CANADIAN SOFTWOOD LUMBER EXPORT TO THE UNITED STATES: A COINTEGRATED AND ERROR-CORRECTED SYSTEM

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ABSTRACT

Canada and United States recently signed a five year deal called "Softwood Lumber Agreement". For almost a decade softwood lumber trade was at the centre of a heated trade dispute between these two countries. The alleged stumpage subsidy in Canada was the main focus of this dispute. Various investigations by the ITA and USITC ignored factors other than the alleged stumpage subsidy in Canada that drive softwood lumber export. This paper investigates the effects of five major excess demand side factors on Canadian softwood lumber exports to the U.S. using Johansen's maximum likelihood cointegration analysis. The results suggest that there is only one long-run equilibrium relationship among Canadian lumber exports, U.S. lumber price, U.S. disposable income, U.S. housing starts, U.S. construction wage rate and the exchange rate. The error-correction models are used to generate both short-run and long-run elasticities. The results suggest that five excess demand side factors explain about 70% of the variations in softwood lumber exports and that 70 to 74 percent of deviations from the long-run equilibrium is corrected within one quarter. Although the results do not negate the possibility of alleged stumpage subsidy influencing exports, they strongly suggest that excess demand side factors are the major determinants of softwood lumber trade between Canada and United States.

Keywords: Elasticities, error-correction models, excess demand side factors, Johansen's cointegration analysis, softwood lumber dispute.

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INTRODUCTION

Canada exported about 70% of its softwood lumber production in 1991 and 77% of its exports were to the United States. Canada is essentially the sole foreign supplier of softwood lumber in the U.S. market. This bilateral softwood lumber trade was worth $2.82 billion in 1991 (USITC, 1992). Such a healthy trade sector between these two neighbouring countries has also been at the centre of a lengthy and heated trade dispute.

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The dispute began with an unsuccessful countervailing duty petition from the U.S. Coalition for Fair Canadian Lumber Imports (a group of 8 trade associations and 350 lumber producing firms) against softwood lumber imports from Canada in 1982. It ended after more than a decade when on January 28, 1994 the Binational Panel formed under the Canada-U.S. Free Trade Agreement (FTA) dismissed the CVD case against Canadian softwood lumber. A chronology and discussion of the various judicial decisions made since the original investigation in 1982 is beyond the scope of this paper. However, a brief background of the dispute is given in section two of this paper.

A number of studies have investigated the price, production, consumption and welfare effects of a tariff on imported softwood lumber from Canada (e.g., Boyd & Krutilla, 1987, 1988; Chen *et al.*, 1988; Wear & Lee, 1993 and Myneni *et al.*, 1994). The results from these studies suggest that the tariff on softwood lumber imports from Canada benefited domestic lumber producers in the U.S. at the expense of U.S. consumers and Canadian lumber producers. The import duty has also resulted in higher prices and considerable efficiency losses in the U.S. lumber market. Note that these studies take the import duty as given and do not investigate subsidy and material injury determination procedures.

Even if lumber production is subsidized in Canada, subsidy alone does not drive lumber exports from Canada to the United States. There are a number of other factors such as exchange rate, housing activity in the U.S., transportation cost, technical improvements resulting in higher fibre recovery in Canadian mills and timber harvest levels in Canada can have significant bearing on softwood lumber export from Canada. Indeed, in contemporary forest economics literature, the dramatic rise in the share of Canadian softwood lumber in the U.S. market during the 1980s has been attributed to such factors as U.S. housing starts and favourable exchange rate movements (Adams & Haynes, 1985; Adams *et al.*, 1986; Constantino & Uhler, 1988; Roberts, 1988; Buongiorno *et al.*, 1988 and Jennings *et al.*, 1991).

The purpose of this paper is to investigate these other factors which affect Canadian lumber exports to the

United States. These factors are generated in the market place and have little to do with the alleged stumpage subsidy in Canada. Johansen's maximum likelihood cointegration analysis and error-correction models are used to determine short-run and long-run effects of various demand side factors on the volume of Canadian softwood lumber export to the United States. The cointegration approach takes into account data nonstationarity and allows us to explore the dynamic relationships among a group of variables without having to impose any *a priori* structural restrictions on the model. This paper, therefore, attempts to contribute to a better understanding of the softwood lumber dispute between Canada and the United States.

Section three presents an analytical framework and describes data. Section four introduces unit roots and cointegration analysis and provides a brief exposition of Johansen's cointegration approach. Section five deals with the error-correction model and discusses empirical results. Section six summarizes the major findings and concludes the paper.

Background

The market share of Canadian softwood lumber in the United States rose sharply from 17% in 1975 to 33% in 1985 (Doran & Nostali, 1987). The increased market share of imported Canadian softwood lumber, during the early 1980s created concern among U.S. lumber producers. In October 1982, the Coalition filed a formal countervailing duty complaint against softwood lumber imported from Canada. The Coalition alleged that Canadian federal and provincial governments subsidize forest products through a number of programs and practices. In a preliminary ruling in November 1982, the USITC found that the U.S. lumber industry has been materially injured by allegedly subsidized softwood lumber imported from Canada. However, in May 1983, the International Trade Administration (ITA) of the Department of Commerce came up with a negative subsidy determination which terminated the case. In May 1986, the Coalition filed a second CVD petition against softwood lumber imports from Canada. This time the ITA came up with a positive determination of subsidy and the subsidy was calculated to equal 15% ad valorem.[1] Following the preliminary determination of subsidy by the ITA, the USITC ruled that the subsidized softwood lumber imported from Canada caused "material injury" to the U.S. lumber industry and immediately imposed a 15% tariff on all softwood lumber imported from Canada.[2] The final determination of the CVD was to be made by December 30, 1986. To avoid the import duty, Canada negotiated a deal called the Memorandum of Understanding (MOU) on softwood lumber with the U.S. and agreed to impose a 15% export tax on certain softwood lumber exports bound for the U.S. market. The MOU took effect on January 8, 1987 and governed the Canada-U.S. softwood lumber trade for almost five years.

