)
	TR	0.0059**	0.00417***	0.0025***	0.0015***	0.0003***	0.0006***	0.0016***	0.00424***	0.0109***
		(0.0019)	(0.0004)	(0.0001)	(0.00007)	(0.00002)	(0.00003)	(0.0001)	(0.0003)	(0.0015)
	Return	−36.10705***	−18.69365***	−11.91755***	−6.90235***	−2.20075***	−4.97775***	−12.314395***	−23.40855***	−52.07375***
		(1.4033)	(0.3598)	(0.1634)	(0.0728)	(0.0212)	(0.0440)	(0.1442)	(0.3618)	(1.6890)
4 y	Vol	−9.51225***	−3.51745***	−1.60055***	−0.60315***	−0.05025***	0.2068***	1.0076***	2.40285***	7.50855***
		(0.1708)	(0.0474)	(0.0220)	(0.0100)	(0.0029)	(0.0059)	(0.0185)	(0.0435)	(0.1905)
	TR	−0.000045◇	0.0027***	0.0012***	0.0011***	0.0001◇	0.00007***	0.0010***	0.0054***	0.0174***
		(0.0017)	(0.0004)	(0.0002)	(0.0001)	(0.00003)	(0.00006)	(0.00018)	(0.0004)	(0.0019)
	Return	−42.2594***	−23.1555***	−14.8026***	−8.5493***	−2.6291***	−6.8665***	−16.6834***	−30.2765***	−61.5915***
		(1.5278)	(0.4335)	(0.2051)	(0.0908)	(0.02554)	(0.06133)	(0.1973)	(0.4869)	(2.0636)
5 y	Vol	−11.4517***	−4.5529***	−2.0271***	−0.7699***	−0.0333***	0.4234***	1.6928***	3.6042***	9.6731***
		(0.1816)	(0.0573)	(0.0283)	(0.0129)	(0.0036)	(0.0085)	(0.0260)	(0.0589)	(0.2279)
	TR	−0.0104***	−0.0016***	−0.0009***	0.0011◇	0.00002***	−0.0005*	0.00036***	0.0071***	0.0268***
		(0.0014)	(0.0004)	(0.0002)	(0.0001)	(0.00003)	(0.00007)	(0.0002)	(0.0004)	(0.0018)
	Return	−46.8499***	−26.1987***	−17.4813***	−10.2383***	−3.8535***	−9.4837***	−20.5886***	−36.3619***	−70.0557***
		(1.7590)	(0.4692)	(0.2295)	(0.1094)	(0.0353)	(0.0865)	(0.2504)	(0.6049)	(2.4837)
7 y	Vol	−15.2753***	−5.8302***	−2.5154***	−0.9282***	−0.0007◇	0.8064***	2.5254***	5.7199***	13.2415***
		(0.2109)	(0.0648)	(0.0334)	(0.0165)	(0.0054)	(0.0128)	(0.0348)	(0.0762)	(0.2800)
	TR	−0.0135***	−0.0050***	−0.0029***	0.0006***	−0.000004◇	−0.0010***	0.0004*	0.00613***	0.0344***
		(0.0012)	(0.0003)	(0.0002)	(0.00009)	(0.00003)	(0.00009)	(0.0002)	(0.0004)	(0.0016)
	Return	−43.8408***	−25.1945***	−16.6703***	−9.3483***	−5.2453***	−10.2858***	−20.1126***	−35.7491***	−67.8181***
		(1.7411)	(0.4505)	(0.2193)	(0.0950)	(0.0478)	(0.0951)	(0.2432)	(0.6098)	(2.3496)
10 y	Vol	−17.2013***	−5.5618***	−2.1117***	−0.7263***	0.0654***	0.9383***	2.4969***	6.2195***	14.2604***
		(0.2135)	(0.0642)	(0.0332)	(0.0148)	(0.0076)	(0.0146)	(0.0350)	(0.0793)	(0.2708)
	TR	−0.0053***	−0.0053***	−0.0036***	0.00001◇	−0.00007*	−0.0007***	0.0013***	0.0048***	0.0314***
		(0.0011)	(0.0003)	(0.0001)	(0.00007)	(0.00004)	(0.00007)	(0.0001)	(0.0004)	(0.0014)
	Return	−37.1418***	−21.2232***	−13.2271***	−8.0651***	−6.0049***	−9.4217***	−16.3020***	−30.2802***	−57.6025***
		(1.5544)	(0.4114)	(0.1789)	(0.0862)	(0.0575)	(0.0942)	(0.1954)	(0.5471)	(2.2594)
20 y	Vol	−18.1273***	−5.7746***	−1.8715***	−0.5699***	0.1209***	0.8852***	2.4357***	6.9969***	18.9665***
		(0.1916)	(0.0599)	(0.0274)	(0.0135)	(0.0091)	(0.0146)	(0.0284)	0.(0727)	(0.2568)
	TR	0.0024*	−0.0022***	−0.0029***	−0.0007***	0.00002◇	0.0005***	0.0018***	0.0003***	0.0064***
		(0.0009)	(0.0002)	(0.0001)	(0.00006)	(0.00004)	(0.00007)	(0.0001)	(0.0003)	(0.0012)
	Return	−32.9638***	−18.0532***	−11.3636***	−7.0519***	−5.4903***	−8.1607***	−13.4919***	−25.3438***	−51.6025***
		(1.4494)	(0.3407)	(0.1573)	(0.0767)	(0.0562)	(0.0861)	(0.1649)	(0.4794)	(1.9147)
30 y	Vol	−19.0081***	−5.2461***	−1.8281***	−0.4864***	0.0826***	0.6800***	1.8968***	6.1094***	19.3046***
		(0.1786)	(0.04976)	(0.02432)	(0.0121)	(0.0089)	(0.0133)	(0.0240)	(0.0638)	(0.2241)
	TR	0.0115***	−0.0013***	−0.0021***	−0.0009***	0.0001***	0.0013***	0.0027***	0.0005*	−0.0030***
		(0.0008)	(0.0002)	(0.0001)	(0.00005)	(0.00004)	(0.00006)	(0.0001)	(0.0003)	(0.0010)

