Analytics of Timber Supply and Forest Taxation under Endogenous Credit Rationing — Separability after All

Markku Ollikainen*

Abstract

In his celebrated essay, Paul Samuelson defined the "heroic" assumptions of perfect foresight and perfect capital and land markets, under which the harvesting decision is separable from forest owners' preferences, and under which the Faustmann rotation model offers an adequate model of harvesting behavior. By exogenously assuming the existence of imperfect capital markets, some analysts have concluded that the separability, Faustmann analysis and conventional wisdom about forest taxation is not valid under imperfect capital markets. In this paper credit rationing is endogenized to analyze the harvesting behavior of nonindustrial private forest owners and the effects of forest taxation on timber supply in a two-period model. It turns out that, contrary to exogenous credit rationing models, the cutting decision is separable from the forest owner's preferences under endogenous credit rationing. The liquidity effects of forest taxes are also absent undermining the ceteris paribus effects derived in exogenous credit rationing models. The paper defines the preferable tax base subject to the government budget constraint in terms of harvesting incentives and derives the optimal forest tax design in terms of the welfare of forest owners. It is shown that, under credit rationing, it is optimal to introduce yield tax at the margin even though the land productivity tax has been chosen optimally. The optimal yield tax rate is greater than zero but less than 100%, which differs from the perfect capital market case, where the 100% yield tax rate is optimal.

Keywords: Credit rationing, default risk, optimal forest taxation.

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Introduction

In his seminal article in this journal, Paul Samuelson defined the "heroic assumptions" behind the Faustmann rotation model. If capital and land markets are perfect and all economic variables known for certain, then the timing of harvests can be solved from the sum of the present value of an infinite series of rotations. The rotation length de-

* Markku Ollikainen, University of Helsinki, Department of Economics, P.O. Box 54, FIN-00014 University of Helsinki. Finland.
E-mail: Markku.Ollikainen@Helsinki.fi

pends on timber price, the interest rate, planting costs, the growth function of trees and other purely economic factors, not on the owner's preferences (Samuelson,1976; Chang, 1982; Hyde, 1980 and Johansson & Löfgren, 1985). Conventional theory of the effects of forest taxes has been derived in this framework: a site productivity tax (as a lump-sum type tax) is neutral, but other forest taxes are distortionary and affect the rotation time (see e.g. Chang, 1982 and Jackson, 1980).

The assumptions behind the Faustmann model imply that the harvesting decision is separable from the forest owner's preferences. Separability implies that the owner's consumption plans and other preference related factors do not affect cutting decisions. Samuelson regarded his assumptions as first approximations which can be relaxed without major changes in the analysis. Recently, this view has been challenged. The assumption of perfect capital markets has been relaxed by assuming that they are imperfect because of credit rationing. This gives a raise the question of how a loan ceiling on borrowing possibilities affects cutting (see Koskela, 1989a and also Kuuluvainen, 1990). Koskela argues that, under credit rationing, the owners cannot necessarily harvest their forest stands at the optimal point of time (as defined by perfect capital market conditions), because they must finance their consumption. Thus, harvesting is not separable from preferences, and the Faustmann analysis is invalid. What is more, he also argues that the effects of forest taxation will differ from those of conventional wisdom.

These new results seem intuitively appealing. When forest owners have an exogenous upper limit on borrowing, which they would like to exceed but cannot, in the absence of other income sources, they can adjust their behavior only through cutting. Thus, they can no longer necessarily guarantee equality between the forest growth rate and the interest rate, as they could under certainty and perfect capital markets. For these reasons the standard results of forest taxation must be corrected to take liquidity effects into account, but the question remains, how seriously should this message be taken? Do we really have to believe, for example, that a site productivity tax is non-neutral in terms of timber supply? The answer is, not necessarily. Credit rationing in Koskela's model was an exogenous assumption, not explained through the behavior of the banks and forest owners. It is important to understand whether these results change if credit rationing is endogenized in the model. Does the separability behind the Faustmann analysis still prevail and, more interestingly, what should the proper design of forest taxation be under credit rationing? Answering these questions is the task of the present paper.

