Online Appendix
Repression of Enslaved Americans' Protest:
A Model of Escape in the Antebellum South

A Optimal investment in pursuit

To find the enslaver M's optimal investment in pursuit, take the first and second derivatives of his utility function with respect to $\kappa(c_S)$. Recall that the M's belief that a runaway who produced c_S is a high type is $\mu(c_S)$, and that the probability of successful escape for a runaway is $q(\kappa) = \frac{\bar{q}}{1+\kappa}$.

$$U_M = \mu(c_S)\cdot(c_h-\gamma)(1-\frac{\bar{q}}{1+\kappa})+(1-\mu(c_S))\cdot c_l\cdot(1-\frac{\bar{q}}{1+\kappa})-\kappa$$
$$\frac{\delta U_M}{\delta\kappa} = \frac{\bar{q}\cdot[\mu(c_S)\cdot(c_h-\gamma)+(1-\mu(c_S))\cdot c_l]}{(1+\kappa)^2}-1$$
$$\kappa = \sqrt{\bar{q}\cdot[\mu(c_S)\cdot(c_h-\gamma)+(1-\mu(c_S))\cdot c_l]}-1$$

$$\frac{\delta^2 U_M}{\delta\kappa^2} = \frac{-\bar{q}\cdot[\mu(c_S)\cdot(c_h-\gamma)+(1-\mu(c_S))\cdot c_l]\cdot(2+2\kappa)}{(1+\kappa)^4}$$

The critical value of $\kappa = \sqrt{\bar{q}\cdot[\mu(c_S)\cdot(c_h-\gamma)+(1-\mu(c_S))\cdot c_l]}-1$, and it is an argmax, because $\frac{\delta^2 U_M}{\delta\kappa^2}\leq 0$. Because $\kappa\in\mathbb{R}_{++}$, if $\sqrt{\bar{q}\cdot[\mu(c_S)\cdot(c_h-\gamma-c_l)+c_l]}-1<0$, the enslaver would set $\kappa=0$.

After observing low output, the enslaver invests $\kappa^* = \max\{0, \sqrt{\bar{q}\cdot[\mu(c_S)\cdot(c_h-\gamma-c_l)+c_l]}-1\}$. Solving for μ, we have that the enslaver invests zero in pursuing low producers when $\mu(c_S) < \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$. Note that if $c_l > \frac{1}{\bar{q}}$, no belief can sustain an investment of zero; M will always invest $\sqrt{\bar{q}\cdot[\mu(c_S)\cdot(c_h-\gamma-c_l)+c_l]}-1$. $\frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$ is always less than 1 because $c_h-\gamma>\frac{1}{\bar{q}}$ by assumption 2. After observing high output, M knows that a runaway is a high type, and so he invests $\kappa^* = \sqrt{\bar{q}\cdot(c_h-\gamma)}-1$.

The optimal values of κ allow us to define the values of probability $q(\kappa)$ realized in equilibrium. See equation 4, and observe that $\bar{q} > \sqrt{\frac{\bar{q}}{\mu(c_l)\cdot(c_h-\gamma-c_l)+c_l}} > q_h$.

$$q(\kappa) = \begin{cases} \bar{q} & \text{, if } c_S = c_l \text{ and } \mu(c_l) \leq \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)} \\ \sqrt{\frac{\bar{q}}{\mu(c_l)\cdot(c_h-\gamma-c_l)+c_l}} & \text{, if } c_S = c_l \text{ and } \mu(c_l) > \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)} \\ q_h := \sqrt{\frac{\bar{q}}{c_h-\gamma}} & \text{, if } c_S = c_h \end{cases} \tag{4}$$

B Posterior beliefs given pure strategies

The following equation specifies the enslaver's posterior belief after observing low output and running.

$$\mu(c_l, e_S = 1) = \frac{(1-\psi)\rho}{(1-\psi)\rho + \pi_l(1-\rho)}$$

If both types produce low and run, the enslaver's posterior belief is ρ.

$$\mu(c_l, e_S = 1|\psi = 0, \pi_l = 1) = \frac{(1-0)\rho}{(1-0)\rho + 1(1-\rho)}$$
$$\mu(c_l, e_S = 1|\psi = 0, \pi_l = 1) = \rho$$

If low types run and high types produce high and run, $\mu(c_h) = 1$ and $\mu(c_l) = 0$.

$$\mu(c_l, e_S = 1|\psi = 1, \pi_l = 1) = \frac{(1-1)\rho}{(1-1)\rho + 1(1-\rho)}$$
$$\mu(c_l, e_S = 1|\psi = 1, \pi_l = 1) = 0$$

By the same logic, if low types run and high types stay, $\mu(c_h) = 1$ and $\mu(c_l) = 0$.

C $\mu(c_l) \leq \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$

For sufficiently low beliefs $\mu(c_l)$ and sufficiently small values of c_l, M's optimal investment in pursuing low producers is zero. Low types, knowing that M invests zero in pursuing low producers, prefer to run if $w_l > r \cdot \frac{1-\bar{q}}{\bar{q}}$ and stay otherwise.

C.1 Low types run

Given $\mu(c_l) \leq \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$, M's optimal investment after observing low output is $\kappa = 0$, and the probability of successful escape after producing low is $\bar{q}$. Low types then prefer to run when $w_l \geq r \cdot \frac{1-\bar{q}}{\bar{q}}$.

If high types conceal, they also face a probability of $\bar{q}$, and if they reveal and run, they face a probability of q_h. I proceed by solving for the outside wage values that make S_h indifferent for each pair of actions. First, observe that high types prefer to stay rather than produce high and run when the outside wage w_h is less than $\gamma + r \cdot \frac{1-q_h}{q_h} := A$.

$$
\begin{aligned}
U_h(\psi = 1, \pi(c_h) = 0) >& \quad U_h(\psi = 1, \pi(c_h) = 1 | \kappa = \sqrt{\bar{q}\cdot(c_h-\gamma)} - 1) \\
2\cdot\gamma >& \quad \gamma + q_h \cdot w_h + (1-q_h)\cdot(\gamma - r) \\
\gamma + r \cdot \frac{1-q_h}{q_h} >& \quad w_h
\end{aligned}
$$

High types prefer to stay rather than conceal and run when the outside wage w_h is less than $\frac{1+\bar{q}}{\bar{q}} \cdot \gamma + \frac{1-\bar{q}}{\bar{q}} \cdot r := B$.

$$
\begin{aligned}
U_h(\psi = 1, \pi(c_h) = 0) >& \quad U_h(\psi = 0, \pi(c_l) = 1 | \kappa = 0) \\
2\cdot\gamma >& \quad \bar{q}\cdot w_h + (1-\bar{q})\cdot(\gamma - r) \\
\frac{1+\bar{q}}{\bar{q}} \cdot \gamma + \frac{1-\bar{q}}{\bar{q}} \cdot r >& \quad w_h
\end{aligned}
$$

High types prefer to produce high and run rather than conceal and run when the outside wage w_h is less than $\frac{1+\bar{q}-q_h}{\bar{q}-q_h} \cdot \gamma - r := C$.

$$\begin{aligned}
U_h(\psi = 1, \pi(c_h) = 1 | \kappa = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1) &> \quad U_h(\psi = 0, \pi(c_l) = 1 | \kappa = 0) \\
\gamma + q_h \cdot w_h + (1 - q_h) \cdot (\gamma - r) &> \quad \bar{q} \cdot w_h + (1 - \bar{q}) \cdot (\gamma - r) \\
(1 + \bar{q} - q_h) \cdot \gamma - r(\bar{q} - q_h) &> \quad (\bar{q} - q_h) \cdot w_h \\
\frac{1 + \bar{q} - q_h}{\bar{q} - q_h} \cdot \gamma - r &> \quad w_h
\end{aligned}$$

In order to compare the utilities in terms of the outside wage, I compare A, B, C. Note that $\frac{\bar{q}}{q_h} = \sqrt{\bar{q} \cdot (c_h - \gamma)}$.