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.1$, $^{\diamond}\ 0.1 < p < 1$

Table 4: This table presents the results of the panel regression for the *diffusion* model, where the dependent variable is the error $\mathcal{E}_{i,t}$ between model and market CDS. The columns contains the estimates for the quantiles $\tau \in \{0.1, ..., 0.9\}$. Standard errors are reported between brackets ().

7 Descriptive Statistics

This section reports the descriptive statistics for the variables we employed in the analysis.

Variable	observations	mean	median	std	skew	kurtosis
CDS market 6m	105756	21.9188	8.3700	73.4032	12.0192	180.7346
CDS market 1y	105756	26.8150	11.1900	81.4541	11.4796	171.6312
CDS market 2y	105756	38.4486	20.1100	87.1869	9.4004	115.8410
CDS market 3y	105756	50.1026	29.6000	91.0793	7.9210	83.6267
CDS market 4y	105756	64.0296	42.3700	92.5535	6.7104	60.9326
CDS market 5y	105756	77.8766	55.3400	95.4347	5.8134	46.9894
CDS market 7y	105756	99.9479	76.6000	96.8531	4.8107	33.5819
CDS market 10y	105756	115.5844	91.5950	96.3777	4.2664	27.5051
CDS market 20y	105756	125.1668	102.4250	95.7793	3.8413	23.0704
CDS market 30y	105756	130.6239	107.5200	95.8474	3.6567	21.3859
Ret	105756	0.0001	0.0003	0.0186	-0.5487	20.8007
Vol	105756	0.2682	0.2365	0.1165	1.9622	6.3121
Carbon price	1259	24.9186	23.4000	17.0163	1.1675	1.1326
TR 6m	1259	6.9138	4.2444	6.9662	3.0766	12.6582
TR 1y	1259	8.4419	5.7428	8.0094	3.1992	13.6579
TR 2y	1259	13.2221	11.4281	9.7826	2.8476	11.2722
TR 3y	1259	18.9923	18.2223	11.2428	2.6586	10.5240
TR 4y	1259	26.5263	25.3949	11.3736	2.5650	10.3813
TR 5y	1259	34.1395	33.0266	11.6649	2.4313	9.5039
TR 7y	1259	47.3614	46.2781	11.3790	2.2279	8.4454
TR 10y	1259	54.7323	54.2707	11.1107	2.0062	7.6375
TR 20y	1259	58.0126	57.5406	10.9802	1.9966	7.6018
TR 30y	1259	60.0840	59.6963	11.0472	1.7956	6.9918
MRI 6m	105756	11.2854	7.7400	12.6435	4.5012	22.3518
MRI 1y	105756	14.5828	10.4800	15.3598	4.4879	23.0778
MRI 2y	105756	24.2714	18.9400	21.2326	3.6527	16.7962
MRI 3y	105756	34.5308	28.3100	27.1528	3.0949	13.5697
MRI 4y	105756	47.9410	42.0400	33.1248	2.5021	8.6156
MRI 5y	105756	61.4777	54.5300	39.2497	2.1292	5.9447
MRI 7y	105756	84.6142	72.9750	46.4385	1.7703	3.8712
MRI 10y	105756	101.1261	87.6500	51.0265	1.5869	3.0508
MRI 20y	105756	109.7678	95.9250	53.6013	1.5815	3.0196
MRI 30y	105756	114.9311	98.7400	54.3299	1.5657	3.0108

Table 5: This table reports some statistics for the CDS market spreads and control variables. Transition risk has been computed by the Wasserstein distance.

The tables below report the correlation between the change in transition risk at different maturities ΔTR_t^m (computed via the Wasserstein distance 6 and by the median CDS spread 7), and the daily first-difference of the Carbon Price.

	Δ TR 6m	Δ TR 1y	Δ TR 2y	Δ TR 3y	Δ TR 4y	Δ TR 5y	Δ TR 7y	Δ TR 10y	Δ TR 20y	Δ TR 6m
ΔCP_t	-0.0939	-0.0789	-0.1067	-0.1200	-0.1223	-0.1170	-0.1171	-0.1144	-0.1079	-0.1041

Table 6: Correlation between ΔCP_t and ΔTR_t, computed by the Wasserstein distance.

	Δ TR 6m	Δ TR 1y	Δ TR 2y	Δ TR 3y	Δ TR 4y	Δ TR 5y	Δ TR 7y	Δ TR 10y	Δ TR 20y	Δ TR 6m
ΔCP_t	-0.0875	-0.0857	-0.1031	-0.0770	-0.0650	-0.0879	-0.1038	-0.1036	-0.0982	-0.0985

Table 7: Correlation between ΔCP_t and ΔTR_t, computed by median CDS spread.

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