A Theory of Trade Policy Transitions
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Appendix for Online Publication

Proof of Proposition 2

We start by showing that for any $i \in \{\alpha, \beta\}$, $(\gamma_a, \gamma_b) \in (\frac{1}{2}, 1)^2$, $\tau_a \in [0, \tau^{aut}]$ and $\tau_b, \tau_b' \in [0, \tau^{aut}]$ it holds

$$v_i(\tau_b', \tau_a) \gtreqless v_i(\tau_b; \tau_a) \iff |\tau_b' - \tau_{bi}| \lesseqgtr |\tau_b - \tau_{bi}|. \tag{10}$$

Fix $i \in \{\alpha, \beta\}$, $(\gamma_a, \gamma_b) \in (\frac{1}{2}, 1)^2$, $\tau_a \in [0, \tau^{aut}]$. Define $\tilde{v}_i(\cdot|\tau_a) : \mathbb{R} \to \mathbb{R}$ to have the exact same functional form as $v_i(\cdot|\tau_a)$ but unrestricted domain. Since $\tilde{v}_i(\cdot|\tau_a)$ is a concave quadratic, it is single peaked and symmetric. Hence, for any $\tau_b, \tau_b' \in \mathbb{R}$, $\tilde{v}_i(\tau_b', \tau_a) \gtreqless \tilde{v}_i(\tau_b; \tau_a)$ if and only if $|\tau_b' - \tau^*| \lesseqgtr |\tau_b - \tau^*|$, where τ^* is the point of global maximum of $\tilde{v}_i(\cdot|\tau_a)$. We have two cases. If $\tau^* \in [0, \tau^{aut}]$ then $\tau_{bi} = \tau^*$, by definition of τ_{bi}, implying (10). If $\tau^* \notin \{0, \tau^{aut}\}$, then $\tau_{bi} = \arg\min_{\tau \in [0, \tau^{aut}]} |\tau^* - \tau|$. Hence either (x) $\hat{\tau} \leq \tau_{bi} < \tau^*$ or (ii) $\hat{\tau} \geq \tau_{bi} > \tau^*$ for each $\hat{\tau} \in [0, \tau^{aut}]$. Let $\tau_b, \tau_b' \in [0, \tau^{aut}]$. Since either (x) or (ii) must hold for both $\hat{\tau} = \tau_b$ and $\hat{\tau} = \tau_b'$, $|\tau_b' - \tau^*| \lesseqgtr |\tau_b - \tau^*|$ if and only if $|\tau_b' - \tau_{bi}| \lesseqgtr |\tau_b - \tau_{bi}|$. Hence $\tilde{v}_i(\tau_b', \tau_a) \gtreqless \tilde{v}_i(\tau_b; \tau_a)$ if and only if $|\tau_b' - \tau_{bi}| \lesseqgtr |\tau_b - \tau_{bi}|$, which implies (10).

Note that (10) means that i's preference ranking over domestic tariffs in $[0, \tau^{aut}]$ only depends on the distance of each tariff from her bliss point τ_{bi}. But for $\tau_b, \tau_b^0 \in [0, \tau^{aut}]$, we have that $|\tau_b - \tau_{bi}| \leq |\tau_b^0 - \tau_{bi}|$ if and only if $\tau_b \in [\min\{2\tau_{bi} - \tau_b^0, \tau_b^0\}, \max\{2\tau_{bi} - \tau_b^0, \tau_b^0\}]$. Hence for any responding party $i \in \{\alpha, \beta\}$, $\tau_b^0, \tau_{bi}, \tau_a \in [0, \tau^{aut}]$, and unilateral tariff proposal $\tau_b^{-i} \in [0, \tau^{aut}]$, it has

$$\rho_i = 1 \iff \tau_b^{-i} \in A_i,$$

where $A_i \equiv [\min\{2\tau_{bi} - \tau_b^0, \tau_b^0\}, \max\{2\tau_{bi} - \tau_b^0, \tau_b^0\}]$ and we assumed acceptance when i is indifferent.

To see why Proposition 2 holds, consider the agenda setter $i \in \{\alpha, \beta\}$ and fix $\tau_b^0, \tau_{b-i}, \tau_a \in [0, \tau^{aut}]$. By (10), τ_b^{i*} is the tariff in $A_{-i} \cap [0, \tau^{aut}]$ that is closest to τ_{bi}. Formally $\tau_b^{i*} = \arg\min_{\tau \in A_{-i} \cap [0, \tau^{aut}]} |\tau - \tau_{bi}|$. If $\tau_{bi} \in A_{-i}$ then $\tau_b^{i*} = \tau_{bi}$. If $\tau_{bi} \notin A_{-i}$ then, given that $\tau_{b-i} \in A_{-i}$ by construction, and $\tau_{b\alpha} < \tau_{b\beta}$, it must be either $i = \alpha$ and $\tau_{b\alpha} < \min\{2\tau_{b\beta} - \tau_b^0, \tau_b^0\}$, or $i = \beta$ and $\tau_{b\beta} > \max\{2\tau_{b\alpha} - \tau_b^0, \tau_b^0\}$. Hence, if $\tau_{bi} \notin A_{-i}$, we have $\tau_b^{i*} = \min\{2\tau_{b-i} - \tau_b^0, \tau_b^0\}$ if $i = \alpha$ and $\tau_b^{i*} = \max\{2\tau_{b-i} - \tau_b^0, \tau_b^0\}$ otherwise.