$$\begin{aligned}
A &> \quad B \\
\gamma + r \cdot \frac{1 - q_h}{q_h} &> \quad \frac{1 + \bar{q}}{\bar{q}} \cdot \gamma + \frac{1 - \bar{q}}{\bar{q}} \cdot r \\
(\frac{1}{q_h} - \frac{1}{\bar{q}}) \cdot r &> \quad \frac{1}{\bar{q}} \cdot \gamma \\
(\frac{\bar{q}}{q_h} - 1) \cdot r &> \quad \gamma \\
(\sqrt{\bar{q} \cdot (c_h - \gamma)} - 1) \cdot r &> \quad \gamma
\end{aligned}$$

$$
\begin{aligned}
B &> C \\
\frac{1+\bar{q}}{\bar{q}} \cdot \gamma + \frac{1-\bar{q}}{\bar{q}} \cdot r &> \frac{1+\bar{q}-q_h}{\bar{q}-q_h} \cdot \gamma - r \\
\frac{1}{\bar{q}} \cdot \gamma + \frac{1}{\bar{q}} \cdot r &> \frac{1}{\bar{q}-q_h} \cdot \gamma \\
\gamma + r &> \frac{\bar{q}}{\bar{q}-q_h} \cdot \gamma \\
\gamma + r &> \frac{\bar{q} \cdot \sqrt{c_h - \gamma}}{\bar{q} \cdot \sqrt{c_h - \gamma} - \sqrt{\bar{q}}} \cdot \gamma \\
r &> \frac{\sqrt{\bar{q}}}{\bar{q} \cdot \sqrt{c_h - \gamma} - \sqrt{\bar{q}}} \cdot \gamma \\
\frac{\bar{q} \cdot \sqrt{c_h - \gamma} - \sqrt{\bar{q}}}{\sqrt{\bar{q}}} \cdot r &> \gamma \\
(\sqrt{\bar{q} \cdot (c_h - \gamma)} - 1) \cdot r &> \gamma
\end{aligned}
$$

$$
\begin{aligned}
A &> C \\
\gamma + r \cdot \frac{1-q_h}{q_h} &> \frac{1+\bar{q}-q_h}{\bar{q}-q_h} \cdot \gamma - r \\
\frac{1}{q_h} \cdot r &> \frac{1}{\bar{q}-q_h} \cdot \gamma \\
(\frac{\bar{q}}{q_h} - 1) \cdot r &> \gamma \\
(\sqrt{\bar{q} \cdot (c_h - \gamma)} - 1) \cdot r &> \gamma
\end{aligned}
$$

Let $\hat{\gamma} := (\sqrt{\bar{q} \cdot (c_h - \gamma)} - 1) \cdot r$. If $\gamma < \hat{\gamma}$, $A > B > C$, and if $\gamma > \hat{\gamma}$, $C > B > A$.

If $\gamma > \hat{\gamma}$ and $w_h < A$ or $\gamma < \hat{\gamma}$ and $w_h < B$, high types prefer to produce high and stay. The enslaver then believes that those who produce low amounts are in fact high types with probability $\mu = 0$, and the enslaver invests zero in pursuing low producers. Similarly, if $\gamma > \hat{\gamma}$ and $C > w_h > A$, then high types prefer to reveal and run, and the enslaver believes that those who produce low amounts are in fact high types with probability $\mu = 0$. He invests zero in pursuing low producers.

If $\gamma > \hat{\gamma}$ and $w_h > C$ or $\gamma < \hat{\gamma}$ and $w_h > B$, high types prefer to conceal and run. The enslaver then believes that those who produce low amounts are in fact high types with probability $\mu = \rho$. The enslaver then invests zero in pursuit when $\rho < \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$. Thus $\rho < \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$ can sustain an equilibrium in which both high and low types produce low and run, and the enslaver does not invest in pursuit, given the stated external wages.

Proposition 1 states each unique equilibrium path of play, followed by the strategy profile and set of beliefs. I suppress the statements $\mu(c_l, e_S = 0) = 0$ and $\mu(c_h, e_S) = 1$, which is true in all equilibria; see equation 3.

Proposition 1. *For* $w_l \geq r \cdot \frac{1-\bar{q}}{\bar{q}}$*,*

a) *there exists a PBE in which low types run, high types produce high and stay, and the enslaver, believing with certainty that those who produce the low amount are low types, invests nothing in pursuing low-producing runaways when* $w_h < B$ *and* $\gamma < \hat{\gamma}$ *or when* $w_h < A$ *and* $\gamma > \hat{\gamma}$*; and*

   $(\psi^* = 1,\ \pi_l^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \kappa^*(c_l) = 0,\ \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h-\gamma)} - 1,\ \mu(c_l, e_S = 1) = 0)$ if $w_h \in (\frac{1-\bar{q}}{\bar{q}}\cdot(r-\gamma), B)$ and $\gamma < \hat{\gamma}$ or $w_h \in (\frac{1-\bar{q}}{\bar{q}}\cdot(r-\gamma), A)$ and $\gamma > \hat{\gamma}$

   $(\psi^* = 1,\ \pi_l^* = 1,\ \pi_h^*(c_l) = 0,\ \pi_h^*(c_h) = 0,\ \kappa^*(c_l) = 0,\ \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h-\gamma)} - 1,\ \mu(c_l, e_S = 1) = 0)$ if $w_h < \frac{1-\bar{q}}{\bar{q}}\cdot(r-\gamma)$

b) *there exists a PBE in which both types produce low and run, and the enslaver, believing it is sufficiently unlikely that low-producing fugitives are in fact high types,* $\rho < \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$*, invests nothing in pursuing low-producing runaways when* $w_h > B$ *and* $\gamma < \hat{\gamma}$ *or when* $w_h > C$ *and* $\gamma > \hat{\gamma}$*; and*

   $(\psi^* = 0,\ \pi_l^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 1,\ \kappa^*(c_l) = 0,\ \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h-\gamma)} - 1,\ \mu(c_l, e_S = 1) = \rho)$ if $w_h > \gamma + r\cdot\frac{1-q_h}{q_h}$ and $\gamma < \hat{\gamma}$ or $w_h > C$ and $\gamma > \hat{\gamma}$

   $(\psi^* = 0,\ \pi_l^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \kappa^*(c_l) = 0,\ \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h-\gamma)} - 1,\ \mu(c_l, e_S = 1) = \rho)$ if $w_h \in (B, \gamma + r\cdot\frac{1-q_h}{q_h})$ and $\gamma < \hat{\gamma}$

c) *there exists a PBE in which low types run, high types produce high and run, and the enslaver, believing with certainty that those who produce the low amount are low types, invests nothing in pursuing low producers and invests* $\sqrt{\bar{q} \cdot (c_h - \gamma)} - 1$ *in pursuing high producers when* $C > w_h > A$ *and* $\gamma > \hat{\gamma}$.

$(\psi^* = 1,\ \pi_l^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 1,\ \kappa^*(c_l) = 0,\ \kappa^*(c_h) = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1,\ \mu(c_l, e_S = 1) = 0)$

C.2 Low types stay

Low types stay when $w_l < r \cdot \frac{1-\bar{q}}{\bar{q}}$. Again, if high types produce low in the first time period, they can run with probability of success $\bar{q}$, and if they produce high, they can run in the second with probability of success q_h.

If $\gamma < \hat{\gamma}$ and $w_h < B$, high types prefer to produce high and stay. Thus, no one runs. However, if $w_h > B$, high types prefer to conceal and run, rather than to stay or to reveal and run. Then, the enslaver believes that low-producing runaways are high types with certainty, $\mu(c_l) = 1$, because low types prefer to stay. He would then want to invest in pursuing low-producing runaways. This contradicts the initial supposition, $\mu(c_l) \leq \frac{1-\bar{q} \cdot c_l}{\bar{q} \cdot (c_h - \gamma - c_l)}$. There is not an equilibrium in which the enslaver knows that low-producing runaways are high types and invests nothing in pursuing them.

If $\gamma > \hat{\gamma}$ and $w_h < A$, high types produce high and stay, and no one runs. If $w_h \in (A, C)$, high types produce high and run, escaping with probability q_h, while enslavers invest $\kappa = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1$ in pursuing them. If $\gamma > \hat{\gamma}$ and $w_h > C$, high types prefer to conceal and run, again resulting in a contradiction.

Proposition 2. *For* $w_l \leq r \cdot \frac{1-\bar{q}}{\bar{q}}$,

a) *there exists a PBE in which low types stay, high types produce high and run, and the enslaver invests* $\kappa = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1$ *in pursuing runaways, believing that they are high types with certainty, if and only if they produced high when* $\gamma > \hat{\gamma}$ *and* $w_h \in (A, C)$*; and*

$(\psi^* = 1,\ \pi_l^* = 0,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 1,\ \kappa^*(c_l) = 0,\ \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h-\gamma)} - 1,\ \mu(c_l, e_S = 1) \leq \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)})$

b) *there exists a PBE in which both types stay and high types produce high if and only if* $\gamma > \hat{\gamma}$ *and* $w_h < A$ *or when* $\gamma < \hat{\gamma}$ *and* $w_h < B$.