On September 3, 1991, the government of Canada unilaterally announced that provincial stumpage charges had increased to the extent that it was no longer necessary to collect the export tax.[3] Following this action, the U.S. government reopened the CVD case against softwood lumber from Canada and imposed temporary import duties ranging up to 15% on softwood lumber imported from certain provinces of Canada. In May 1992, the ITA reported the results of it's final subsidy determination and calculated a subsidy margin of 6.51%. In July 1992, the USITC ruled that subsidized softwood lumber imported from Canada caused material injury to lumber producers in the United States. Consequently, a 6.51% ad valorem duty went into effect on May 28, 1992 (USITC, 1992).

Although the magnitude of final tariff was less than 50% of its initial value, the government of Canada appealed the ITA and USITC decisions to a Binational Panel under Article 1904 of the Canada-United States Free Trade Agree-

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[1] The Trade and Tariff Act of 1984 may have contributed to the reversal of the ITA's decision about Canadian softwood lumber. Two provisions of this legislation are particularly notable. First, the Act provided a reinterpretation of the statute which allowed the ITA to find a product to be subsidized if it was produced from subsidized inputs. Second, the Act required all agencies administering U.S. trade laws to give technical assistance to U.S. firms on how to make successful antidumping and countervailing duty petitions (CBO, 1994, p. 28).

[2] Note, under the U.S. Law against unfair trade practices both subsidy and material injury must be found before imposing a countervailing duty on an import.

[3] The MOU was terminated on October 4, 1991.

ment (FTA). On July 26, 1993, the Binational Panel remanded the ITA and USITC decisions. In particular, the Panel asked the Department of Commerce to recalculate the subsidy margin and the USITC to provide additional statistical evidence to support it's determination of material injury. Both the ITA and USITC responded to the request. The ITA revised it's subsidy estimate to 11.54% and the USITC reaffirmed it's original determination of material injury. After reviewing the responses from the ITA and USITC, the Binational Panel ruled that the analysis of the determination of subsidy was flawed and that the USITC's determination of material injury to the U.S. lumber industry was not based on sound statistical evidence. As a result, the Panel dismissed the CVD case against Canadian softwood lumber on January 28, 1994 (USITC, 1994).[4]

AN ANALYTICAL FRAMEWORK

A simple trade model involving a single commodity and two countries is used in this section to establish theoretical relationships among the variables. In this model, lumber produced in Canada and in the United States are assumed to be homogeneous products. In Figure 1, Canada is the exporting region while the United States is the importing region. The intersection between the excess supply from Canada and excess demand from the U.S. determines the equilibrium price of lumber and the volume of trade. For simplicity, it is assumed that exchange rates are set exogenously to the lumber sector.[5]

Initially the exchange rate between the two countries is at par (as shown by the 45^0 line $0A$ in panel (c) of Figure 1). The volume of trade is equal to $0Q_0$. If a change in exchange rate causes the value of the U.S. dollar to ap-

[4] Despite the dismissal of the CVD case against Canadian softwood lumber by the Binational Panel, the softwood lumber dispute did not go away. Trade tension started again at the end of 1995. To cap these tensions, Canada and U.S. signed a new five year deal called "Softwood Lumber Agreement". Under this agreement, Canada will export 14.7 billion board feet of softwood lumber to the U.S. market each year duty free. Any additional volume of export will be taxed at the rate of $50 per 1000 board feet. The agreement took effect on April 1, 1996.

[5] Because the lumber sector in the United States and Canada is a very small part of the overall economy and accounts for a small part of total trade, the exogeneity assumption seems reasonable.

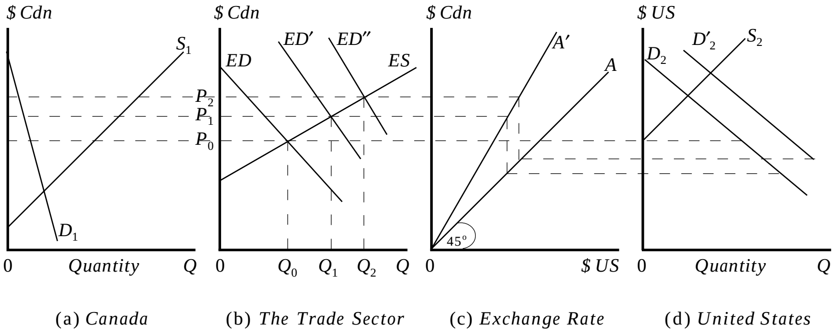

FIGURE 1. CANADA-U.S. SOFTWOOD LUMBER TRADE

preciate such that $0A$ rotates to $0A'$, the excess demand curve rotates upward from ED to ED' in Figure 1. This results in an expansion of trade to $0Q_1$ and an increase in the exporters price to $0P_1$. If the number of housing starts increase in the U.S., other things remaining unchanged, the demand function for lumber in the U.S. shifts to D_2'. This causes the excess demand function to shift to ED''. Consequently, lumber trade expands to $0Q_2$ and the export price increases to $0P_2$. Similar trade expansion and export price increase effects can also be obtained for an increase in per capita disposable income in the United States. If, however, the real wage of common labour in the U.S. construction industry goes up, the excess demand function will shift downward. Other things remaining constant, this will result in lower export prices and reduced lumber exports from Canada. Finally, the price of softwood lumber in the U.S. market will rise either due to a shift of the demand function to the right or due to a leftward shift of the supply function. In either case, the excess demand function will shift to the right causing trade expansion and higher export prices for Canadian softwood lumber. Thus, the demand function for Canadian softwood lumber in the U.S. market (i.e., the excess demand function) can be expressed as:

$$CEXL = f(USPL,\ USHS,\ USPY,\ USCW,\ EXRT)$$

$$\frac{\partial CEXL}{\partial USPL} > 0; \qquad \frac{\partial CEXL}{\partial USHS} > 0; \qquad \frac{\partial CEXL}{\partial USPY} > 0;$$

$$\frac{\partial CEXL}{\partial USCW} < 0; \qquad \text{and} \qquad \frac{\partial CEXL}{\partial EXRT} > 0, \tag{1}$$

where

CEXL is total softwood lumber exports from Canada to the U.S.,

USPL is the price of lumber in the U.S.,

USHS is the total number of housing starts in the U.S.,

USPY is the per capita disposable income in the U.S.,

USCW is the wage rate of common labour in the U.S. construction industry and

EXRT is the Canada-U.S. exchange rate.

Note that in multivariate cointegration analysis each variable is treated as endogenous in the system at least initially. Moreover, if there is a long-run relationship among the variables, the cointegration analysis also provides a mechanism for testing which variables are exogenous to the estimated long-run relationship. These results can be used to estimate a single-equation error-correction model.

DATA DESCRIPTION

Canadian lumber exports to the U.S. is defined as thousands of cubic meters of lumber and is compiled from Statistics Canada Catalogue Nos. 35-002 and 35-003. The exchange rate is expressed as Canadian dollars per U.S. dollar. The exchange rate data are from various issues of the Bank of Canada Review. The *USHS* is defined as the total number (thousands of units) of housing starts in the U.S. and are taken from the U.S. Federal Reserve Bulletin. Both per capita disposable income and Consumer Price Index (CPI) [1986=100] are from the Survey of Current Business published by the U.S. Department of Commerce. The *USCW* represents the hourly wage rate of common labour in the construction industry. This is a simple average of wages actually paid in 20 cities in the United States. The data are collected from various editions of Business Statistics published by the U.S. Department of Commerce.[6]

[6] Since lumber is one of the four major components of the Construction Cost Index and the Building Cost Index in the U.S., neither is used as a proxy for related goods of lumber in this study. Instead, common labour is used as a related good of lumber. Between 1971 and 1991 common labour represented 76–77 out of every 100 in the Construction Cost Index in the U.S. (U.S. Department of Commerce 1992, p. 151).

Finally, *USPL* is the producer price of lumber in the U.S. and is taken from producer price indices, U.S. Department of Labour. The data are quarterly, seasonally adjusted, and span 1973:4 to 1992:3.[7] The per capita disposable income and the hourly wage rate of common labour were converted to their real values using CPI as the deflator. The U.S. lumber price index was re-indexed to 1986 = 100 and was converted to it's real value by using Producer Price Index (all commodities) [1986 = 100] as the deflator. The PPI data are taken from the Survey of Current Business published by the U.S. Department of Commerce.

INTEGRATION AND COINTEGRATION ANALYSIS

The first step in cointegration analysis is to verify the integrating properties of the variables in the model. The integration properties of all variables in question are examined by using unit root tests. A unit root test examines if the variables are integrated of first-order (i.e., there is unit root in the data set). A variable X_t is said to have a unit root in its autoregressive process if it has the following univariate representation:

$$(1-L)X_t = \phi_1(1-L)X_{t-1} + \ldots\ldots + \phi_p(1-L)X_{t-p} + \varepsilon_t \qquad (2)$$

where

ε is a stationary stochastic process, $\Sigma\phi_i < 1$, and $L^k X_t \equiv X_{t-k}$. A number of statistics have been proposed in the literature as tests for the existence of unit roots. Many of these are variants of the t-like tests proposed by Dickey & Fuller (1979). To test for the presence of a unit root in X_t, an augmented Dickey-Fuller (ADF) test can be computed by running the following regression:

$$(1-L)X_t = \alpha + \beta X_{t-1} + \sum_{i=1}^{p} \phi_i (1-L)X_{t-1} \qquad (3)$$

[7] A journal reviewer correctly points out that if data were available in original form (i.e., seasonally not adjusted) we could test for seasonal as well as non-seasonal unit roots in each series using procedures outlined in Hylleberg *et al.* (1990). Moreover, the estimated cointegrated relationship may be somewhat weaker than it could have been due to the removal of seasonal components from the data. Unfortunately, data were available in seasonally adjusted forms and it was difficult and time consuming to retrieve the original data.

TABLE 1: RESULTS OF UNIT ROOT TEST ON INDIVIDUAL SERIES.

SERIES	ESTIMATED COEFFICIENT	LAG-LENGTH	ADF-STATISTIC*
Canadian Lumber Exports	−0.200	8	−2.587
Canada-U.S. Exchange Rate	−0.041	3	−2.210
U.S. Lumber Price	−0.096	5	−2.112
U.S. Housing Starts	−0.150	2	−2.712
U.S. Construction Wage Rate	−0.066	4	−2.047
U.S. Disposable Income	−0.014	8	−0.722
First-Difference Series			
Canadian Lumber Exports	−1.629	8	−4.136
Canada-U.S. Exchange Rate	−0.572	3	−3.889
U.S. Lumber Price	−0.983	5	−5.073
U.S. Housing Starts	−0.810	2	−7.870
U.S. Construction Wage Rate	−0.595	4	−4.002
U.S. Disposable Income	−0.938	8	−3.679

* The critical value of ADF at 5 percent level of error probability is −3.14.