The credit rationing literature offers many possibilities for this kind of examination. Credit rationing is broadly defined as a situation in which there is an excess demand for loans, because quoted loan rates are below the Walrasian market-clearing level (Jaffee & Stiglitz, 1990). There are three different hypotheses concerning the way credit rationing appears in the capital markets. They are the following: credit rationing as the quantitative limit on the amount of borrowing, (called loan ceiling in this paper) (Stiglitz & Weiss, 1981), an endogenously determined wedge between the borrowing and lending rate (King, 1986), and a nonlinear interest rate, i.e., an interest rate that increases as a function of the amount borrowed (Jaffee & Russel, 1976; Keeton, 1979).

In this paper, credit rationing is endogenized by using the nonlinear interest rate hypothesis, because the harvesting problem can be quite conveniently included in the framework. The basic features of the model to be developed here are the following and similar in some respects to those of Webb (1984) and de Meza & Webb (1992) (for further applications of the two-period model, see Mont-gomery & Adams (1995)). Forest owners are assumed to be identical in that among other things, they have similar attitudes towards risk-taking. A forest owner asks for a loan during the first period in order to finance consumption plans while intending to pay it back with future timber sale revenues. Future timber price is uncertain but the owner and the bank know the probability distribution of the timber price. Information in the model is thus imperfect but symmetrical.[1] If the timber price realization is high enough

[1] Symmetrical information is enough for the emergence of credit rationing. It is also a very plausible assumption in the Scandinavian countries and the USA, which have well-functioning roundwood markets and established procedures for forecasting future timber prices.

then the owner is able to pay back the loan, but under sufficiently low realizations he defaults. Banks can determine the probability of default as a function of the probability distribution. The default risk affects the profits of banks and is the source of credit rationing. The study is organized as follows. The basic model is developed and the equilibrium solution analyzed in section two. The effects of forest taxation are analyzed from various viewpoints in section three. After having calculated the comparative statics of forest taxes, the incentive effects in terms of timber supply are solved. The focus is then extended towards a novel analysis, the welfare effects of forest taxes.

THE MODEL

The Representative Forest Owner as a Borrower

The cutting possibilities of the owner are defined over periods one and two. Given cutting during the first period (x), the growth function of forest ($g(Q)$) and the original volume of timber (Q), then future cutting (z), is uniquely determined. Equation (1) shows that the more the forest owner cuts today, the less is available tomorrow, i.e. $dz/dx = -(1 + g')$.

$$z = (Q - x) + g(Q - x) \quad \text{with} \quad g'(Q - x) > 0, g''(Q - x) < 0. \qquad (1)$$

During the first period, the representative forest owner receives timber income from cutting. If p_i, $i = 1,2$, denotes pre-tax timber prices, this pre-tax income is $p_1 x$. The owner faces two different forms of forest tax: site productivity and yield taxes (T_i,τ_i), the rates of which differ between periods 1 and 2. Site productivity tax is a lumpsum tax, which is the benchmark case in the forest taxation literature. It can be regarded as a simple property tax, which is independent of the level of timber selling or silvicultural activities. Yield tax is chosen as a representative of general harvest taxes and is levied on timber sale revenue.[2] After-tax timber prices are therefore defined as

[2] The assumption of differing tax rates come from exogenous credit rationing models, but the comparative statics of constant forest tax rates will also be analyzed.

$$p_i^* = p_i(1-\tau_i),\ i = 1,2.$$

The owner demands a bank loan B at the quoted interest rate $R = (1 + r)$. Consumption during the first period can be written as equation (2).

$$c_1 = p_1^* x + B - T_1 \tag{2}$$

During the second period, the owner receives after-tax timber-selling income $p_2^* z$, and pays site productivity tax T_2. Let $K = \tilde{p}_2^*$ denote the highest possible (finite) realization of p_2^*. Then the expected future timber price

$$E[p_2^*] = \int_0^K p_2^* f(p_2^*) dp_2^* .$$

Under a favorable realization of the future timber price, the forest owner earns a positive income over interest payments. Assuming limited liability, under low realization, the owner defaults on the bank loan. It is assumed that he is guaranteed a minimum security income by the state, which can be omitted from the calculations without losing generality.