In conclusion,

$$\tau_b^{\alpha*} = \begin{cases} \tau_{b\alpha} & \text{if } \tau_{b\alpha} \geq \min\{2\tau_{b\beta} - \tau_b^0, \tau_b^0\} \\ \min\{2\tau_{b\beta} - \tau_b^0, \tau_b^0\} & \text{if } \tau_{b\alpha} < \min\{2\tau_{b\beta} - \tau_b^0, \tau_b^0\} \end{cases}$$

$$\tau_b^{\beta*} = \begin{cases} \tau_{b\beta} & \text{if } \tau_{b\beta} \leq \max\{2\tau_{b\alpha} - \tau_b^0, \tau_b^0\} \\ \max\{2\tau_{b\alpha} - \tau_b^0, \tau_b^0\} & \text{if } \tau_{b\beta} > \max\{2\tau_{b\alpha} - \tau_b^0, \tau_b^0\} \end{cases}$$

where we used the fact that $\tau_{b\alpha} < \tau_{b\beta} \leq \max\{2\tau_{b\beta} - \tau_b^0, \tau_b^0\}$ and $\tau_{b\beta} > \tau_{b\alpha} \geq \min\{2\tau_{b\alpha} - \tau_b^0, \tau_b^0\}$. Since $\tau_{b\alpha} < \tau_{b\beta}$, the above definitions of $\tau_b^{\alpha*}$ and $\tau_b^{\beta*}$ are a compact form of the ones in Proposition 2.

Proof of Proposition 3

$T^{\alpha*}$ solves α's problem

$$\max_{\hat{T} \geq 0} v_\alpha(0,0) - \hat{T}$$
$$\text{subject to } v_\beta(0,0) + \hat{T} \geq v_\beta(\tau_b^0, \tau_a).$$

Given that $v_\alpha(0,0)$ does not depend on T, the problem can be rewritten as

$$\min_{\hat{T} \geq 0} \hat{T}$$
$$\text{subject to } \hat{T} \geq v_\beta(\tau_b^0, \tau_a) - v_\beta(0,0).$$

We have two cases. If $v_\beta(\tau_b^0, \tau_a) - v_\beta(0,0) \leq 0$ then $T^0 \geq 0$ implies $\hat{T} \geq v_\beta(\tau_b^0, \tau_a) - v_\beta(0,0)$; hence, we can ignore the latter constraint and the solution is $\hat{T} = 0$. If $v_\beta(\tau_b^0, \tau_a) - v_\beta(0,0) > 0$ then $\hat{T} \geq v_\beta(\tau_b^0, \tau_a) - v_\beta(0,0)$ implies $T^0 > 0$; hence we can ignore the non-negativity constraint and the solution becomes $\hat{T} = v_\beta(\tau_b^0, \tau_a) - v_\beta(0,0)$.

To sum up,

$$T^{\alpha*} = \begin{cases} 0, & \text{if } v_\beta(\tau_b^0, \tau_a) - v_\beta(0,0) \leq 0 \\ v_\beta(\tau_b^0, \tau_a) - v_\beta(0,0), & \text{if } v_\beta(\tau_b^0, \tau_a) - v_\beta(0,0) > 0, \end{cases}$$

This proves the first part of Proposition 3. The proof of the second part follows analogously.

Proof of Proposition 4

Fix $(\gamma_a, \gamma_b) \in (\frac{1}{2}, 1)^2$. Let $\Delta^{FT}(\tau_b, \tau_a) = v_\alpha(0,0) + v_\beta(0,0) - (v_\alpha(\tau_b, \tau_a) + v_\beta(\tau_b, \tau_a))$ be country A aggregate welfare gain from free trade when unilateral tariffs are (τ_b, τ_a).