$(\psi^* = 1,\ \pi_l^* = 0,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \kappa^*(c_l) = 0,\ \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h-\gamma)} - 1,\ \mu(c_l, e_S = 1) \leq \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)})$ if $w_h \in (\frac{1-\bar{q}}{\bar{q}}\cdot(r-\gamma), B)$ and $\gamma < \hat{\gamma}$ or $w_h \in (\frac{1-\bar{q}}{\bar{q}}\cdot(r-\gamma), A)$ and $\gamma > \hat{\gamma}$

$(\psi^* = 1,\ \pi_l^* = 0,\ \pi_h^*(c_l) = 0,\ \pi_h^*(c_h) = 0,\ \kappa^*(c_l) = 0,\ \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h-\gamma)} - 1,\ \mu(c_l, e_S = 1) \leq \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)})$ if $w_h < \frac{1-\bar{q}}{\bar{q}}\cdot(r-\gamma)$

D $\mu(c_l) > \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$

D.1 Low types run

Low types run if $w_l > r\cdot(\frac{\mu(c_l)\cdot(c_h-\gamma-c_l)+c_l}{\bar{q}} - 1)$. If high types produce high in the first time period, $\mu(c_l) = 0$, and low-producing runaways can escape successfully with probability $\sqrt{\frac{\bar{q}}{c_l}}$ if $c_l > \frac{1}{\bar{q}}$. If high types produce high in the first time period and $c_l < \frac{1}{\bar{q}}$, there is not an equilibrium in which the enslaver invests some positive amount in pursuing low producers. Given that low types run, if high types produce low and run, $\mu(c_l) = \rho$, and low-producing runaways can escape successfully with probability $\sqrt{\frac{\bar{q}}{\rho\cdot(c_h-\gamma-c_l)+c_l}}$ if $\rho > \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$.

As in appendix C.1, I compare S_h's utility from each action, given that low types run. Recall that $q_h := \sqrt{\frac{\bar{q}}{c_h-\gamma}}$. Again, high types prefer to stay rather than produce high and run if $w_h < A$. High types prefer to stay than to conceal and run when w_h is less than $\gamma - r + (\gamma + r)\cdot\sqrt{\frac{\rho\cdot(c_h-\gamma-c_l)+c_l}{\bar{q}}} := B'$.

$$
\begin{aligned}
U_h(\psi=1,\pi(c_h)=0) > &\quad U_h(\psi=0,\pi(c_l)=1|\kappa=\sqrt{\bar{q}\cdot[\rho\cdot(c_h-\gamma-c_l)+c_l]}-1) \\
2\cdot\gamma > &\quad (\sqrt{\frac{\bar{q}}{\rho\cdot(c_h-\gamma-c_l)+c_l}})\cdot(w_h-\gamma+r)+\gamma-r \\
\gamma-r+(\gamma+r)\cdot\sqrt{\frac{\rho\cdot(c_h-\gamma-c_l)+c_l}{\bar{q}}}) > &\quad w_h
\end{aligned}
$$

High types prefer to produce high and run rather than conceal and run when the outside wage w_h is less than $(\sqrt{\frac{\bar{q}}{\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l}}-\sqrt{\frac{\bar{q}}{c_h-\gamma}})^{-1}\cdot\gamma+\gamma-r := C'$.

$$
\begin{aligned}
U_h(\psi=1,\pi(c_h)=1|\kappa=\sqrt{\bar{q}\cdot(c_h-\gamma)}-1) > &\quad U_h(\psi=0,\pi(c_l)=1|\kappa=\sqrt{\bar{q}\cdot[\rho\cdot(c_h-\gamma-c_l)+c_l]}-1) \\
\gamma+q_h\cdot w_h+(1-q_h)\cdot(\gamma-r) > &\quad (\sqrt{\frac{\bar{q}}{\rho\cdot(c_h-\gamma-c_l)+c_l}})\cdot(w_h-\gamma+r)+\gamma-r \\
(\sqrt{\frac{\bar{q}}{\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l}}-\sqrt{\frac{\bar{q}}{c_h-\gamma}})^{-1}\cdot\gamma+\gamma-r > &\quad w_h
\end{aligned}
$$

In order to compare the utilities in terms of the outside wage, I compare A, B', C'.

$$
\begin{aligned}
A > &\quad B' \\
\gamma+r\cdot\frac{1-q_h}{q_h} > &\quad \gamma-r+(\gamma+r)\cdot\sqrt{\frac{\rho\cdot(c_h-\gamma-c_l)+c_l}{\bar{q}}} \\
(\frac{1}{q_h}-\sqrt{\frac{\rho\cdot(c_h-\gamma-c_l)+c_l}{\bar{q}}})\cdot r > &\quad \sqrt{\frac{\rho\cdot(c_h-\gamma-c_l)+c_l}{\bar{q}}}\cdot\gamma \\
(\sqrt{\frac{\bar{q}}{\rho\cdot(c_h-\gamma-c_l)+c_l}}\cdot\sqrt{\frac{c_h-\gamma}{\bar{q}}}-1)\cdot r > &\quad \gamma
\end{aligned}
$$

$$\begin{aligned}
B' &> C' \\
\gamma - r + (\gamma + r) \cdot \sqrt{\frac{\rho \cdot (c_h - \gamma - c_l) + c_l}{\bar{q}}} &> \left(\sqrt{\frac{\bar{q}}{\rho \cdot (c_h - \gamma) + (1-\rho) \cdot c_l}} - \sqrt{\frac{\bar{q}}{c_h - \gamma}}\right)^{-1} \cdot \gamma + \gamma - r \\
\left(\sqrt{\frac{\bar{q}}{\rho \cdot (c_h - \gamma - c_l) + c_l}} \cdot \sqrt{\frac{c_h - \gamma}{\bar{q}}} - 1\right) \cdot r &> \gamma \\
r \cdot \sqrt{\frac{(1-\rho) \cdot (c_h - \gamma - c_l)}{\rho \cdot (c_h - \gamma - c_l) + c_l}} &> \gamma
\end{aligned}$$

$$\begin{aligned}
A &> C' \\
\gamma + r \cdot \left(\sqrt{\frac{c_h - \gamma}{\bar{q}}} - 1\right) &> \left(\sqrt{\frac{\bar{q}}{\rho \cdot (c_h - \gamma) + (1-\rho) \cdot c_l}} - \sqrt{\frac{\bar{q}}{c_h - \gamma}}\right)^{-1} \cdot \gamma + \gamma - r \\
\left(\sqrt{\frac{\bar{q}}{\rho \cdot (c_h - \gamma) + (1-\rho) \cdot c_l}} \cdot \sqrt{\frac{c_h - \gamma}{\bar{q}}} - 1\right) \cdot r &> \gamma
\end{aligned}$$

Let $\hat{\gamma}' := r \cdot \sqrt{\frac{(1-\rho)\cdot(c_h-\gamma-c_l)}{\rho\cdot(c_h-\gamma-c_l)+c_l}}$. If $\gamma < \hat{\gamma}'$, $A > B' > C'$, and if $\gamma > \hat{\gamma}'$, $C' > B' > A$. Observe that $\hat{\gamma}' < \hat{\gamma}$ when $\mu(c_l) > \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$, and that $C' > C$ and $B' > B$ when $c_l > \frac{1}{\bar{q}}$.