Let $\underline{p}_2^*$ be that critical price which is just high enough to allow the forest owner to pay back the loan and interest on it according to equation (3). This equation shows that by assumption, under a default, the government has the right to take its taxes before the bank has the right to reclaim its money. The critical $\underline{p}_2^* = [RB + T_2]/z$ depends on B, which is endogenous, making the critical price endogenous too. The higher the interest rate, the size of the loan, the current harvest and the site productivity tax, the higher the critical value, $\underline{p}_2^*$ and the probability of default, other things being equal. This is counter-affected by future cutting so that the higher the z, the lower the value of $\underline{p}_2^*$.

$$\underline{p}_2^* z - T_2 - RB = 0 \tag{3}$$

Given the critical value of $\underline{p}_2^*$, equation (4) defines the owner's expected consumption during the second period.

$$c_2 = \int_{\underline{p}_2^*}^{K} \left[p_2^* z - T_2 - (1+r)B\right] f\left(p_2^*\right) dp_2^* \tag{4}$$

Assume that the owner's preferences are described by an additive utility function $u(c_1) + \beta u(c_2)$, where $\beta=(1+\rho)^{-1}$ is the time preference factor. Substituting equations (2) and (4) for c_1 and c_2 into the target function indicates that the owner's problem is to choose the size of bank loan and timber cutting so as to maximize his utility from consumption in equation (5).[3]

$$\underset{\{x,B\}}{\text{Max}}\; E[U] = u\left(p_1^* x + B - T_1\right) + \beta \int_{\underline{p}_2^*}^{K} u\left(p_2^* z - T_2 - (1+r)B\right) f\left(p_2^*\right) dp_2^* \tag{5}$$

Assume, first, that the interest rate is fixed, i.e. that the owner can freely borrow at a constant interest rate. This case will help to later trace out how the existence of credit rationing changes the owner's cutting behavior. In what follows, the partial derivatives are denoted by primes for functions with one argument and by subscripts for functions with many arguments. The optimal choice of current cutting and borrowing satisfies the following first-order conditions.

$$E[U]_x = p_1^* u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2) p_2^* (1+g') f\left(p_2^*\right) dp_2^* = 0 \tag{6a}$$

$$E[U]_B = u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2) R f\left(p_2^*\right) dp_2^* = 0 \tag{6b}$$

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[3] Note that (5) could equivalently be written as

$u\left(p_1^* x + B - T_1\right) + \beta E\left[u\left(p_2^* z - T_2 - (1+r)B\right)\right]$ by defining $E\left[u(c_2)\right] = \int_{\underline{p}_2^*}^{K} u(c_2) f\left(p_2^*\right) dp_2^*$.

However, because the definition of the default risk through $\underline{p}_2^*$ is a crucial part of the later analysis, the density function formulation is used throughout the paper.

$E[U]_B = 0$ implies that $u'(c_1) = \beta \int_{\underline{p}_2^*}^{K} u'(c_2) R f(p_2^*) dp_2^*$. Using this in $E[U]_x = 0$ gives the familiar cutting rule under price uncertainty: the marginal return on cutting Rp_1^*, must be equal to the opportunity cost of cutting $(1+g')p_2^*$ over all favorable realizations of p_2^*.

$$\beta \int_{\underline{p}_2^*}^{K} u'(c_2)\left[Rp_1^* - p_2^*(1+g')\right] f(p_2^*) dp_2^* = 0 \tag{7}$$

This rule is analyzed in detail in Koskela (1989b) and related to various other modifications in Ollikainen (1993). Two properties of rule (7) will be important for later analysis. First, price uncertainty means that the cutting decision does not separate from preferences. Second, because of risk-aversion, the owner cuts more timber today relative to conditions under certainty. Both properties become apparent by comparing (7) with the corresponding rule under certainty, where the probability distribution is absent, and $Rp_1^* = p_2^*(1+g')$ holds at the margin.