First, note that $\Delta^{FT}(\tau_b, \tau_a)$ takes the following form:

$$\begin{aligned}\Delta^{FT}(\tau_b;\tau_a) =& \frac{(1-\gamma_a+\gamma_b)\tau_b-(1+\gamma_a-\gamma_b)\tau_a}{8}-\frac{\tau_A^2+\tau_a^2}{16}+\\ &-\tau_b\left(\frac{1-\gamma_a+\gamma_b-\tau_b}{4}-1+\gamma_a\right)+\frac{\gamma_a}{4}\tau_a-\frac{3(1-\gamma_a)}{4}\tau_b;\end{aligned}$$

it can be easily verified that $\Delta^{FT}(\tau_b;\tau_a)$ has the following properties, which will be useful in proving proposition 4:

1. $\Delta^{FT}(\tau_b;\tau_a)$ is continuous in τ_b and τ_a.
2. $\Delta^{FT}(\tau_b;\tau_a)$ is strictly increasing in τ_a on $[0,\tau^{aut}]$.
3. $\Delta^{FT}(\tau_b;\tau_a)$ is a strictly convex quadratic equation in τ_b, with minimum at $\tau_b=\frac{\tau^{aut}}{3}$ and maximum at $\tau_b=\tau^{aut}$.
4. $\Delta^{FT}(\frac{\tau^{aut}}{3},\tau^{aut})>0$, $\Delta^{FT}(\frac{\tau^{aut}}{3},0)<0$.

Second, note that for all $(\tau_b;\tau_a)\in[0,\tau^{aut}]^2$ it holds $\frac{\partial v_\beta(\tau_b;\tau_a)}{\partial\tau_a}=\frac{1+\tau_a+\gamma_a-\gamma_b}{16}>0$ and $\tau_{b\beta}\geq\tau_b^{\beta*}(\tau_b^0)>0$, implying $v_\beta(0,0)-v_\beta(\tau_b^{\beta*}(\tau_b^0),\tau_a)<0$. Assuming that agenda setter β chooses $\mathcal{I}_{FT}^\beta=1$ if indifferent between $\mathcal{I}_{FT}^\beta=1$ and $\mathcal{I}_{FT}^\beta=0$, for all $(\tau_b^0,\tau_a)\in[0,\tau^{aut}]^2$ it holds

$$\begin{aligned}\mathcal{I}_{FT}^{\beta*}=1 &\iff v_\beta(0,0)+T^{\beta*}\geq v_\beta(\tau_b^{\beta*}(\tau_b^0),\tau_a)\\ &\iff v_\beta(0,0)-v_\beta(\tau_b^{\beta*}(\tau_b^0),\tau_a)\\ &\quad+\max\left\{0,v_\alpha(0,0)-v_\alpha(\tau_b^{\beta*}(\tau_b^0),\tau_a)+v_\alpha(\tau_b^{\beta*}(\tau_b^0),\tau_a)-v_\alpha(\tau_b^0,\tau_a)\right\}\geq0\\ &\iff\Delta^{FT}(\tau_b^{\beta*}(\tau_b^0),\tau_a)+v_\alpha(\tau_b^{\beta*}(\tau_b^0),\tau_a)-v_\alpha(\tau_b^0,\tau_a)\geq0,\end{aligned}$$

where the last implication follows from $v_\beta(0,0)-v_\beta(\tau_b^{\beta*}(\tau_b^0),\tau_a)<0$. Similarly, assuming that agenda setter α chooses $\mathcal{I}_{FT}^\alpha=1$ if indifferent between $\mathcal{I}_{FT}^\alpha=1$ and $\mathcal{I}_{FT}^\alpha=0$, for all $(\tau_b^0,\tau_a)\in[0,\tau^{aut}]^2$ it holds

$$\begin{aligned}\mathcal{I}_{FT}^{\alpha*}=1 &\iff v_\alpha(0,0)-T^{\alpha*}\geq v_\alpha(\tau_b^{\alpha*}(\tau_b^0),\tau_a)\\ &\iff v_\alpha(0,0)-v_\alpha(\tau_b^{\alpha*}(\tau_b^0)+\min\left\{0,v_\beta(0,0)-v_\beta(\tau_b^0,\tau_a)\right\}\geq0\\ &\iff\Delta^{FT}(\tau_b^{\alpha*}(\tau_b^0),\tau_a)\\ &\quad+\min\left\{v_\beta(\tau_b^{\alpha*}(\tau_b^0),\tau_a)-v_\beta(0,0),v_\beta(\tau_b^{\alpha*}(\tau_b^0),\tau_a)-v_\beta(\tau_b^0,\tau_a)\right\}\geq0.\end{aligned}$$

This second set of results allows us reformulate Proposition 4 as follows. For all $\tau_b;\tau_a\in$

$[0, \tau^{aut}]$, let

$$\begin{aligned} d_\alpha(\tau_b; \tau_a) &= \min\{v_\beta(\tau_b^{\alpha*}(\tau_b), \tau_a) - v_\beta(0,0), v_\beta(\tau_b^{\alpha*}(\tau_b), \tau_a) - v_\beta(\tau_b, \tau_a)\}, \\ d_\beta(\tau_b; \tau_a) &= v_\alpha(\tau_b^{\beta*}(\tau_b), \tau_a) - v_\alpha(\tau_b, \tau_a), \\ \Delta_i(\tau_b; \tau_a) &= \Delta^{FT}(\tau_b^{i*}(\tau_b), \tau_a) + d_i(\tau_b; \tau_a), \text{ for } i \in \{\alpha, \beta\}; \end{aligned}$$

proving Proposition 4 is equivalent to showing that, for all $i \in \{\alpha, \beta\}$ and all $\tau_b^0 \in [0, \tau^{aut}]$, there exist $\bar{\tau}_a^i(\tau_b^0) \in [0, \tau^{aut})$ such that:

$$\Delta_i(\tau_b^0, \tau_a) \geq 0 \iff \tau_a \geq \bar{\tau}_a^i(\tau_b^0). \tag{11}$$