$$\begin{aligned}
\hat{\gamma}' &< \hat{\gamma} \\
\sqrt{\frac{c_h - \gamma}{\mu(c_l) \cdot (c_h - \gamma) + (1 - \mu(c_l)) \cdot c_l}} - 1 &< \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1 \\
\frac{c_h - \gamma}{\mu(c_l) \cdot (c_h - \gamma) + (1 - \mu(c_l)) \cdot c_l} &< \bar{q} \cdot (c_h - \gamma) \\
\bar{q} &> \frac{1}{\mu(c_l) \cdot (c_h - \gamma) + (1 - \mu(c_l)) \cdot c_l} \\
\mu(c_l) \cdot (c_h - \gamma - c_l) + c_l &> \frac{1}{\bar{q}} \\
\mu(c_l) &> \frac{1 - \bar{q} \cdot c_l}{\bar{q} \cdot (c_h - \gamma - c_l)}
\end{aligned}$$

$$
\begin{aligned}
C' &> C \\
\left(\sqrt{\frac{\bar{q}}{\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l}}-\sqrt{\frac{\bar{q}}{c_h-\gamma}}\right)^{-1}\cdot\gamma+\gamma-r &> \frac{1}{\bar{q}-\sqrt{\frac{\bar{q}}{c_h-\gamma}}}\cdot\gamma+\gamma-r \\
\left(\sqrt{\frac{\bar{q}}{\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l}}-\sqrt{\frac{\bar{q}}{c_h-\gamma}}\right)^{-1} &> \frac{1}{\bar{q}-\sqrt{\frac{\bar{q}}{c_h-\gamma}}} \\
\bar{q}-\sqrt{\frac{\bar{q}}{c_h-\gamma}} &> \sqrt{\frac{\bar{q}}{\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l}}-\sqrt{\frac{\bar{q}}{c_h-\gamma}} \\
\bar{q} &> \frac{1}{\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l} \\
\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l &> \frac{1}{\bar{q}}
\end{aligned}
$$

$$
\begin{aligned}
B' &> B \\
\gamma-r+(\gamma+r)\cdot\sqrt{\frac{\rho\cdot(c_h-\gamma-c_l)+c_l}{\bar{q}}} &> (\gamma+r)\cdot\frac{1}{\bar{q}}+\gamma-r \\
\sqrt{\rho\cdot(c_h-\gamma-c_l)+c_l} &> \sqrt{\frac{1}{\bar{q}}} \\
\rho\cdot(c_h-\gamma-c_l)+c_l &> \frac{1}{\bar{q}}
\end{aligned}
$$

If $\gamma > \hat{\gamma}'$ and $w_h < A$ or $\gamma < \hat{\gamma}'$ and $w_h < B'$, high types prefer to stay. The enslaver then believes that those who produce low amounts are in fact high types with probability $\mu = 0$. If $c_l > \frac{1}{\bar{q}}$, then the enslaver invests $\kappa = \sqrt{\bar{q}\cdot c_l} - 1$ on pursuing low producers. If $w_l > (\sqrt{\frac{c_l}{\bar{q}}} - 1)\cdot r$, low types run.

The same logic holds if $\gamma > \hat{\gamma}'$ and $C' > w_h > A$, except here, high types prefer to reveal and run. Again, the enslaver believes that those who produce low amounts are in fact high types with probability $\mu = 0$. He invests $\kappa = \sqrt{\bar{q} \cdot c_l} - 1$ in pursuing low producers and $\kappa = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1$ in pursuing high producers. Low types run if $w_l > (\sqrt{\frac{c_l}{\bar{q}}} - 1) \cdot r$.

If $\gamma > \hat{\gamma}'$ and $w_h > C'$ or $\gamma < \hat{\gamma}'$ and $w_h > B'$, high types strictly prefer to conceal and run. The enslaver then believes that those who produce low amounts are in fact high types with probability $\mu = \rho$. The enslaver invests $\kappa = \sqrt{\bar{q} \cdot \rho \cdot (c_h - \gamma)} - 1$ in pursuing low producers. Low types then want to run if $w_l > (\sqrt{\frac{\rho \cdot (c_h - \gamma - c_l) + c_l}{\bar{q}}} - 1) \cdot r$. Note that $(\sqrt{\frac{\rho \cdot (c_h - \gamma - c_l) + c_l}{\bar{q}}} - 1) \cdot r > (\sqrt{\frac{c_l}{\bar{q}}} - 1) \cdot r$, and so if the conditions are such that high types want to conceal and run, but $w_l \in ((\sqrt{\frac{c_l}{\bar{q}}} - 1) \cdot r, (\sqrt{\frac{\rho \cdot (c_h - \gamma - c_l) + c_l}{\bar{q}}} - 1) \cdot r)$, there is not an equilibrium in pure strategies, because given that high types are concealing and running, low types do not want to run.

Proposition 3. *For* $w_l > r \cdot \sqrt{\frac{\mu(c_l) \cdot (c_h - \gamma - c_l) + c_l}{\bar{q}}} - r$*,*

a) *there exists a PBE in which both types produce low and run, and the enslaver, believing it is sufficiently likely that low-producing fugitives are in fact high types,* $\rho > \frac{1 - \bar{q} \cdot c_l}{\bar{q} \cdot (c_h - \gamma - c_l)}$*, invests* $\kappa = \sqrt{\bar{q} \cdot [\rho \cdot (c_h - \gamma - c_l) + c_l]} - 1$ *in pursuing low-producing runaways when* $\gamma < \hat{\gamma}'$ *and* $w_h > B'$ *or when* $\gamma > \hat{\gamma}'$ *and* $w_h > C'$*; and*

   $(\psi^* = 0,\ \pi_l^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 1,\ \kappa^*(c_l) = \sqrt{\bar{q} \cdot [\rho \cdot (c_h - \gamma - c_l) + c_l]} - 1,\ \kappa^*(c_h) = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1,\ \mu(c_l, e_S = 1) = \rho)$ if $w_h > \gamma + r \cdot \frac{1 - q_h}{q_h}$ and $\gamma < \hat{\gamma}'$ or $w_h > C'$ and $\gamma > \hat{\gamma}'$

   $(\psi^* = 0,\ \pi_l^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \kappa^*(c_l) = \sqrt{\bar{q} \cdot [\rho \cdot (c_h - \gamma - c_l) + c_l]} - 1,\ \kappa^*(c_h) = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1,\ \mu(c_l, e_S = 1) = \rho)$ if $w_h \in (B', \gamma + r \cdot \frac{1 - q_h}{q_h})$ and $\gamma < \hat{\gamma}'$

b) *there exists a PBE in which high types produce high and stay, low types run, and the enslaver, believing that fugitives are not high types, invests* $\kappa = \sqrt{\bar{q} \cdot c_l} - 1$ *in pursuing low-producing runaways when* $c_l > \frac{1}{\bar{q}}$ *and either* $\gamma > \hat{\gamma}'$ *and* $w_h < A$ *or* $\gamma < \hat{\gamma}'$ *and* $w_h < B'$*; and*

   $(\psi^* = 1,\ \pi_l^* = 1,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \kappa^*(c_l) = \sqrt{\bar{q} \cdot c_l} - 1,\ \kappa^*(c_h) = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1,\ \mu(c_l, e_S = 1) = 0)$ if $w_h \in ((r - \gamma) \cdot \frac{1 - \sqrt{\frac{\bar{q}}{c_l}}}{\sqrt{\frac{\bar{q}}{c_l}}}, A)$ and $\gamma > \hat{\gamma}'$ or $w_h \in ((r - \gamma) \cdot \frac{1 - \sqrt{\frac{\bar{q}}{c_l}}}{\sqrt{\frac{\bar{q}}{c_l}}}, B')$ and $\gamma < \hat{\gamma}'$

($\psi^* = 1$, $\pi_l^* = 1$, $\pi_h^*(c_l) = 0$, $\pi_h^*(c_h) = 0$, $\kappa^*(c_l) = \sqrt{\bar{q} \cdot c_l} - 1$, $\kappa^*(c_h) = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1$, $\mu(c_l, e_S = 1) = 0$) if $w_h < (r - \gamma) \cdot \frac{1 - \sqrt{\frac{\bar{q}}{c_l}}}{\sqrt{\frac{\bar{q}}{c_l}}}$

c) *there exists a PBE in which high types produce high and run, low types run, and the enslaver, believing that fugitives are not high types, invests $\kappa = \sqrt{\bar{q} \cdot c_l} - 1$ in pursuing low-producing runaways when $c_l > \frac{1}{\bar{q}}$, $\gamma > \hat{\gamma}'$, and $w_h \in (A, C')$.*

($\psi^* = 1$, $\pi_l^* = 1$, $\pi_h^*(c_l) = 1$, $\pi_h^*(c_h) = 1$, $\kappa^*(c_l) = \sqrt{\bar{q} \cdot c_l} - 1$, $\kappa^*(c_h) = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1$, $\mu(c_l, e_S = 1) = 0$)

D.2 Low types stay

Given that low types stay, if high types produce low and run, the enslaver believes that low producing runaways are high types. This is consistent with the condition $\mu(c_l) > \frac{1 - \bar{q} \cdot c_l}{\bar{q} \cdot (c_h - \gamma - c_l)}$ and with the intuitive criterion (Cho and Kreps 1987). The high type then faces the same probability of successful escape regardless of her first period action, and so reveal and run strictly dominates conceal and run.