Finally, it is useful to derive the isoutility curves of the forest owner. These curves will be employed in the graphic illustration of equilibrium in the credit market and the comparative statics of timber supply in the following sections. Isoutility curves show the combinations of r and B that keep the forest owner's utility constant for a given level of cutting, x and given exogenous parameters. They have the properties described in Remark 1, which is proved in Appendix 1. Figure 1 describes the isoutility curves (denoted by k_1 and k_2) and also illustrates the relationship between isoutility curves and the demand-for-loans curve, $d(B)$, which cuts the isoutility curves at their top.

REMARK 1: *The isoutility curves of the representative forest owner in {r,B}-space increase up to the point where they cut the demand curve for loans, and decrease thereafter. Moreover, as a low interest rate is in the owner's interests, the lower the isoutility curves, the higher the utility with which they are associated.*

The Banking Sector

By assumption, perfect competition and free entry prevails in the banking sector. Risk-neutral banks maximize their expected profits given that it is uncertain whether the forest owner will be able to pay back the loan. Banks get their funds from competitive markets at some constant cost, at the deposit rate $I = (1 + i)$. They will get all their loans back with interest (RB) if the realization of p_2^* is equal to or higher than $\underline{p}_2^*$. If p_2^* is smaller than $\underline{p}_2^*$, banks only receive a part of their expected income. It is assumed that the government has the right to collect its taxes from the defaulting owner's income before banks get their share. The expected profit function of a representative risk-neutral bank can thus be written as follows

$$E[\pi] = RB\int_{\underline{p}_2^*}^{K} f\left(p_2^*\right)dp_2^* + \int_0^{\underline{p}_2^*}\left[p_2^* z - T_2\right]f\left(p_2^*\right)dp_2^* - IB.$$

This expression can be manipulated into the more convenient form given by equation (8).[4] The target function indicates that the bank will get its money back at the quoted rate R with the probability of $1 - F(\underline{p}_2^*)$, and will suffer some losses with the probability of $F(\underline{p}_2^*)$.

$$E[\pi] = RB\left[1 - F\left(\underline{p}_2^*\right)\right] + \int_0^{\underline{p}_2^*}\left[p_2^* z - T_2\right]f\left(p_2^*\right)dp_2^* - IB \tag{8}$$

Under perfect competition and free entry, bank profits will be zero and banks are obliged to behave under the zero-profit condition, $E[\pi] = 0$. The implications of this condi-

[4] To get equation (8), add to and at the same time subtract the term

$$RB\int_0^{\underline{p}_2^*} f(p_2^*)dp_2^*$$

in E[p]. This produces

$$E[\pi] = RB(\int_0^{K} f(p_2^*)dp_2^* - \int_0^{\underline{p}_2^*} f(p_2^*)dp_2^*) + \int_0^{\underline{p}_2^*}(p_2^* z - T_2)f(p_2^*)dp_2^* - IB.$$

Integrating the terms in braces produces the cumulative distribution function of $\underline{p}_2^*$ denoted by $F(\underline{p}_2^*)$ in equation (8).

tion become apparent through the zero-profit curve, the properties of which are given in Remark 2. For proof, see Appendix 1. The bank's zero-profit curve $E[\pi] = 0$ is the loan supply curve and is shown in Figure 1.