We start by showing that for all $i \in \{\alpha, \beta\}$ and $\tau_b^0 \in [0, \tau^{aut}]$, $\Delta_i(\tau_b^0, \tau_a)$ is strictly increasing in τ_a. To see this, note that (i) $\frac{\partial^2 v_{-i}}{\partial \tau_b \partial \tau_a} = 0$ and therefore $v_{-i}(\tau_b^{i*}(\tau_b), \tau_a) - v_{-i}(\tau_b, \tau_a)$ is constant in τ_a; and (ii) $v_\beta(\tau_b^{\alpha*}(\tau_b), \tau_a) - v_\beta(0,0)$ is strictly increasing in τ_a because $\frac{\partial v_\beta}{\partial \tau_a} > 0$. Hence, for each $i \in \{\alpha, \beta\}$, d_i is weakly increasing in τ_a. This, together with property 2 of Δ^{FT} implies that Δ_i is strictly increasing in τ_a on $[0, \tau^{aut}]$. The strict monotonicity result has three immediate consequences: for any given $\tau_b^0 \in [0, \tau^{aut})$ and $i \in \{\alpha, \beta\}$, (i) there exists at most one value $\hat{\tau}_a^i(\tau_b^0) \in [0, \tau^{aut})$ such that $\Delta_i(\tau_b^0, \hat{\tau}_a^i(\tau_b^0)) = 0$; (ii) if $\hat{\tau}_a^i(\tau_b^0)$ exists, then (11) holds by setting $\bar{\tau}_a^i(\tau_b^0) = \hat{\tau}_a^i(\tau_b^0)$; (iii) if $\Delta_i(\tau_b^0, 0) \geq 0$ then (11) holds by setting $\bar{\tau}_a^i(\tau_b^0) = 0$. It remains to show is that if $\Delta_i(\tau_b^0, 0) < 0$ then $\hat{\tau}_a^i(\tau_b^0)$ exists in $(0, \tau^{aut})$. This can be shown using the intermediate value theorem. Fix $i \in \{\alpha, \beta\}$ and τ_b^0 such that $\Delta_i(\tau_b^0, 0) < 0$. By definition of τ_b^{i*}, $d_i(\tau_b^0, \tau_a) \geq 0$ for each $\tau_a \in [0, \tau^{aut}]$. Moreover, properties 3 and 4 of Δ^{FT} imply that $\Delta^{FT}(\tau_b^{i*}(\tau_b^0), \tau^{aut}) > 0$. It follows that $\Delta_i(\tau_b^0, \tau^{aut}) = \Delta^{FT}(\tau_b^{i*}(\tau_b^0), \tau^{aut}) + d_i(\tau_b^0, \tau^{aut}) > 0$. Finally, note that Δ_i is continuous in its arguments, due to the continuity of Δ^{FT}, τ_b^{i*} and d_i. Hence by the intermediate value theorem, there exist $\hat{\tau}_a^i \in (0, \tau^{aut})$ such that $\Delta_i(\tau_b^0, \hat{\tau}_a^i) = 0$. This completes the proof of Proposition 4.

Proof of Proposition 5

First consider α's problem

$$\begin{aligned} \tau_b^{\alpha^*} = \arg\max_{\hat{\tau} \geq 0} \;& v_\alpha(\hat{\tau}, 0) \\ \text{subject to } & v_\beta(\hat{\tau}, 0) \geq v_\beta(0,0) + T^0 \\ & \hat{\tau} \leq \tau^{aut}. \end{aligned}$$

We know $v_i(\cdot; \tau_a)$ is single-peaked, so the KKT sufficient condition is satisfied. The Lagrangian of the problem is

$$\mathcal{L}(\hat{\tau}, \lambda_1, \lambda_2) = v_\alpha(\hat{\tau}, 0) + \lambda_1(v_\beta(\hat{\tau}, 0) - v_\beta(0,0) - T^0) + \lambda_2(\tau^{aut} - \hat{\tau}).$$