The null hypothesis that there is a unit root in X_t. The series X_t is said to be stationary if the estimate of β is negative and statistically significant. However, the t-ratio on estimated β does not have a standard t-distribution. The critical values provided by Dickey & Fuller (1979) need to be used.[8]

Table 1 reports the results of ADF tests for unit root in the six variables. Akaike's FPE criterion was used to determine the appropriate lag-length truncation in each case.[9]

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[8] In a recent paper, Handa & Ma (1989) examine the power properties of four alternative tests for the random walk hypothesis. The results of their simulation experiments show that the conventional Dickey-Fuller test has higher power than the Phillips-Perron test. The decision to use the ADF test in this paper is based on the above finding.

[9] The FPE value for an order of h is given by:

$$\text{FPE}(h) = [(T+h+1)/(T-h-1)]*\text{RSS}(h)/T,$$

where T is the total number of observations and RSS is the sum of squared residuals.

The null hypothesis of a unit root in the univariate representation cannot be rejected for any one of these six variables at 95% level of significance. Note also that each series becomes stationary after first-differencing. So, the results are consistent with the hypothesis that unit root nonstationarity characterizes each of the six variables in this study.

Given the presence of unit root in each series, a precondition for the existence of a stable steady-state relationship is cointegration among these variables. A vector of variables is said to be cointegrated if each variable in the vector has a unit root in its univariate representation, but some linear combination of these variables is stationary (Engle & Granger, 1987).

Six alternative approaches have been proposed in the literature for testing cointegration. These are:

(i) the Dickey-Fuller test on cointegration regression residuals (Engle & Granger, 1987);

(ii) the cointegration regression Durbin-Watson test (Engle & Granger, 1987);

(iii) the Park J_1 superfluous variable addition test based on the canonical cointegration regression (Park, 1990, 1992);

(iv) the Hansen fully modified regression estimator L_c test (Hansen, 1992);

(v) the dynamic ordinary least square procedure developed for testing common trends (Stock & Watson, 1988) and,

(vi) the maximum likelihood cointegration approach (Johansen, 1988, 1991).

The first five of these tests are based on some variations of regression analysis (conventional and modified), while the last one is based on the VAR model. While the first four tests involve single-equation estimation, the last two involve multiple-equation method of identifying cointegration relationships.

The full-system approach developed by Johansen is based on the estimation of a VAR system by maximum likelihood method. This approach essentially extends the Engle-Granger procedure to a multivariate context where there may exist more than a single cointegrating relationship among a set of *n* variables.[10] The maximum likelihood procedure gives estimates of a system's cointegrating vector and their weights. These estimates can be used to test relevant economic hypotheses. Moreover, the maximum likelihood estimates are symmetrically distributed, median unbiased and have mixed normal distributions (Johansen, 1992; Toda & Phillips, 1993). It is due to these attractive features that the Johansen's approach is used in this paper.

JOHANSEN'S APPROACH: A BRIEF EXPOSITION

Following Johansen (1988, 1991) and Johansen & Juselius (1990, 1992), this approach is based on a *k*th order unrestricted VAR representation of X_t such that:

$$X_t = \pi_1 X_{t-1} + \pi_2 X_{t-2} + \ldots\ldots + \pi_k X_{t-k} + \mu + \Phi D_t + \varepsilon_t, \qquad (4)$$

where X_t is a vector of p $I(1)$ variables, D_t are three seasonal dummies, π_i are $(p \times p)$ matrices of parameters, μ is a $(p \times 1)$ vector of constant terms, and $\varepsilon_t \sim$ NID $(0, \Omega)$. Using $\nabla = 1 - L$, where L is the lag operator, the model in equation (4) can be reparameterized as,

$$\nabla X_t = \mu + \Gamma_1 \nabla X_{t-1} + \Gamma_2 \nabla X_{t-2} + \ldots\ldots + \Gamma_{k-1} \nabla X_{t-k+1} - \Pi X_{t-k} + \Phi D_t + \varepsilon_t, \qquad (5)$$

where,

$$\Gamma_i = -I + \pi_1 + \ldots\ldots + \pi_i \quad \text{and} \quad -\Pi = I - \pi_1 - \pi_2 - \ldots\ldots - \pi_k; \quad \forall i = 1, 2, \ldots\ldots, k-1.$$

[10] Stock & Watson (1988) also extend the Engle-Granger approach to a multivariate context. According to Stock and Watson, if a vector process has $n-k$ common trends, then the $n-k$ principal components with largest variance should correspond to the unit root processes or "common trends". Since the normalized variance of a sum-of-squares and cross-product matrix is equal to the sum of eigenvalues of this matrix, Stock and Watson suggest a test based on computing the largest eigenvalues of the sum-of-squares and cross-product matrix.

The reparameterized model is a traditional first difference VAR model except for the term ΠX_{t-k}. The coefficient matrix of X_{t-k}, Π, contains information about the long-run relationships among variables in the data vector. If Π has a full rank, then X_t is a stationary process. In this case, an undifferenced VAR model is appropriate. If Π has a zero rank, then Π is a null matrix and X_t is an integrated process; only in this case, a traditional VAR model in first-difference is appropriate (Orden & Fisher, 1991). If, however, $0<(\text{rank}(\Pi) = r) < p$, cointegration holds, and Π can be represented as the product of two $(p\times r)$ matrices α and β such that, $\Pi = \alpha\beta'$. The β's are the cointegrating vectors and α's are the weights. In this case, $\beta' X_t$ is stationary and a dynamic error-correction model is appropriate. The long-run equilibrium is unique only when $r = 1$.