REMARK 2: *The bank zero-profit curve, $E[\pi] = 0$ in $\{r,B\}$-space is convex, indicating that when the loan size increases, the interest rate has to be raised in order to keep the expected profits at zero.*

Differentiating (8) with respect to the interest rate, cutting and taxes reveals how the interest rate charged by banks changes with changes in loan size cutting and forest taxes. An increase in future (current) forest taxes increases (has no effect on) the quoted rate. Increasing harvesting and the size of the loan tends to increase the quoted rate as suggested by equation (9), where $\alpha = B(1 - F(\underline{p}_2^*))$

$$\frac{dr}{dB} = r_B = -\frac{R\left(1 - F\left(\underline{p}_2^*\right)\right) - I}{\alpha} > 0$$

$$\frac{dr}{dx} = r_x = \frac{\int_0^{\underline{p}_2^*} (1 + g')p_2^* f\left(p_2^*\right) dp_2^*}{\alpha} > 0$$

$$\frac{dr}{dT_2} = r_{T_2} = \frac{\int_0^{\underline{p}_2^*} f\left(p_2^*\right) dp_2^*}{\alpha} > 0$$

$$\frac{dr}{d\tau_2} = r_{\tau_2} = \frac{\int_0^{\underline{p}_2} p_2 z f\left(p_2^*\right) dp_2^*}{\alpha} > 0. \tag{9}$$

Loan Contract and Equilibrium in the Credit Market: Cutting, Borrowing and the Quoted Rate

Borrowers are assumed to be identical and the banking sector competitive with free entry. Since the bank knows the probability of default, and free competition in the banking sector prevails, the optimal contract between the bank and the owner can be defined by maximizing the objective function of the representative forest owner subject to the zero-profit condition of the bank. Thus, in this case, the social planner problem and the determination of equilibrium in the credit market have the same form. An equivalent way to determine the optimal contract and market equilibrium is to write the interest rate as a function of the zero-profit condition in problem (4), as is done in (10).

$$\begin{aligned}\operatorname*{Max}_{\{x,B\}}\ P &= u\left(p_1^* x + B - T_1\right) \\ &\quad + \beta \int_{\underline{p}_2^*}^{K} u\left[p_2^* z - T_2 - \left(1 + r\left(x, B, T_2, \tau_2\right)\right)B\right] f\left(p_2^*\right) dp_2^* \qquad (10)\end{aligned}$$

Differentiating (10) with respect to x and B by applying the Leibnitz rule of differentiation of integral functions, recalling the endogeneity of the critical price $\underline{p}_2^*$, and setting the resulting derivatives equal to zero produces

$$P_x = p_1^* u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2)\left[(1+g')p_2^* + r_x B\right] f\left(p_2^*\right) dp_2^* = 0$$

and

$$P_B = u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2)\left[R + r_B B\right] f\left(p_2^*\right) dp_2^* = 0,$$

where the fact that the derivative of P with respect to critical price $\underline{p}_2^*$ is zero has been utilized. Using expressions (9) for r_x and r_B in the first-order conditions (10) yields

$$P_x = p_1^* u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2) \Bigg[(1+g') p_2^* + \frac{\int_0^{\underline{p}_2^*} (1+g') p_2^* f(p_2^*) dp_2}{1 - F\left(\underline{p}_2^*\right)} \Bigg] f(p_2^*) dp_2^* = 0$$

and

$$P_B = u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2) \left[R - \frac{R\left(1 - F\left(\underline{p}_2^*\right)\right) - I}{1 - F\left(\underline{p}_2^*\right)} \right] f(p_2^*) dp_2^* = 0.$$

Adding (subtracting) the terms in P_x = 0 (in P_B = 0) and noting that $\left(1 - F\left(\underline{p}_2^*\right)\right) = \int_{\underline{p}_2}^{K} f(p_2^*) dp_2^*$ produces the first-order conditions (11).

$$P_x = p_1^* u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2)(1+g')) E\left[p_2^*\right] f(p_2^*) dp_2^* \left[1 - F\left(\underline{p}_2^*\right)\right]^{-1} = 0, \quad (11a)$$

$$P_B = u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2) I f(p_2^*) dp_2^* \left[1 - F\left(\underline{p}_2^*\right)\right]^{-1} = 0, \quad (11b)$$

where $E\left[p_2^*\right] = \int_0^K p_2^* f\left(p_2^*\right) dp_2^*$.