The KKT conditions are

$$\frac{\partial \mathcal{L}}{\partial \hat{\tau}} = \frac{\partial v_\alpha}{\partial \hat{\tau}} + \lambda_1 \frac{\partial v_\beta}{\partial \hat{\tau}} - \lambda_2 \leq 0, \qquad \hat{\tau} \geq 0, \hat{\tau}\frac{\partial \mathcal{L}}{\partial \hat{\tau}} = 0,$$
$$v_\beta(\hat{\tau}, 0) - v_\beta(\tau_b^0, 0) - T^0 \geq 0, \qquad \lambda_1 \geq 0, \lambda_1(v_\beta(\hat{\tau}, 0) - v_\beta(0, 0) - T^0) = 0,$$
$$\tau^{aut} - \hat{\tau} \geq 0, \qquad \lambda_2 \geq 0, \lambda_2(\tau^{aut} - \hat{\tau}) = 0.$$

First, note that $\frac{\partial \mathcal{L}}{\partial \hat{\tau}}(\hat{\tau}, 0, 0) = \frac{\partial v_\alpha}{\partial \hat{\tau}} = 0$ solves party α's unconstrained utility maximization problem and implies $\hat{\tau} = \tau_{b\alpha} \in [0, \tau^{aut}]$. Second, $\bar{\tau}_{b\alpha}(T^0)$, if exists, is defined as the smallest $\hat{\tau}$ such that β's incentive constraint binds, so $\frac{\partial v_\beta}{\partial \hat{\tau}}$ is increasing at $\bar{\tau}_{b\alpha}(T^0)$, that is, $\bar{\tau}_{b\alpha}(T^0) \leq \tau_{b\beta}$. Further recall that by definition, $\tau_{b\alpha} < \tau_{b\beta}$.

There are two cases based on the existence and location of $\bar{\tau}_{b\alpha}(T^0)$:

1. The problem is infeasible when the constraints lead to an empty set. When $\bar{\tau}_{b\alpha}(T^0)$ doesn't exist on $[0, \tau^{aut}]$, that is when $T^0 > v_\beta(\tau_{b\beta}, 0) - v_\beta(0, 0)$, then β's incentive constraint is violated in α's problem, therefore, α's problem has no solution. The set of parameters making the problem infeasible is

$$T^0 > v_\beta(\tau_{b\beta}, 0) - v_\beta(0, 0).$$

2. When the problem is feasible, that is when $T^0 \leq v_\beta(\tau_{b\beta}, 0) - v_\beta(0, 0)$:

   (a) When $\bar{\tau}_{b\alpha}(T^0) \leq \tau_{b\alpha}$. In this case $\tau_{b\alpha} < \tau_{b\beta}$ and the monotonicity of v_β on $[0, \tau_{b\beta}]$ imply $v_\beta(\bar{\tau}_{b\alpha}(T^0), 0) \leq v_\beta(\tau_{b\alpha}, 0) < v_\beta(\tau_{b\beta}, 0)$. Therefore it holds

$$T^0 \leq v_\beta(\tau_{b\alpha}, 0) - v_\beta(0, 0) < v_\beta(\tau_{b\beta}, 0) - v_\beta(0, 0).$$

   If $\bar{\tau}_{b\alpha}(T^0) < \tau_{b\alpha}$, the above condition translates into the KKT condition being interior solution with constraints unbinding: $\tau_b^{\alpha^*} \in (0, \tau^{aut})$ (that is $\lambda_2 = 0$ and $\frac{\partial \mathcal{L}}{\partial \hat{\tau}} = 0$), and β's constraint is slack $\lambda_1 = 0$. In this case, the optimal solution is pinned down by the first order condition of unconstrained problem $\frac{\partial \mathcal{L}}{\partial \hat{\tau}}(\hat{\tau}, 0, 0) = 0$ and thus $\hat{\tau} = \tau_{b\alpha}$ if $\tau_{b\alpha} \in (0, \tau^{aut})$. If instead $\bar{\tau}_{b\alpha}(T^0) = \tau_{b\alpha}$, the problem is solved by $\hat{\tau} = \tau_{b\alpha}$.

   (b) When $\tau_{b\alpha} < \bar{\tau}_{b\alpha}(T^0) \leq \tau_{b\beta}$, that is when

$$v_\beta(\tau_{b\alpha}, 0) - v_\beta(0, 0) < T^0 \leq v_\beta(\tau_{b\beta}, 0) - v_\beta(0, 0).$$

   This condition translates into the KKT condition being interior solution with β's incentive constraint binding: $\tau_b^{\alpha^*} \in (0, \tau^{aut})$ (that is $\lambda_2 = 0$ and $\frac{\partial \mathcal{L}}{\partial \hat{\tau}} = 0$), and β's constraint binds $\lambda_1 \geq 0$ and $v_\beta(\hat{\tau}, 0) - v_\beta(0, 0) = T^0$. To check $\frac{\partial \mathcal{L}}{\partial \hat{\tau}} = 0$, we need $\frac{\partial v_\alpha}{\partial \hat{\tau}} + \lambda_1 \frac{\partial v_\beta}{\partial \hat{\tau}} = 0$ for $\lambda_1 \geq 0$, which requires $\frac{\partial v_\alpha}{\partial \hat{\tau}}$ and $\frac{\partial v_\beta}{\partial \hat{\tau}}$ to have opposite signs at $\bar{\tau}_{b\alpha}(T^0)$. Because $\tau_{b\alpha} < \tau_{b\beta}$, this requires $\bar{\tau}_{b\alpha}(T^0) \in [\tau_{b\alpha}, \tau_{b\beta}]$, which is satisfied in this case. Hence $\hat{\tau} = \bar{\tau}_{b\alpha}(T^0)$.