The maximum likelihood estimation of Π consists of two sets of regressions; one set generates the residuals R_{0t} from the regression of X_t on X_{t-1},... $X_{t-\mathrm{k}+1}$, and the other set generates R_{kt} from the regression of $X_{t-\mathrm{k}}$ on X_{t-1},... X_{t-k+1} (see Johansen, 1991 for details). The concentrated likelihood function in terms of the product moment matrices of the residuals can be expressed as:

$$L(\beta) = \left|\hat{\Omega}(\beta)\right|^{-\frac{T}{2}} = \left|S_{00} - S_{0k}\beta\left(\beta' S_{kk}\beta\right)^{-1}\beta' S_{k0}\right|^{-\frac{T}{2}} \qquad (6)$$

Where S_{00}, $S_{0\mathrm{k}}$, S_{k0}, and S_{kk} are the product moment matrices of the residuals defined as:

$$S_{ij} = T^{-1}\sum_{t=1}^{T} R_{it}R'_{jt} \qquad \forall i, j = 0, k.$$

It is clear from equation (6) that maximizing the concentrated likelihood function is equivalent to minimizing $|\Omega(\beta)|$. This minimization amounts to solving the following eigenvalue problem:

$$\left|\lambda I - C^{-1}S_{k0}S_{00}^{-1}S_{0k}C'^{-1}\right| = 0, \qquad (7)$$

where C is a ($p \times p$) matrix such that $S_{kk} = C'C$. The vector of eigenvalues is given by λ while the corresponding eigenvectors can be derived as $v_i = C'^{-1}e_i$, where e_i's are the eigenvectors from equation (7). The estimates of α and Ω can be obtained by using the estimated value of β.

The rank of the coefficient matrix Π is the number of linearly independent cointegrating relations among the variables in X_t. The objective of testing for cointegration is to determine the rank of Π by testing whether the estimated eigenvalues of Π are significantly different from zero. The null hypothesis that there are r cointegrating vectors is tested using two likelihood ratio tests called the trace test and the maximum eigenvalue test. If H_1: is a special case of H_2: for $r = p$, then the trace statistic is defined as:[11]

$$-2\ln\left(Q; H_2 \middle| H_1\right) = -T \sum_{i=r+1}^{p} \ln\left(1 - \hat{\lambda}_i\right) \tag{8}$$

Similarly, the maximum eigenvalue statistic for testing $H_2(r)$ in $H_2(r + 1)$ can be defined as:

$$-2\ln\left(Q; r \middle| r+1\right) = -T\ln\left(1 - \hat{\lambda}_{r+1}\right) \tag{9}$$

The asymptotic distributions of these likelihood ratio tests do not follow the standard Chi-squared distribution. They involve stochastic integral of Brownian motions and can be represented as multivariate versions of the Dickey-Fuller distribution. The critical values for these tests are generated from numerically simulated distributions of Brownian motions and are reported in Johansen & Juselius (1990) and in Osterwald-Lenum (1992).

Johansen's approach also provides a convenient framework for testing linear hypotheses expressed in terms of restrictions on coefficients μ, α and β. Theoretical and em-

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[11] Note H_1: considers the unrestricted VAR model in levels as given in equation (4) as the true model while H_2: considers the model in first-difference form as given in equation (5) as the true model. If there is no linear trend in the non-stationary component of the model and the coefficient matrix has the full column rank (i.e., $r = p$), H_1: and H_2: are equivalent to each other.

pirical economic knowledge can be used to formulate the restrictions. These restrictions essentially limit the space spanned by the r cointegrating vectors to lie in the s-dimensional space. If $s = r$, then the cointegration space is said to be fully specified (Johansen, 1991). Let H_3: $\beta = H\delta$ represents a formulation of a linear restriction on the cointegrating vectors, where H is a $(p \times s)$ matrix of restrictions designed to restrict the space spanned by β to lie in s-dimensional space and δ is a set of cointegrating vectors (see Johansen, 1991 for details). The likelihood ratio test can be computed as:

$$-2\ln\left(Q; H_3 \middle| H_2\right) = T\sum_{i=1}^{r} \ln\left[\left(1-\hat{\lambda}_{3.i}\right) / \left(1-\hat{\lambda}_i\right)\right] \qquad (10)$$

This statistic is also distributed as χ^2 with $r(p - s)$ degrees of freedom.

COINTEGRATION AND ERROR-CORRECTION MODELS

There is a close relationship between cointegration and error-correction models. In general, linear combinations of $I(1)$ variables will also be $I(1)$. If the linear combinations of such variables happen to be $I(0)$, then the variables are said to be cointegrated. In the ***"****Granger Representation Theorem****"***, Engle & Granger (1987) show that if all variables in a vector stochastic process X_t are $I(1)$ and they are cointegrated, then there exists an error-correction representation such as:

$$A(L)(1-L)X_t = -\gamma\, e_{t-1} + \varepsilon_t, \qquad (11)$$

where L is a lag operator, $A(L)$ is a polynomial in L of the form $[\beta_0 + \beta_1 L + \beta_2 L^2 + \ldots\ldots]$ and ε_t is a stationary multivariate disturbance. It is assumed that $A(0) = I$, all elements in $A(1)$ are finite and $\gamma \neq 0$. The cointegrating vector is β, where $e_t = \beta' X_t$ is $I(0)$. The long-run equilibrium is interpreted as $\beta' X_t = 0$; thus, e_t is a measure of the error or deviation from the equilibrium. Because the series are cointegrated, the error-correction term (ECT) is stationary. Hence, the least squares standard errors of the error-correction model will estimate the true standard errors consistently (Engle & Granger, 1987).

Advances in cointegration methodology by Engle & Granger (1987) and Johansen (1988, 1991), thus, provide the necessary tools to apply error-correction models that explicitly take into account the dynamics of short-run adjustments toward a long-run equilibrium. When all variables in an equation are $I(1)$, stochastic trends seem to cause them to wander randomly. The variables will eventually return to an equilibrium path and follow one another along this path if they are cointegrated. Cointegration analysis, therefore, links the concept of equilibrium relationships among economic variables postulated by economic theory to a statistical model among those variables. In doing so, it provides a theoretically consistent and econometrically efficient approach to measure economic relationships.