The second-order conditions for the maximization problem are stated in equation (12). They hold because of assumptions of the concavity of the forest growth function and the utility function of the forest owner.

$$P_{xx} < 0, \quad P_{BB} < 0, \quad P_{xB} < 0, \tag{12a}$$

$$\Delta = P_{xx} P_{BB} - \left(P_{xB}\right)^2 = \beta g'' P_{BB} \left[1 - F\left(\underline{p_2^*}\right)\right]^{-1} \int_{\underline{p_2^*}}^{K} u'\left(c_2\right) E\left[p_2^*\right] f\left(p_2^*\right) dp_2^* > 0. \tag{12b}$$

The first-order conditions are worth closer analysis. Solving $u'(c_1)$from $P_B = 0$ and using it in condition $P_x = 0$ produces

$$\int_{\underline{p_2^*}}^{K} u'\left(c_2\right)\left\{p_1^* I - \left(1 + g'\right) E\left[p_2^*\right]\right\} f\left(p_2^*\right) dp_2^* \left[1 - F\left(\underline{p_2^*}\right)\right]^{-1} = 0.$$

Given that the probability of success is positive and $u'(c_2)$ is always positive, the terms in braces must, therefore, be zero for the condition to hold. The following new cutting rule now emerges.

$$p_1^* I = \left(1 + g'\right) \int_0^K p_2^* f\left(p_2^*\right) dp_2^* = \left(1 + g'\right) E\left[p_2^*\right] = 0. \tag{13}$$

Equation (13) suggests that the cutting decision separates from the preferences of the owner. This results is really surprising when contrasted to earlier results. I have already shown that under perfect capital market conditions and price uncertainty, preferences affect the owner's cutting decision. Moreover, nonseparability also occurs in loan ceiling models. The questions to be answered are, then, why does endogenous credit rationing lead to separability and why does the amount of cutting still differ from that of perfect capital market models? Let us start with the first question. Why does the level of cutting depend on the deposit rate i, and the whole distribution of future timber price $E\left[p_2^*\right]$, although they do not even belong to the owner's target function? Why does risk-aversion have no role in optimal cutting? The answer to the first question is as follows. The risk-neutral bank's zero-profit curve determines the size of the cut through the loan contract. This suggests that the difference from exogenous loan ceiling models must lie in the treatment of the default risk. In loan ceiling models, the upper limit of borrowing is used as a means of rationing and of preventing the default risk from going too high. In the nonlinear interest rate model, higher loans are given at a higher quoted rate. The quoted rate the bank charges, reflects the default risk, which depends on the size of the loan, the amount of timber left for the second period, and on the distribution of future timber price. The price distribution is exogenous, but timber volume can be affected. By fixing current cutting in the loan contract, the bank ensures that there will be enough timber for the repayment during the second period. As the bank is risk-neutral, it uses the deposit rate and the expectation value of the future timber price in determining the size of the harvest.

The answer to the second question is straightforward. By assumption the deposit rate reflects the competitive interest rate. Thus, the marginal revenue of cutting in (13) is the same as in perfect capital market models ($i = r$) but there is a difference in the opportunity cost of harvesting. As the bank takes into account the whole distribution of the future timber price, the opportunity cost of cutting decreases relative to perfect capital market conditions and conventional price uncertainty under risk-neutrality. The reason for this lies in the fact that yield tax cuts the price distribution at the upper integration level in (13) so that $\int_0^{\tilde{p}_2^*} p_2^* f\left(p_2^*\right) dp_2^* < \int_0^{\tilde{p}_2} p_2^* f(p_2) dp_2$. Therefore, current cutting will be higher under equilibrium credit rationing relative to exogenous credit rationing. If institutional arrangements allow the banks to get their money first in the case of default, then the yield tax level would not affect the expected price distribution, and cutting would be the same as in perfect capital markets. Notice finally, that the site productivity tax does not affect cutting at the margin, i.e. it is neutral, while the yield tax is distortionary. Only if banks were allowed to get their money first in the default, would a constant yield tax be neutral in terms of timber supply. Thus the institutional arrangements in the case of default crucially affect timber supply under (endogenous) credit rationing.