Next, β's problem is solved similarly. With $\bar{\tau}_{b\beta}(T_0)$ being defined as the maximum $\hat{\tau}$

satisfying α's constraint in β's problem, then $\frac{\partial v_\alpha}{\partial \hat{\tau}}$ is decreasing at $\bar{\tau}_{b\beta}(T^0)$, that is, $\bar{\tau}_{b\beta}(T^0) \geq \tau_{b\alpha}$. Further recall that by definition, $\tau_{b\alpha} < \tau_{b\beta}$. There are two cases based on the location of $\bar{\tau}_{b\beta}(T^0)$:

1. When $\bar{\tau}_{b\beta}(T^0) \geq \tau_{b\beta}$, that is, when

$$T_0 \geq v_\alpha(0,0) - v_\alpha(\tau_{b\beta}, 0),$$

   α's constraint is satisfied at $\tau_{b\beta}$ so $\hat{\tau} = \tau_{b\beta}$.

2. When $\tau_{b\beta} > \bar{\tau}_{b\beta}(T^0) \geq \tau_{b\alpha}$, that is,

$$v_\alpha(0,0) - v_\alpha(\tau_{b\beta}, 0) > T_0$$

   α's constraint in β's problem binds, so $\hat{\tau} = \bar{\tau}_{b\beta}(T^0)$, with $v_\alpha(\bar{\tau}_{b\beta}, 0) = v_\alpha(0,0) - T^0$.

In conclusion,

$$\tau_b^{\alpha^*} = \begin{cases} \tau_{b\alpha}, & \text{if } T^0 \leq v_\beta(\tau_{b\alpha}, 0) - v_\beta(0,0) \\ \bar{\tau}_{b\alpha}(T^0), & \text{if } v_\beta(\tau_{b\alpha}, 0) - v_\beta(0,0) < T^0 \leq v_\beta(\tau_{b\beta}, 0) - v_\beta(0,0) \\ \text{no solution}, & \text{if } T^0 > v_\beta(\tau_{b\beta}, 0) - v_\beta(0,0) \end{cases}$$

and

$$\tau_b^{\beta^*} = \begin{cases} \bar{\tau}_{b\beta}(T^0), & \text{if } T^0 < v_\alpha(0,0) - v_\alpha(\tau_{b\beta}, 0), \\ \tau_{b\beta}, & \text{if } T^0 \geq v_\alpha(0,0) - v_\alpha(\tau_{b\beta}, 0) \end{cases}$$

Proof of Proposition 6

Fix $(\gamma_a, \gamma_b) \in (\frac{1}{2}, 1)^2$. If $T^0 = 0$ and $\tau_{b\alpha} = 0$ then $\mathcal{I}_{FT}^{i*} = 1$ for each $i \in \{\alpha, \beta\}$. In what follows we rule out this specific case, assuming $\max\{T^0, \tau_{b\alpha}\} > 0$.

First, note that the following holds. Let $i \in \{\alpha, \beta\}$ be the agenda setter and $T^0 \in \mathbb{R}^+$ be the free trade status quo transfer. A switch away from free trade, $\mathcal{I}_{FT}^{i*} = 0$, occurs if and only if each of the following two conditions holds:

1. $\bar{\tau}_{bi}(T^0)$ exists in $[0, \tau^{aut}]$
2. $v_i(\tau_b^{i*}, 0) \geq v_i(0,0) - \mathbb{1}_{\{i=\alpha\}} T^0 + \mathbb{1}_{\{i=\beta\}} T^0$

Condition 1 is equivalent to $T^0 \leq v_\beta(\tau_{b\beta}, 0) - v_\beta(0,0)$ when the agenda setter is $i = \alpha$, and ensures that there exist a unilateral tariff such that β is willing to accept the switch away from free trade. Condition 2 means that the agenda setter $i \in \{\alpha, \beta\}$ is willing to switch from free trade to the unilateral tariff τ_b^{i*} defined in Proposition 5.

Now, consider the problem when the agenda setter is $i = \alpha$.

- If $T^0 \leq v_\beta(\tau_{b\alpha}, 0) - v_\beta(0,0)$ then condition 1 is always satisfied by definition of $\tau_{b\beta}$. By Proposition 5, in this case $\tau_b^{\alpha*} = \tau_{b\alpha}$. Since $v_\alpha(\tau_{b\alpha}, 0) \geq v_\alpha(0,0)$ by definition of $\tau_{b\alpha}$, condition 2 also holds, and hence $\mathcal{I}_{FT}^{\alpha*} = 0$.