In this model, cointegration analysis is used to determine the long-run relationship among the observed values of Canadian softwood lumber exports to the U.S. and five other economic time series. The long-run parameters obtained from the VAR model are then used to compute the deviations of the observed values from the estimated long-run cointegrated relationship. Finally, these residuals are used to estimate the error-correction model.

RESULTS OF COINTEGRATION TEST

Before estimation, the six variables in the model were ordered on the basis of economic reasoning. The U.S. disposable income was placed at the top of the order followed by the U.S. housing starts, the exchange rate, the wage rate of common labour in the U.S. construction industry, the U.S. lumber price and Canadian softwood lumber exports to the United States.[12] Note that this ordering is for convenience.

[12] The per capita disposable income and the number of housing starts in the U.S. depend on the overall economic condition in the United States. The wage rate of common labour also depend on the overall economic condition and the demand for housing in the U.S., but as the price of a related input (i.e., labour) it can influence both lumber imports and lumber price in the U.S. Since lumber is just one of many inputs used in the housing industry, contemporaneous causality is expected from housing starts to lumber prices in the U.S. Also because the U.S. economy is at least ten times larger than the Canadian economy, the bilateral exchange rate can have little effect on disposable income and housing starts in the U.S. The ordering of the variables in the VAR model is based on this understanding of the economic system under study.

The estimated maximum likelihood cointegration relationship is invariant to the ordering of the variables (Hamilton, 1994, p. 589).

Since Johansen's approach essentially uses a VAR formulation and the results from VAR models are sensitive to lag-length choice (Hafer & Sheehan, 1991), due emphasis is given to lag-length selection in this model. Although there are at least five different approaches to determine lag specification, Sims (1980) modified likelihood ratio test is used to select the appropriate lag-length; only this approach allows testing cross-equation restrictions.[13] Sims modified likelihood ratio test examines the equivalence of models with different lag-lengths, and has an asymptotic Chi-squared distribution with degrees of freedom equal to the number of restrictions imposed on the model. The equivalence of a one-lag model and a two-lag model could not be rejected (χ^2 = 41.90, 36 df). The equivalence hypothesis also could not be rejected for a two-lag model and a three-lag model (χ^2 = 34.83, 36 df), and for a three-lag model and a four-lag model (χ^2 = 40.03, 36 df). Based on these results, a VAR model incorporating only one lag of each variable was selected.

Table 2 presents the results of Johansen's maximum likelihood cointegration test for the number of independent cointegration relationships in the six-dimensional system. Both the trace test and the maximum eigenvalue test suggest the presence of only one cointegration relationship in this system.[14] This cointegrating vector represents a stable equilibrium relationship to which the variables have a tendency to return in the long-run (Engle & Granger, 1987).

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[13] Sims modified likelihood ratio test can be defined as:

$$L = (T-k)\{\ln|D_r| - \ln|D_u|\} \sim \chi^2{}_d,$$

where T is the total number of observations, k is the number of variables in each unrestricted equation, d is the number of restrictions, and D_r and D_u are the restricted and unrestricted covariance matrices respectively. The other approaches are an F-test, Akaike's Final Prediction Error (FPE) criterion, Schawarz's Bayesian Information Criterion (BIC), and the Bayesian Estimation Criterion (BEC). See Hafer & Sheehan (1991) for details on these approaches.

[14] Notice that the critical values for the trace statistic are at 90 percent instead of 95 percent level of significance. This is because the power of the trace test is likely to be lower than that of the maximum eigenvalue test. See Johansen & Juselius (1990, pp. 183–192) for details.

TABLE 2. MAXIMUM LIKELIHOOD COINTEGRATION RESULTS: TESTING FOR THE NUMBER OF COINTEGRATING VECTORS.

EIGENVALUES (λ's):	0.447	0.289	0.215	0.139	0.027	0

NULL HYPOTHESIS	TRACE STATISTIC	TRACE (0.90)	λ_{MAX} STATISTIC	λ_{MAX} (0.95)
r = 0	102.83*	100.14	45.07**	42.48
r ≤ 1	57.76	73.40	25.88	36.41
r ≤ 2	31.88	50.74	18.41	30.33
r ≤ 3	13.47	31.42	11.39	23.78
r ≤ 4	2.08	16.06	2.08	16.87
r ≤ 5	0.00	2.57	0.00	3.74

The critical values, trace (0.90) and λ_{max} (0.95) are taken from Osterwald-Lenum (1992), page 468. * and ** indicate statistical significance at 90% and 95% respectively.

RESULTS OF THE ERROR-CORRECTION MODELS

Given that the variables in equation (1) are cointegrated, the short-run dynamic excess demand for Canadian softwood lumber in the U.S. can be characterized by an error-correction model. In such a model, the growth rate of Canadian softwood lumber exports to the U.S. depends on the error-correction term and the growth rates of other relevant variables. Since the cointegrating vector is unique, the residuals from the cointegrating relationship represent equilibrium errors in this model. The model can also be used to generate both short-run and long-run elasticities.[15] This is important because the response of the excess demand for Canadian softwood lumber to a change in lumber price in the U.S. and other factors may be dispersed over more than one period and thus, can generate different values of short-run and long-run elasticities.

[15] The derivation of the long-run elasticities is given in the Appendix. Following Krinsky & Robb (1990) the standard errors of the estimated long-run elasticities are obtained through linear approximations. Since the estimated long-run elasticities are non-linear functions of the estimated parameters from the error-correction model (i.e., $e = f(t)$), the variance-covariance matrix of the estimated elasticities is given by: $v(e) = (\partial f/\partial t)'*v(t)*(\partial f/\partial t)$. Note that $(\partial f/\partial t)$ is the gradient vector and $v(t)$ is the variance-covariance matrix of the estimated parameters from the error-correction model.