High types prefer to stay rather than to reveal and run if $w_h < A$. Low types then stay if $w_l < r \cdot (\frac{\mu(c_l) \cdot (c_h - \gamma - c_l) + c_l}{\bar{q}} - 1)$, for $\mu(c_l) > \frac{1 - \bar{q} \cdot c_l}{\bar{q} \cdot (c_h - \gamma - c_l)}$. Because both types stay, there is no investment in pursuit in equilibrium, and any belief $\mu(c_l)$ is consistent with Bayesian updating. Given that low types stay, high types produce high and run if $w_h > \gamma + r \cdot \frac{1 - q_h}{q_h}$. The enslaver then invests $\kappa = \sqrt{\bar{q} \cdot (c_h - \gamma)} - 1$ in pursuing these high producers, believing that they are high types with certainty.

Proposition 4. *For $w_l < r \cdot \sqrt{\frac{\mu(c_l) \cdot (c_h - \gamma - c_l) + c_l}{\bar{q}}} - r$,*

a) *there exists a PBE in which high types produce high and stay, low types produce low and stay, and if the enslaver observed a low producer run, he would believe it to be a high type, when $w_h < A$; and*

$(\psi^* = 1,\ \pi_l^* = 0,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 0,\ \kappa^*(c_l) = \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h - \gamma)} - 1,\ \mu(c_l, e_s = 1) = 1)$ if $w_h \in ((r-\gamma)\cdot\frac{1-q_h}{q_h}, A)$

$(\psi^* = 1,\ \pi_l^* = 0,\ \pi_h^*(c_l) = 0,\ \pi_h^*(c_h) = 0,\ \kappa^*(c_l) = \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h - \gamma)} - 1,\ \mu(c_l, e_s = 1) = 1)$ if $w_h < (r-\gamma)\cdot\frac{1-q_h}{q_h}$

b) *there exists a PBE in which low types stay, high types produce high and run, the enslaver invests $\kappa = \sqrt{\bar{q}\cdot(c_h-\gamma)} - 1$ in pursuing high-producing runaways, and if he were to observe a low producer run, he would believe it to be a high type, when $w_h > A$.*

$(\psi^* = 1,\ \pi_l^* = 0,\ \pi_h^*(c_l) = 1,\ \pi_h^*(c_h) = 1,\ \kappa^*(c_l) = \kappa^*(c_h) = \sqrt{\bar{q}\cdot(c_h - \gamma)} - 1,\ \mu(c_l, e_s = 1) = 1)$

Works Cited

Cho, In-Koo and Kreps, David M. 1987. "Signaling Games and Stable Equilibria." *The Quarterly Journal of Economics* 102 (2): 179-222.

D.3 Supplementary figures

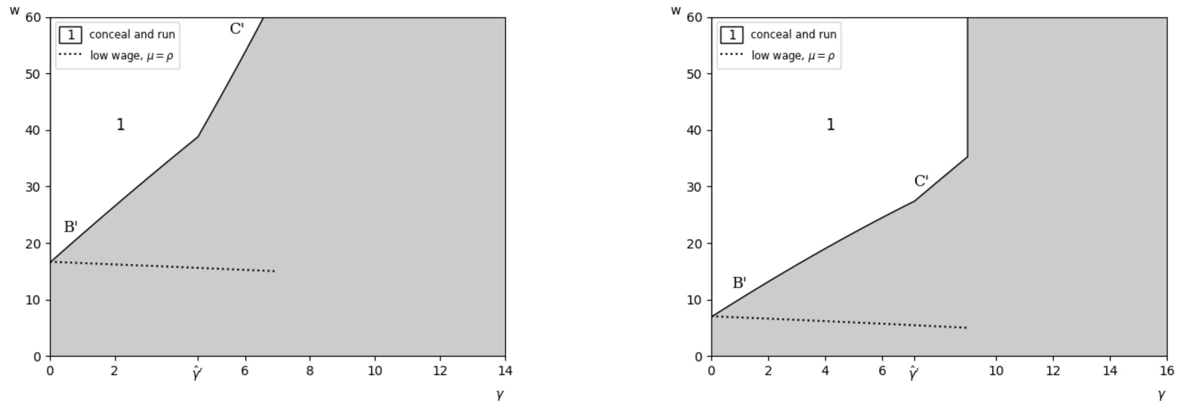
See figure 6 for equilibria when $\mu(c_l) > \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$, S_l runs, and $c_l < \frac{1}{\bar{q}}$. Because $c_l < \frac{1}{\bar{q}}$, M does not invest in pursuing known low types. He only expends resources to pursue low producers if he believes it is sufficiently likely that the runaway is a high type, $\mu(c_l) > \frac{1-\bar{q}\cdot c_l}{\bar{q}\cdot(c_h-\gamma-c_l)}$.

E Mixed strategies

Below I derive the existence conditions for equilibria in mixed strategies. These equilibria are not generic, and so the body of the paper presents the results in pure strategies. For the complete specifications and figures depicting the mixed-strategy equilibria, please contact the author.

Figure 6: Equilibria in which M expends resources to pursue low producers, S_l runs, $c_l < \frac{1}{\bar{q}}$

Panel A. $\bar{q} = 0.25$ Panel B. $\bar{q} = 0.50$



Constant parameters are set at the following values: $c_h = 20$, $c_l = \frac{1}{\bar{q}} - 1$, and $r = 5$, $\rho = .1$. Given that S_h conceals ($\mu(c_l) = \rho$), $w_l > 5 \cdot \sqrt{\frac{0.1 \cdot (20 - \gamma - c_l) + c_l}{\bar{q}}} - 5$ sustains the equilibrium in which S_l runs.

E.1 High types mix

E.1.1 Case 1: Conceal and run = Stay

High types are indifferent between concealing and running, and staying if $2\gamma = \frac{\bar{q}}{1+\kappa} \cdot w_h + (1 - \frac{\bar{q}}{1+\kappa}) \cdot (\gamma - r)$. Let $\hat{\kappa}_1$ be investment in pursuit that makes S_h indifferent between concealing and running, and staying, $\hat{\kappa}_1 := \frac{\bar{q} \cdot (w_h - \gamma + r)}{\gamma + r} - 1$.

$$
\begin{aligned}
2\gamma &= \frac{\bar{q}}{1+\kappa} \cdot w_h + (1 - \frac{\bar{q}}{1+\kappa}) \cdot (\gamma - r) \\
\gamma + r &= \frac{\bar{q}}{1+\kappa} \cdot (w_h - \gamma + r) \\
(1+\kappa) \cdot (\gamma + r) &= \bar{q} \cdot (w_h - \gamma + r) \\
\kappa &= \frac{\bar{q} \cdot (w_h - \gamma + r)}{\gamma + r} - 1
\end{aligned}
$$

Revealing and running is strictly dominated if $w_h < \gamma + r \cdot \frac{1 - q_h}{q_h}$.

$$\begin{aligned}
\frac{\bar{q}}{1+\hat{\kappa}_1}\cdot w_h + (1-\frac{\bar{q}}{1+\hat{\kappa}_1})\cdot(\gamma-r) &> \gamma+\sqrt{\frac{\bar{q}}{c_h-\gamma}}\cdot w_h + (1-\sqrt{\frac{\bar{q}}{c_h-\gamma}})\cdot(\gamma-r)\\
\frac{\bar{q}}{1+\hat{\kappa}_1}\cdot(w_h-\gamma+r) &> \gamma+\sqrt{\frac{\bar{q}}{c_h-\gamma}}\cdot(w_h-\gamma+r)\\
\gamma+r &> \gamma+\sqrt{\frac{\bar{q}}{c_h-\gamma}}\cdot(w_h-\gamma+r)\\
r\cdot(1-\sqrt{\frac{\bar{q}}{c_h-\gamma}}) &> \sqrt{\frac{\bar{q}}{c_h-\gamma}}\cdot(w_h-\gamma)\\
w_h &< \gamma+r\cdot\frac{1-q_h}{q_h}
\end{aligned}$$

Given $\hat{\kappa}_1$, S_l runs if the outside option is greater than $r\cdot\frac{w_h-2\gamma}{\gamma+r}$.

$$\begin{aligned}
0 &< q(\hat{\kappa}_1)\cdot w_l-(1-q(\hat{\kappa}_1))\cdot r\\
r &< \frac{\bar{q}}{\frac{\bar{q}\cdot(w_h-\gamma+r)}{\gamma+r}}\cdot(w_l+r)\\
r\cdot(1-\frac{\gamma+r}{w_h-\gamma+r}) &< \frac{\gamma+r}{w_h-\gamma+r}\cdot w_l\\
r\cdot\frac{w_h-2\gamma}{\gamma+r} &< w_l
\end{aligned}$$

Recall from appendix A that the utility-maximizing investment is $\kappa^*=\max\{0,\sqrt{\bar{q}\cdot[\mu(c_S)\cdot(c_h-\gamma-c_l)+c_l]}-1\}$. If $\frac{\bar{q}\cdot(w_h-\gamma+r)}{\gamma+r}-1=0$, then $\kappa^*=0$ when S_h mixes between concealing and running, and staying. S_l runs when $w_l>r\cdot\frac{1-\bar{q}}{\bar{q}}$. There is a knife-edge equilibrium in mixed strategies, and it is depicted in figure 2 as line B when $\gamma<\hat{\gamma}$.