These observations have been summarized as Proposition 1.

PROPOSITION 1: *Under endogenous credit rationing modeled on the nonlinear interest hypothesis, the cutting decision is separable from the forest owner's preferences and is determined only by relative prices, the deposit rate and the tree growth function. If in the case of default, the government collects taxes before (after) the banks get their money, current timber supply is higher (the same) than (as) under certainty or risk-neutrality.*

The contract between the bank and the representative owner can be interpreted as representing the credit market equilibrium described in the north-east part of Figure 1. The loan (i.e. zero-profit) supply and demand curves intersect each other at point (r^*,B^*). This, however, is not the equilibrium of the model, which occurs at point (r',B'), where the isoutility curve k_1 is tangential to the isoprofit curve. At interest rate r', the forest owner would be willing to demand B'', which is his optimum, but he is not allowed to as the banks would ask for a higher quoted rate for B''. For this reason, Jaffee & Russel (1976) call this equilibrium (r',B') a rationing equilibrium. It is interesting to ask why borrowers are satisfied with equilibrium. The answer lies in the trade-off between loan size and loan rate. Increasing the loan size is connected with an increasing loan rate. Borrowers thus seek the optimal trade-off between the two and find it at the tangent point of the curve and highest possible isoutility curve, point (r',B'), even though they are moving away from the demand

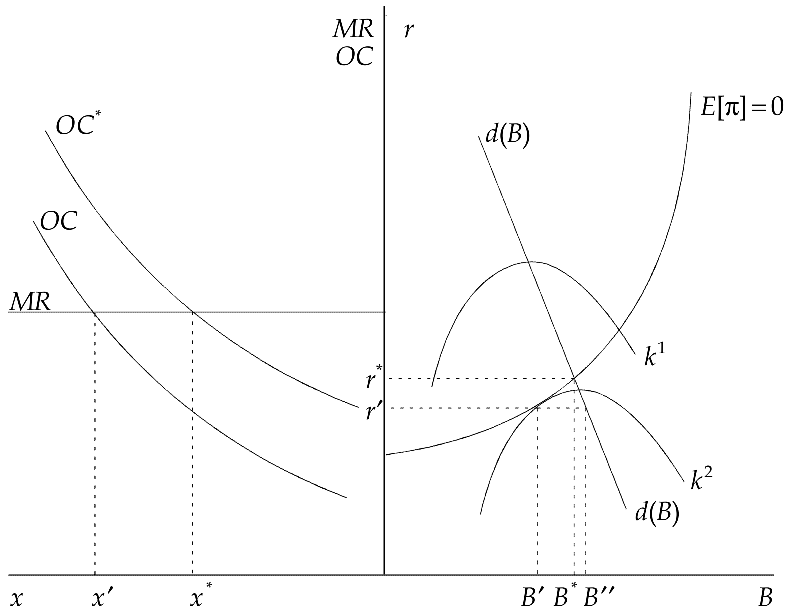

FIGURE 1: EQUILIBRIUM INTEREST RATE, LOAN SIZE AND HARVESTING: PERFECT AND IMPERFECT CAPITAL MARKETS

curve. The north-west part of Figure 1 describes how the short-term timber supply is related to the capital market conditions according to the harvesting rule (13). The marginal revenue of harvesting, $p_1^* I$, is described by a straight line *MR* and the opportunity cost of harvesting, $(1+g')E\left[p_2^*\right]$ by a convex curve *OC*. Optimal harvesting under endogenous credit rationing is determined by their intersection leading to the supply of *x*¢. If capital markets were perfect, the opportunity cost curve would be the higher OC^*, and the supply correspondingly the smaller x^*, as was proved above.