TABLE 3. SHORT-RUN AND LONG-RUN ELASTICITIES FROM THE ERROR-CORRECTION MODEL (T-VALUES WITHIN PARANTHESIS).

EXPLANATORY VARIABLES	MODEL ONE Short-run	MODEL ONE Long-run	MODEL TWO Short-run	MODEL TWO Long-run
Constant	4.33* (6.41)	—	4.09* (6.99)	—
U.S. Disposable Income	2.81* (2.62)	3.80* (2.54)	2.68* (2.60)	3.850* (2.50)
U.S. Housing Starts	0.285* (2.62)	0.385* (2.46)	0.281* (2.61)	0.403* (2.47)
Exchange Rate	0.162 (0.226)	0.22 (0.225)	0.066 (0.094)	0.095 (0.094)
U.S. Const. Wage Rate	−4.13* (−3.42)	−5.58* (−2.97)	−3.56* (−3.99)	−5.110* (−3.53)
U.S. Lumber Price	1.02* (3.21)	1.38* (2.74)	0.998* (3.53)	1.430* (3.02)
Error-Correction Term	−0.74** (−6.41)	—	−0.697** (−7.02)	—
Time Trend	0.007** (5.60)	—	0.007** (5.91)	—
Seasonal Dummies:				
D_1	−0.001 (−0.037)	—	—	—
D_2	0.101* (2.99)	—	0.096** (5.91)	—
D_3	0.029 (0.039)	—	—	—
R_{adj}^2	0.66		0.67	
F-Statistic	14.31**		17.79**	
Durbin's h-Statistic	1.60		1.87	
L-M Test for Normality of the Residuals (χ^2_{23})	17.77		18.10	

* and ** indicate significance at 95 percent and 99 percent levels respectively.

Table 3 reports the results of the error-correction models. Model One includes three seasonal dummy variables while Model Two includes only one dummy variable (i.e., D_2 which is significant). All coefficients have the expected signs and all but one are statistically significant. The Durbin's h-statistics are indicative of no autocorrelation while the Lagrange Multiplier tests support that the residuals are normally distributed.

The error-correction term has a negative and statistically significant coefficient. The negative coefficient of the error-correction term ensures that the long-run equilibrium is achieved. However, the adjustment toward equilibrium is not instantaneous. Between seventy to seventy four percent of any quarters' deviation from the equilibrium is corrected in the next quarter's growth rate of Canadian softwood lumber exports to the United States (Table 3). The negative and statistically significant coefficient of the error-correction term also confirms the cointegration relationship found earlier and the validity of the error-correction representation.

Estimated elasticities indicate that labour is a complement to imported softwood lumber and the demand for Canadian lumber in the U.S. is highly income elastic both in the short-run and in the long-run. Estimated elasticities also indicate that lumber has a unitary price elasticity in the short-run but an elastic demand in the long-run. Finally, the exchange rate has expected positive effect on Canadian softwood lumber exports to the United States but it is not statistically significant. This confirms the results of Buongiorno *et al.* (1988) and Jennings *et al.* (1991).

A number of previous studies have investigated softwood lumber trade between these two countries and produced diverse results. For example, Buongiorno *et al.* (1979) estimate import demand equations for Canadian softwood lumber in the U.S. market and report short-run and long-run elasticity of lumber imports with respect to exchange rates as –0.35 and –0.45. These authors also report long-run elasticities of lumber imports with respect to housing starts and lumber price as 0.51 and 1.16 respectively. Singh & Nautiyal (1986) estimate short-run elasticities of Canadian lumber exports to the U.S. with respect U.S. housing starts and U.S. lumber price as 1.24

and 0.56 respectively. Adams *et al*. (1986) report a single-period elasticity of lumber imports with respect to exchange rate as 0.46 and a multi-period elasticity of 0.40. Finally, Buongiorno *et al*. (1988) use a bi-variate causality model and derive long-run multipliers between Canadian softwood lumber imports in the U.S., U.S. lumber price and exchange rate to be 1.48 and 0.46 respectively. Due to differences in data and methodology, the short-run and long-run elasticities reported in this paper are not directly comparable to those reported in previous studies. Despite these differences, however, the long-run price elasticities obtained in this paper are almost identical to that found by Buongiorno *et al*. (1988). Also the finding that the bilateral exchange rate does not matter is the same as those found by Buongiorno *et al*. (1988) and Jennings *et al*. (1991). Finally, both the short-run and long-run elasticities are quite high for disposable income and wage rates. These results seem to corroborate the finding of a recent European study that trade elasticities are generally higher than elasticities obtained domestically (Brooks *et al*., 1995).

What are the implications of these results for the softwood lumber dispute? The results show that there is a stable long-run relationship among five excess demand side variables and Canadian softwood lumber exports to the United States. The excess demand side factors explain almost 70% variation in the Canadian softwood lumber exports. Moreover, the error-correction models show that when changes in excess demand side factors create a disequilibrium in the softwood lumber export market in quarter t–1, the growth in softwood lumber export in quarter t corrects between 70 to 74% of the deviation from the long-run equilibrium. So, the response of softwood lumber export from Canada to changes in the excess demand side factors is quite substantial. Although there is still room for alleged stumpage subsidy to influence softwood lumber exports from Canada to the United States, the results of this study strongly suggest that excess demand side factors are more powerful determinants of softwood lumber trade between these two countries than the alleged stumpage subsidy in Canada. For a better understanding of the pattern of softwood lumber trade between Canada and the United States (which is at the centre of the dispute), all relevant excess demand side factors need to be considered along with the excess supply side factors (alleged stumpage subsidy being one of those) in the analysis. Perhaps due to time and other resource constraints, such a rigorous approach has not been followed by the ITA in it's determination of the level of subsidy and the USITC in it's determination of positive "material injury" to the U.S. softwood lumber industry (see USITC, 1992).