Now consider the case in which $\frac{\bar{q}\cdot(w_h-\gamma+r)}{\gamma+r}-1>0$. Let $\hat{\mu}_1$ be the posterior belief such that $\hat{\kappa}_1$ maximizes M's utility. Setting $\hat{\kappa}_1$ equal to κ^*, we have that $\hat{\mu}_1=\frac{\bar{q}\cdot(w_h-\gamma+r)^2-c_l\cdot(\gamma+r)^2}{(\gamma+r)^2\cdot(c_h-\gamma-c_l)}$. See below.

$$\begin{aligned}
\frac{\bar{q}\cdot(w_h-\gamma+r)}{\gamma+r}-1 &= \sqrt{\bar{q}\cdot[\hat{\mu}_1\cdot(c_h-\gamma-c_l)+c_l]}-1 \\
[\bar{q}\cdot(w_h-\gamma+r)]^2 &= (\gamma+r)^2\cdot\bar{q}\cdot[\hat{\mu}_1\cdot(c_h-\gamma-c_l)+c_l] \\
\bar{q}\cdot(w_h-\gamma+r)^2-c_l\cdot(\gamma+r)^2 &= (\gamma+r)^2\cdot\hat{\mu}_1\cdot(c_h-\gamma-c_l) \\
\frac{\bar{q}\cdot(w_h-\gamma+r)^2-c_l\cdot(\gamma+r)^2}{(\gamma+r)^2\cdot(c_h-\gamma-c_l)} &= \hat{\mu}_1
\end{aligned}$$

The denominator is positive by assumption 1. In order for the numerator to be positive, $\bar{q}\cdot(w_h-\gamma+r)^2 > c_l\cdot(\gamma+r)^2$. In order for the denominator to be greater than the numerator, $(\gamma+r)^2\cdot(c_h-\gamma) > \bar{q}\cdot(w_h-\gamma+r)^2$.

If low types run and high types conceal and run with probability ψ, the enslaver's posterior belief that a low-producing running is a high type is $\frac{\rho\cdot(1-\psi)}{\rho\cdot(1-\psi)+1-\rho}$. Setting that equal to $\hat{\mu}_1$, we solve for ψ. Let ψ_1^* represent the probability with which S_h conceals and runs when she is indifferent between concealing and running, and staying, $\psi_1^* := \frac{\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2}-c_l-\rho\cdot(c_h-\gamma-c_l)}{\rho\cdot(\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2}-c_h+\gamma)}$. See derivation below.

$$\begin{aligned}
\frac{\rho\cdot(1-\psi)}{\rho\cdot(1-\psi)+1-\rho} &= \frac{\bar{q}\cdot(w_h-\gamma+r)^2-c_l(\gamma+r)^2}{(\gamma+r)^2\cdot(c_h-\gamma-c_l)} \\
(\gamma+r)^2\cdot(c_h-\gamma-c_l)\cdot\rho\cdot(1-\psi) &= (\bar{q}\cdot(w_h-\gamma+r)^2-c_l(\gamma+r)^2)\cdot(1-\psi\cdot\rho) \\
(c_h-\gamma-c_l)\cdot\rho\cdot(1-\psi) &= (\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2}-c_l)\cdot(1-\psi\cdot\rho) \\
\psi\cdot\rho\cdot(\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2}-c_h+\gamma) &= \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2}-c_l-\rho\cdot(c_h-\gamma-c_l) \\
\psi &= \frac{\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2}-c_l-\rho\cdot(c_h-\gamma-c_l)}{\rho\cdot(\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2}-c_h+\gamma)}
\end{aligned}$$

From the conditions on $\hat{\mu}_1$, we know that $\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2} < c_h-\gamma$, and so the denominator on ψ_1^* is negative. Therefore, the denominator needs to be less than the numerator, which in turn needs to be less than zero, in order for ψ_1^* to be an interior probability. The denominator is less than the numerator when $c_l < \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2}$, which was established in the conditions on $\hat{\mu}_1$.

$$\begin{aligned}
\rho \cdot \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2} + \rho\cdot(\gamma-c_h) &< \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2} - c_l + \rho\cdot(c_l+\gamma-c_h) \\
(1-\rho)\cdot c_l &< (1-\rho)\cdot\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2} \\
c_l &< \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2}
\end{aligned}$$

The numerator is negative when $\bar{q}\cdot(w_h-\gamma+r)^2 < ((1-\rho)\cdot c_l + \rho\cdot(c_h-\gamma))\cdot(\gamma+r)^2$; see below.

$$\begin{aligned}
\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2} - c_l + \rho\cdot(c_l+\gamma-c_h) &< 0 \\
\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+r)^2} &< (1-\rho)\cdot c_l + \rho\cdot(c_h-\gamma) \\
\sqrt{\bar{q}}\cdot(w_h-\gamma+r) &< \sqrt{(1-\rho)\cdot c_l + \rho\cdot(c_h-\gamma)}\cdot(\gamma+r) \\
w_h &< \sqrt{\frac{(1-\rho)\cdot c_l + \rho\cdot(c_h-\gamma)}{\bar{q}}}\cdot(\gamma+r)+\gamma-r
\end{aligned}$$

There is a mixed strategy equilibrium when the following conditions hold, in which low types run and high types mix between concealing and running and staying.

- Revealing and running is strictly dominated if $w_h < \gamma + r\cdot\frac{1-q_h}{q_h}$.
- S_l runs if $w_l > r\cdot\frac{w_h-2\gamma}{\gamma+r}$.
- Rewriting in terms of w_h, $\hat{\mu}_1$ is an interior value: $w_h \in (\sqrt{\frac{c_l}{\bar{q}}}\cdot(\gamma+r)+\gamma-r, \frac{1}{q_h}\cdot(\gamma+r)+\gamma-r)$.
- ψ_1^* is an interior value: $w_h < \sqrt{\frac{(1-\rho)\cdot c_l+\rho\cdot(c_h-\gamma)}{\bar{q}}}\cdot(\gamma+r)+\gamma-r$

- $\gamma + r \cdot \frac{1-q_h}{q_h}$ is less than $\frac{1}{q_h} \cdot (\gamma + r) + \gamma - r$, so we can synthesize the conditions as $w_h \in (\sqrt{\frac{c_l}{\bar{q}}} \cdot (\gamma + r) + \gamma - r, \min\{\gamma + r \cdot \frac{1-q_h}{q_h}, \sqrt{\frac{(1-\rho)\cdot c_l + \rho \cdot (c_h - \gamma)}{\bar{q}}} \cdot (\gamma + r) + \gamma - r\})$.