- If $v_\beta(\tau_{b\alpha},0) - v_\beta(0,0) < T^0 \le v_\beta(\tau_{b\beta},0) - v_\beta(0,0)$, then condition 1 is satisfied, hence it suffices to check when condition 2 holds for $i = \alpha$. By Proposition 5 it has $\tau_b^{\alpha*} = \bar{\tau}_{b\alpha}(T^0) > \tau_{b\alpha}$, and therefore $v_\beta(\tau_b^{\alpha*},0) = v_\beta(0,0) + T^0$. Since $T^0 = v_\beta(\tau_b^{\alpha*},0) - v_\beta(0,0)$, we can rewrite condition 2 as

$$\Delta^{UT}(\bar{\tau}_{b\alpha}(T^0),0) \ge 0,$$

where $\Delta^{UT}(\bar{\tau}_{b\alpha}(T^0),0) = -\Delta^{FT}(\bar{\tau}_{b\alpha}(T^0),0)$ is the welfare gain in country A when the policy switches from free trade to the unilateral tariff $\bar{\tau}_{b\alpha}(T^0)$. Using the expression of Δ^{FT} reported at the beginning of the proof of Proposition 4, it is easily verified that

$$\Delta^{UT}(\bar{\tau}_{b\alpha}(T^0),0) \ge 0 \iff \bar{\tau}_{b\alpha}(T^0) \le \tfrac{2}{3}\tau^{aut}.$$

We have two cases.

  1. If $\tau_{b\beta} \le \frac{2}{3}\tau^{aut}$ then $\bar{\tau}_{b\alpha}(T^0) \le \frac{2}{3}\tau^{aut}$ follows from the assumption that $T^0 \le v_\beta(\tau_{b\beta},0) - v_\beta(0,0)$, which guarantees that $\bar{\tau}_{b\alpha}(T^0) \le \tau_{b\beta}$ exists. Hence condition 2 also holds and $\mathcal{I}_{FT}^{\alpha*} = 0$.
  2. If instead $\tau_{b\beta} > \frac{2}{3}\tau^{aut}$, condition 2 is satisfied if and only if $\bar{\tau}_{b\alpha}(T^0) \le \frac{2}{3}\tau^{aut}$, which is equivalent to $T^0 \le v_\beta(\frac{2}{3}\tau^{aut},0) - v_\beta(0,0)$ since $\bar{\tau}_{b\alpha}(T^0)$ is strictly increasing in T^0. Hence, in this case $\mathcal{I}_{FT}^{\alpha*} = 0$ if and only if $T^0 \le v_\beta(\frac{2}{3}\tau^{aut},0) - v_\beta(0,0)$.

- Finally, if $T^0 > v_\beta(\tau_{b\beta},0) - v_\beta(0,0)$ then condition 1 does not hold and therefore $\mathcal{I}_{FT}^{\alpha*} = 1$.

Given that $\tau_{b\alpha} < \min\left\{\frac{2}{3}\tau^{aut}, \tau_{b\beta}\right\}$ the previous results imply that

$$\mathcal{I}_{FT}^{\alpha*} = 0 \iff T^0 \le v_\beta(\min\left\{\tfrac{2}{3}\tau^{aut}, \tau_{b\beta}\right\},0) - v_\beta(0,0).$$

Consider now the problem when the agenda setter is $i = \beta$. Note that $\bar{\tau}_{b\beta}$ always exists, so $\mathcal{I}_{FT}^{\beta*} = 0$ if and only if condition 2 holds.

- If $T^0 < v_\alpha(0,0) - v_\alpha(\tau_{b\beta},0)$ by Proposition 5 we have that $\tau_b^{\beta*} = \bar{\tau}_{b\beta}(T^0) < \tau_{b\beta}$ and therefore $v_\alpha(0,0) - T^0 = v_\alpha(\tau_b^{\beta*},0)$. Since $T^0 = v_\alpha(0,0) - v_\alpha(\tau_b^{\beta*},0)$, we can rewrite condition 2 as

$$\Delta^{UT}(\bar{\tau}_{b\beta}(T^0),0) \ge 0,$$

and, by the same argument followed above, it has

$$\Delta^{UT}(\bar{\tau}_{b\beta}(T^0),0) \ge 0 \iff \bar{\tau}_{b\beta}(T^0) \le \frac{2}{3}\tau^{aut}.$$

We have two cases.