CONCLUDING REMARKS

The Canadian softwood lumber exports to the U.S. had been at the centre of a long trade dispute between these two countries. On January 28, 1994 the Binational Panel, formed under the Canada-United States Free Trade agreement, in its final ruling rejected the subsidy determination by the ITA and the determination of material injury to the domestic lumber producers in the U.S. caused by subsidized stumpage in Canada by the USITC. The CVD case was dismissed and import duties collected by the United States were refunded to Canadian softwood lumber exporters. On April 1, 1996 the governments of Canada and United States signed a new five year deal called "Softwood Lumber Agreement" which will allow Canada to export 14.7 billion board feet of softwood lumber to the U.S. each year duty free. Any additional export will be subject to prohibitive tariff (@ $50 per 1000 board feet). While the dispute is now over, there are indications that softwood lumber dispute between these two countries may erupt again in the future. It is, therefore, important to understand the context of the dispute and the causes. It is also important to understand the factors contributing to the ever changing pattern of softwood lumber trade between these two countries.

Even if stumpage is subsidized in Canada, subsidy alone does not drive Canadian softwood lumber exports to the United States. There are a number of excess demand side factors which affect Canadian softwood lumber exports to the United States. In general, competitive market forces generate changes in these factors and these changes have no connection with the alleged stumpage subsidy in Canada. The objective of this study was to investigate the effects of five excess demand side factors on Canadian softwood lumber exports to the United States using multivariate cointegration analysis.

Johansen's maximum likelihood cointegration analysis is used to determine the number of independent cointegration relationships among the variables in a six-dimensional system. The results suggest that there is only one cointegration relationship among Canadian softwood lumber exports to the U.S., U.S. lumber price, U.S. disposable income, U.S. housing starts, U.S. construction wage rate and the bilateral exchange rate. This implies that there is a stable equilibrium relationship to which these variables have a tendency to return in the long-run. To evaluate the dynamics of short-run adjustments toward a long-run equilibrium, two error-correction models are specified and estimated. In these models, the growth rate of Canadian softwood lumber exports to the U.S. depends on the growth rate of relevant explanatory variables and on the error-correction term. These error-correction models are used to generate both short-run and long-run elasticities. The results suggest that the demand for Canadian softwood lumber in the U.S. is highly income and price elastic in the long-run. In the short-run, however, it has unitary price elasticity but elastic income elasticity. The results also suggest that the bilateral exchange rate does not matter. Although the error-correction term is negative and significant, adjustment towards the long-run equilibrium is not instantaneous. Seventy to seventy four percent of any quarters' deviation from the equilibrium is corrected in the next quarter's growth rate of Canadian softwood lumber exports.

Five excess demand side factors explain about 70% of the variations in Canadian softwood lumber export to the United States. While this finding does not negate the possibility of alleged stumpage subsidy influencing Canadian softwood lumber exports, it strongly suggests that the demand side factors are the major determinants softwood lumber trade between these two countries and not the stumpage subsidy. For a better understanding of the evolution of softwood lumber trade between Canada and the United States, these factors need to be considered in the analysis along with the relevant supply side factors.

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APPENDIX

Derivation of Long-run Elasticities from Error-Corrected Models

Since there is only one cointegrated relationship, the estimated coefficients can be normalized, without any loss of information, by the coefficient of total softwood lumber exports to the United States. Consequently, the long-run steady state relationship can be represented as:

$$X = KY_1^a Y_2^b Y_3^c Y_4^d Y_5^e, \tag{A.1}$$

where X is a vector of Canadian softwood lumber exports to the U.S. and Y_1 to Y_5 are five explanatory variables identified in section two of the paper and *a*, *b*,...,*e* etc. are the respective coefficients. The steady state relationship in (A.1) can be linearized in the logarithms of the variables such as:

$$x = v_1 + yv, \tag{A.2}$$

where x is the logarithm of Canadian softwood lumber exports to the U.S., v_1 is the logarithm of the intercept, y is the logarithm of the row vector containing the explanatory variables and v is the column vector, $[a, b, c, d, e]'$, containing the long-run elasticities.

The convergence to long-run equilibrium requires some sort of short-run dynamic adjustments. To illustrate the mechanism of convergence, assume the case of an AR(1) process such as:

$$x_t = \alpha x_{t-1} + \mu + y_t\theta + \varepsilon, \tag{A.3}$$

where $|\alpha|<1$, μ is the intercept, ε is a column vector of short-run elasticities and θ is an iid error term with zero mean and a constant variance. Given (A.3), the steady state solution can be obtained as a long-run dynamic steady state in which all equilibrium values grow at a constant rate. To see this, manipulate and rearrange (A.3) to obtain:

$$\nabla x_t = \mu + \nabla y_t\theta + (\alpha-1)\left[x_{t-1} - (1-\alpha)^{-1} y_{t-1}\theta\right] + \varepsilon \tag{A.4}$$

If Canadian lumber export to the U.S., x, rises above its long-run equilibrium level at time $(t - 1)$, the term in the brackets becomes positive. Since $(\alpha - 1) < 0$, it reduces the growth rate of the observed x at time K and thus, x approaches towards its steady state equilibrium. Due to this reason, the above equation is called an error-correction model. The term in the brackets represents the error-correction mechanism.

The long-run elasticities can be derived from the estimated parameters of the error-correction model (i.e., equation (A.4)). In equilibrium both x and y must satisfy equation (A.2) and one can obtain the following restriction that connects (A.2) with the error-correction model:

$$v = (1-\alpha)^{-1}\theta \qquad \text{(A.5)}$$

This restriction implies that the long-run elasticities will always be higher than the short-run elasticities.