E.1.2 Case 2: Conceal and run = Reveal and run

High types are indifferent between concealing and running, and revealing and running if $\gamma + q_h \cdot w_h + (1-q_h) \cdot (\gamma - r) = \frac{\bar{q}}{1+\kappa} \cdot w_h + (1 - \frac{\bar{q}}{1+\kappa}) \cdot (\gamma - r)$. Let $\hat{\kappa}_2$ be investment in pursuit that makes S_h indifferent between concealing and running, and staying, $\hat{\kappa}_2 \coloneqq \frac{\bar{q} \cdot (w_h - \gamma + r)}{\gamma + q_h \cdot (w_h - \gamma + r)} - 1$.

$$\begin{aligned}
\gamma + q_h \cdot w_h + (1-q_h) \cdot (\gamma - r) &= \frac{\bar{q}}{1+\kappa} \cdot w_h + (1 - \frac{\bar{q}}{1+\kappa}) \cdot (\gamma - r) \\
\gamma + q_h \cdot (w_h - \gamma + r) &= \frac{\bar{q}}{1+\kappa} \cdot (w_h - \gamma + r) \\
(1+\kappa) \cdot (\gamma + q_h \cdot (w_h - \gamma + r)) &= \bar{q} \cdot (w_h - \gamma + r) \\
\kappa &= \frac{\bar{q} \cdot (w_h - \gamma + r)}{\gamma + q_h \cdot (w_h - \gamma + r)} - 1
\end{aligned}$$

Staying is strictly dominated if $\gamma < q_h \cdot w_h + (1-q_h) \cdot (\gamma - r)$. Given $\hat{\kappa}_2$, S_l runs if the outside option is greater than $r \cdot \frac{(1-q_h)\cdot(w_h - \gamma + r) - \gamma}{\gamma + q_h \cdot (w_h - \gamma + r)}$.

$$\begin{aligned}
0 &< q(\hat{\kappa}_2) \cdot w_l - (1 - q(\hat{\kappa}_2)) \cdot r \\
r \cdot (\frac{1}{q(\hat{\kappa}_2)} - 1) &< w_l \\
r \cdot \frac{(1-q_h) \cdot (w_h - \gamma + r) - \gamma}{\gamma + q_h \cdot (w_h - \gamma + r)} &< w_l
\end{aligned}$$

Recall from appendix A that the utility-maximizing investment is $\kappa^* = \max\{0, \sqrt{\bar{q} \cdot [\mu(c_S) \cdot (c_h - \gamma - c_l) + c_l]} - 1\}$. If $\frac{\bar{q} \cdot (w_h - \gamma + r)}{\gamma + q_h \cdot (w_h - \gamma + r)} - 1 \leq 0$, then $\kappa^* = 0$ when S_h mixes between concealing and running, and revealing and running. S_l runs when $w_l > r \cdot \frac{1-\bar{q}}{\bar{q}}$. There is a knife-edge equilibrium in mixed strategies, and it is depicted in figure 2 as line C.

Now consider the case in which $\frac{\bar{q}\cdot(w_h-\gamma+r)}{\gamma+q_h\cdot(w_h-\gamma+r)}-1>0$. Let $\hat{\mu}_2$ be the posterior belief such that $\hat{\kappa}_2$ maximizes M's utility. Setting $\hat{\kappa}_2$ equal to κ^*, we have that $\hat{\mu}_2=\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2\cdot(c_h-\gamma-c_l)}-\frac{c_l}{c_h-\gamma-c_l}$. See below.

$$\begin{aligned}\frac{\bar{q}\cdot(w_h-\gamma+r)}{\gamma+q_h\cdot(w_h-\gamma+r)}-1 &= \sqrt{\bar{q}\cdot[\hat{\mu}_2\cdot(c_h-\gamma-c_l)+c_l]}-1\\ \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2} &= \hat{\mu}_2\cdot(c_h-\gamma-c_l)+c_l\\ \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2\cdot(c_h-\gamma-c_l)}-\frac{c_l}{c_h-\gamma-c_l} &= \hat{\mu}_2\end{aligned}$$

$\hat{\mu}_2$ is positive when $\gamma<(\sqrt{\frac{\bar{q}}{c_l}}-q_h)\cdot(w_h-\gamma+r)$; see below.

$$\begin{aligned}\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2\cdot(c_h-\gamma-c_l)}-\frac{c_l}{c_h-\gamma-c_l} &> 0\\ \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2} &> c_l\\ w_h-\gamma+r &> \sqrt{\frac{c_l}{\bar{q}}}\cdot(\gamma+q_h\cdot(w_h-\gamma+r))\\ (\sqrt{\frac{\bar{q}}{c_l}}-q_h)\cdot(w_h-\gamma+r) &> \gamma\end{aligned}$$

$\hat{\mu}_2$ is always less than 1, because γ and q_h are positive; see below.

$$\begin{aligned}\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2\cdot(c_h-\gamma-c_l)}-\frac{c_l}{c_h-\gamma-c_l} &< 1\\ \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2} &< c_h-\gamma\\ \frac{w_h-\gamma+r}{\gamma+q_h\cdot(w_h-\gamma+r)} &< \frac{1}{q_h}\\ w_h-\gamma+r &< \frac{\gamma}{q_h}+w_h-\gamma+r\end{aligned}$$

As before, the enslaver's posterior belief that a low-producing runaway is a high type is $\frac{\rho\cdot(1-\psi)}{\rho\cdot(1-\psi)+1-\rho}$. Setting that equal to $\hat{\mu}_2$, we solve for ψ to determine the probability with which S_h conceals and runs, given that she is indifferent between concealing and running and revealing and running. Let ψ_2^* represent the probability with which S_h reveals and runs when she is indifferent between concealing and running and revealing and running, $\psi_2^* := \frac{\bar{q}\cdot(w_h-\gamma+r)^2-(c_l\cdot(1-\rho)+\rho\cdot(c_h-\gamma))\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2}{\rho\cdot(\bar{q}\cdot(w_h-\gamma+r)^2-(c_h-\gamma)\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2)}$. See derivation below.

$$\begin{aligned}
\frac{\rho\cdot(1-\psi)}{\rho\cdot(1-\psi)+1-\rho} &= \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2\cdot(c_h-\gamma-c_l)}-\frac{c_l}{c_h-\gamma-c_l}\\
\rho\cdot(1-\psi)\cdot(c_h-\gamma-c_l) &= (\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2}-c_l)\cdot(1-\rho\cdot\psi)\\
\rho\cdot\psi\cdot(\frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2}-c_h+\gamma) &= \frac{\bar{q}\cdot(w_h-\gamma+r)^2}{(\gamma+q_h\cdot(w_h-\gamma+r))^2}-c_l-\rho\cdot(c_h-\gamma-c_l)\\
\psi &= \frac{\bar{q}\cdot(w_h-\gamma+r)^2-(c_l\cdot(1-\rho)+\rho\cdot(c_h-\gamma))\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2}{\rho\cdot(\bar{q}\cdot(w_h-\gamma+r)^2-(c_h-\gamma)\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2)}
\end{aligned}$$

ψ_2^* is less than one when $\gamma > (\sqrt{\frac{\bar{q}}{c_l}}-q_h)\cdot(w_h-\gamma+r)$, which contradicts the condition that makes $\hat{\mu}_2$ positive. The derivation below assumed that the denominator and numerator are positive. If they are both negative, then the condition holds. The denominator is negative when $(c_h-\gamma)\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2) > \rho\cdot\bar{q}\cdot(w_h-\gamma+r)^2$. The numerator is negative when $(c_h-\gamma)\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2) > \bar{q}\cdot(w_h-\gamma+r)^2-c_l\cdot(1-\rho)\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2$.

$$\begin{aligned}
\frac{\bar{q}\cdot(w_h-\gamma+r)^2-(c_l\cdot(1-\rho)+\rho\cdot(c_h-\gamma))\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2}{\rho\cdot(\bar{q}\cdot(w_h-\gamma+r)^2-(c_h-\gamma)\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2)} &< 1\\
\bar{q}\cdot(w_h-\gamma+r)^2-c_l\cdot(1-\rho)\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2 &< \rho\cdot\bar{q}\cdot(w_h-\gamma+r)^2\\
\bar{q}\cdot(w_h-\gamma+r)^2 &< c_l\cdot(\gamma+q_h\cdot(w_h-\gamma+r))^2\\
(\sqrt{\frac{\bar{q}}{c_l}}-q_h)\cdot(w_h-\gamma+r) &< \gamma
\end{aligned}$$

So, there is a mixed strategy equilibrium when the following conditions hold, in which low types run and high types mix between concealing and running and revealing and running.