  1. If $\tau_{b\beta} \le \frac{2}{3}\tau^{aut}$ then $\bar{\tau}_{b\beta}(T^0) < \frac{2}{3}\tau^{aut}$, $\Delta^{UT}(\bar{\tau}_{b\beta}(T^0),0) > 0$ and $\mathcal{I}_{FT}^{\beta*} = 0$.
  2. If instead $\tau_{b\beta} > \frac{2}{3}\tau^{aut}$ then condition 2 is satisfied if and only if $\bar{\tau}_{b\beta}(T^0) \le \frac{2}{3}\tau^{aut}$. Since $\bar{\tau}_{b\beta}(T^0)$ is strictly increasing in T^0 when T^0 is in the considered range, in this case $\bar{\tau}_{b\beta}(T^0) \le \frac{2}{3}\tau^{aut}$ if and only if $T^0 \le v_\alpha(0,0) - v_\alpha(\frac{2}{3}\tau^{aut},0)$. Hence, when $\tau_{b\beta} > \frac{2}{3}\tau^{aut}$, $\mathcal{I}_{FT}^{\beta*} = 0$ if and only if $T^0 \le v_\alpha(0,0) - v_\alpha(\frac{2}{3}\tau^{aut},0)$.

- If $T^0 \geq v_\alpha(0,0) - v_\alpha(\tau_{b\beta},0)$ then $\tau_b^{\beta *} = \tau_{b\beta}$, and condition 2 becomes $T^0 \leq v_\beta(\tau_{b\beta},0) - v_\beta(0,0)$. Hence in this case $\mathcal{I}_{FT}^{\beta *} = 0$ if and only if $T^0 \leq v_\beta(\tau_{b\beta},0) - v_\beta(0,0)$. Note that this condition is never satisfied for $\tau_{b\beta} > \frac{2}{3}\tau^{aut}$. In fact, $\tau_{b\beta} > \frac{2}{3}\tau^{aut}$ implies $\Delta^{UT}(\tau_{b\beta},0) < 0$, which means that $v_\beta(\tau_{b\beta},0) - v_\beta(0,0) < v_\alpha(0,0) - v_\alpha(\tau_{b\beta},0)$, and consequently $v_\beta(\tau_{b\beta},0) - v_\beta(0,0) < v_\alpha(0,0) - v_\alpha(\tau_{b\beta},0) \leq T^0$.

We conclude that if $\tau_{b\beta} \leq \frac{2}{3}\tau^{aut}$

$$\mathcal{I}_{FT}^{\beta *} = 0 \iff T^0 \leq v_\beta(\tau_{b\beta},0) - v_\beta(0,0),$$

while if $\tau_{b\beta} > \frac{2}{3}\tau^{aut}$

$$\mathcal{I}_{FT}^{\beta *} = 0 \iff T^0 \leq v_\alpha(0,0) - v_\alpha(\tfrac{2}{3}\tau^{aut},0).$$

Finally note that $\Delta^{UT}(\frac{2}{3}\tau^{aut},0) = 0$, which implies $v_\alpha(0,0) - v_\alpha(\frac{2}{3}\tau^{aut},0) = v_\beta(\frac{2}{3}\tau^{aut},0) - v_\beta(0,0)$.

It follows that Proposition 6 holds setting

$$\bar{T}^0 = v_\beta(\min\{\frac{2}{3}\tau^{aut}, \tau_{b\beta}\},0) - v_\beta(0,0)$$

for $i \in \{\alpha, \beta\}$.

Proof of Proposition 7

Let $\gamma_a \in (\frac{1}{2},1)$ and $i \in \{\alpha,\beta\}$. By solving the algebra, one obtains that (i) $v_\beta(\tau_{b\beta},0) - v_\beta(0,0) = \frac{[5(1-\gamma_a)+\gamma_b]^2}{96}$; (ii) $v_\beta(\frac{2}{3}\tau^{aut},0) - v_\beta(0,0) = \frac{[\gamma_b-(1-\gamma_a)](1-\gamma_a)}{4}$; and (iii) $\frac{2}{3}\tau^{aut} \geq \tau_{b\beta}$ if and only if $\gamma_b \geq 7(1-\gamma_a)$. Hence, we can rewrite the expression of $\bar{T}^0$ derived in the proof of proposition 6 as follows:

$$\bar{T}^0 = \begin{cases} \frac{[5(1-\gamma_a)+\gamma_b]^2}{96} & \text{if} \quad \gamma_b \geq 7(1-\gamma_a) \\ \frac{[\gamma_b-(1-\gamma_a)](1-\gamma_a)}{4} & \text{if} \quad \gamma_b < 7(1-\gamma_a). \end{cases}$$

Note that $\frac{[5(1-\gamma_a)+\gamma_b]^2}{96}$ and $\frac{[\gamma_b-(1-\gamma_a)](1-\gamma_a)}{4}$ are both strictly increasing in γ_b since $\gamma_a \in (0,1)$. Moreover, $\bar{T}^0$ is continuous in γ_b at $\gamma_b = 7(1-\gamma_a)$, since in that case $\frac{[5(1-\gamma_a)+\gamma_b]^2}{96} = \frac{[\gamma_b-(1-\gamma_a)](1-\gamma_a)}{4} = \frac{3}{2}(1-\gamma_a)^2$. Hence $\bar{T}^0$ is strictly increasing in γ_b.