- $\hat{\mu}_2$ and ψ_2^* are interior: $(c_h - \gamma) \cdot (\gamma + q_h \cdot (w_h - \gamma + r))^2) > \max\{\bar{q} \cdot (w_h - \gamma + r)^2 - c_l \cdot (1 - \rho) \cdot (\gamma + q_h \cdot (w_h - \gamma + r))^2, \rho \cdot \bar{q} \cdot (w_h - \gamma + r)^2\}$ and $\gamma < (\sqrt{\frac{\bar{q}}{c_l}} - q_h) \cdot (w_h - \gamma + r)$.
- Staying is strictly dominated: $\gamma < q_h \cdot w_h + (1 - q_h) \cdot (\gamma - r)$
- M invests some positive amount: $\frac{\bar{q} \cdot (w_h - \gamma + r)}{\gamma + q_h \cdot (w_h - \gamma + r)} > 1$
- S_l runs: $w_l > r \cdot \frac{(1 - q_h) \cdot (w_h - \gamma + r) - \gamma}{\gamma + q_h \cdot (w_h - \gamma + r)}$

E.2 Low types mix

S_l is indifferent between running and staying when $0 = q(\kappa) \cdot w_l - r \cdot (1 - q(\kappa))$. Let $\hat{\kappa}_3$ be the investment in pursuit that makes S_l indifferent between running and staying, $\hat{\kappa}_3 := \frac{\bar{q} \cdot (w_l + r)}{r} - 1$. See derivation below.

$$
\begin{aligned}
0 &= \frac{\bar{q}}{1+\kappa} \cdot w_l - r \cdot (1 - \frac{\bar{q}}{1+\kappa}) \\
\bar{q} \cdot (w_l + r) &= (1+\kappa) \cdot r \\
\frac{\bar{q} \cdot (w_l + r)}{r} - 1 &= \kappa
\end{aligned}
$$

When the enslaver invests $\frac{\bar{q} \cdot (w_l + r)}{r} - 1$ in pursuing low producers, low-producing runaways succeed with probability $\frac{r}{w_l + r}$. Given this probability of success, S_h prefers to conceal and run than to stay when $w_h > (\gamma + r) \cdot \frac{w_l + r}{r} + \gamma - r$. See below.

$$\begin{aligned} 2\gamma < && \frac{r}{w_l+r}\cdot w_h + (1-\frac{r}{w_l+r})\cdot(\gamma-r) \\ \gamma+r < && \frac{r}{w_l+r}\cdot(w_h-\gamma+r) \\ (\gamma+r)\cdot\frac{w_l+r}{r}+\gamma-r < && w_h \end{aligned}$$

S_h prefers to conceal and run than to reveal and run when $w_h > \gamma\cdot\frac{w_l\cdot(1-q_h)+r\cdot(2-q_h)}{r-q_h\cdot(w_l+r)} - r$. See below.

$$\begin{aligned} \gamma+q_h\cdot w_h+(1-q_h)\cdot(\gamma-r) < && \frac{r}{w_l+r}\cdot w_h + (1-\frac{r}{w_l+r})\cdot(\gamma-r) \\ \gamma+q_h\cdot(w_h-\gamma+r) < && \frac{r}{w_l+r}\cdot(w_h-\gamma+r) \\ \gamma < && (\frac{r}{w_l+r}-q_h)\cdot(w_h-\gamma+r) \\ \gamma\cdot\frac{w_l\cdot(1-q_h)+r\cdot(2-q_h)}{r-q_h\cdot(w_l+r)}-r < && w_h \end{aligned}$$

Recall from appendix A that the utility-maximizing investment is $\kappa^* = \max\{0, \sqrt{\bar{q}\cdot[\mu(c_S)\cdot(c_h-\gamma-c_l)+c_l]}-1\}$. If $\frac{\bar{q}\cdot(w_l+r)}{r} = 1$, then $\kappa^* = 0$ when S_l mixes between running and staying.

Now consider the case in which $\frac{\bar{q}\cdot(w_l+r)}{r} - 1 > 0$. Let $\hat{\mu}_3$ be the posterior belief such that $\hat{\kappa}_3$ maximizes M's utility. Setting $\hat{\kappa}_3$ equal to κ^*, we have that $\hat{\mu}_3 := \frac{\bar{q}\cdot(\frac{w_l+r}{r})^2-c_l}{c_h-\gamma-c_l}$. See below.

$$\begin{aligned} \frac{\bar{q}\cdot(w_l+r)}{r}-1 = && \sqrt{\bar{q}\cdot[\hat{\mu}_3\cdot(c_h-\gamma-c_l)+c_l]}-1 \\ \bar{q}\cdot(\frac{w_l+r}{r})^2 = && \hat{\mu}_3\cdot(c_h-\gamma-c_l)+c_l \\ \frac{\bar{q}\cdot(\frac{w_l+r}{r})^2-c_l}{c_h-\gamma-c_l} = && \hat{\mu}_3 \end{aligned}$$

$\hat{\mu}_3 \in (0,1)$ when $(\frac{w_l}{r}+1)^2 \cdot \bar{q} \in (c_l, c_h - \gamma)$.

Given that low types are indifferent between running and staying, and high types prefer to conceal and run, the enslaver's posterior belief that a low-producing runaway is a high type is $\frac{\rho}{\rho+(1-\rho)\cdot\pi_l}$. Setting that equal to $\hat{\mu}_3$, we solve for π_l to determine the probability with which S_l runs, $\pi_l^* = \frac{\rho\cdot[c_h-\gamma-(\frac{w_l}{r}+1)^2\cdot\bar{q}]}{((\frac{w_l}{r}+1)^2\cdot\bar{q}-c_l)\cdot(1-\rho)}$. See derivation below.

$$\begin{aligned}
\frac{\rho}{\rho+(1-\rho)\cdot\pi_l} &= \frac{(\frac{w_l}{r}+1)^2\cdot\bar{q}-c_l}{c_h-\gamma-c_l} \\
\rho\cdot(c_h-\gamma-c_l)+ & \quad ((\frac{w_l}{r}+1)^2\cdot\bar{q}-c_l)\cdot(\rho+(1-\rho)\cdot\pi_l) \\
\rho\cdot[c_h-\gamma-(\frac{w_l}{r}+1)^2\cdot\bar{q}] &= ((\frac{w_l}{r}+1)^2\cdot\bar{q}-c_l)\cdot(1-\rho)\cdot\pi_l \\
\frac{\rho\cdot[c_h-\gamma-(\frac{w_l}{r}+1)^2\cdot\bar{q}]}{((\frac{w_l}{r}+1)^2\cdot\bar{q}-c_l)\cdot(1-\rho)} &= \pi_l
\end{aligned}$$

π_l^* is positive when $c_h - \gamma > (\frac{w_l}{r}+1)^2 \cdot \bar{q} > c_l$, which is consistent with the conditions for $\hat{\mu}_3 \in (0,1)$. π_l^* is less than one when $c_h - \gamma < (\frac{w_l}{r}+1)^2 \cdot \frac{\bar{q}}{\rho} - \frac{c_l}{\rho} + c_l$; see below.

$$\begin{aligned}
((\frac{w_l}{r}+1)^2\cdot\bar{q}-c_l)\cdot(1-\rho) &> \rho\cdot[c_h-\gamma-(\frac{w_l}{r}+1)^2\cdot\bar{q}] \\
(\frac{w_l}{r}+1)^2\cdot\bar{q}-c_l+\rho\cdot c_l &> \rho\cdot(c_h-\gamma) \\
(\frac{w_l}{r}+1)^2\cdot\bar{q} &> \rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l
\end{aligned}$$

Because $c_h - \gamma > c_l$, $\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l > c_l$, and so we can synthesize the conditions on π_l^* and $\hat{\mu}_3$ as $(\frac{w_l}{r}+1)^2 \cdot \bar{q} \in (\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l, c_h-\gamma)$.

So there is a mixed strategy equilibrium when the following conditions hold, in which low types are indifferent between running and staying, high types conceal and run, and the enslaver invests $\hat{\kappa}_3$ in pursuing low-producing runaways.

- $\hat{\mu}_3$ and π_l^* are interior values: $(\frac{w_l}{r}+1)^2 \cdot \bar{q} \in (\rho\cdot(c_h-\gamma)+(1-\rho)\cdot c_l, c_h-\gamma)$

- M invests some positive amount: $\frac{\bar{q}\cdot(w_l+r)}{r} - 1 > 0$

- S_h conceals and runs: $w_h > \max\{\ (\gamma + r)\cdot\frac{w_l+r}{r} + \gamma - r, \gamma\cdot\frac{w_l\cdot(1-q_h)+r\cdot(2-q_h)}{r-q_h\cdot(w_l+r)} - r\}$