COMPARATIVE STATICS OF FOREST TAXATION UNDER ENDOGENOUS CREDIT RATIONING

In this section, the effects of forest taxes on timber supply, borrowing and the interest rate are analyzed in more detail. This is interesting per se — e.g. to see whether liquidity effects exogenous credit rationing models do emerge — and the results will be used later to examine the incentive and welfare effects of government tax policy. The comparative statics will be developed for both transitory (differing tax rates) and steady-state (constant tax rates) cases. The former facilitates comparisons with the results of exogenous credit rationing models and the latter with those of the traditional rotation models.

Transitory Effects of Forest Taxes

Assume that the bank and the owner can and do renegotiate a new contract without any cost whenever the exogenous parameters of the model change. This allows one to solve the conventional ceteris paribus effects of forest taxes on cutting, borrowing and interest rate. Note that the comparative static results of the interest rate can be derived directly from the zero-profit condition. Denote taxes generally by θ. Then, differentiating $E[\pi]$ with respect to x, B and θ produce $dr/d\theta = r_\theta + r_B B_\theta + r_x x_\theta$ as the very formulation from which the results are easily calculated. The first term, r_θ, is the direct effect of a change in θ on the quoted rate. The next two are indirect effects arising from changes in optimal cutting and borrowing.

The Wealth Effect of Site Productivity Tax

A change in site productivity tax during the first period will not change current cutting, i.e. the wealth effect is zero in (14), as was expected because of separability. It is more profitable for the owner to increase his borrowing and pay a higher quoted rate. The quoted rate goes up because of increased borrowing $r_B B_{T_1} > 0$ (as $r_x x_{T_1} = r_{T_1} = 0$).

$$x_{T_1} = 0$$

$$B_{T_1} = \frac{u''(c_1)}{P_{BB}} > 0$$

$$\frac{dr}{dT_1} = -\frac{\left[R\left(1 - F\left(\underset{-}{p_2^*}\right)\right) - I\right]u''(c_2)}{B\left[1 - F\left(\underset{-}{p_2^*}\right)\right]P_{BB}} > 0 \tag{14}$$

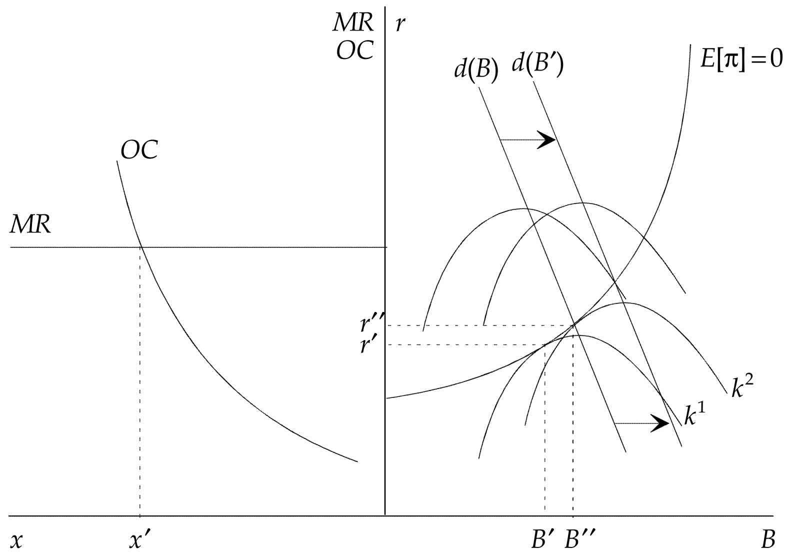

FIGURE 2: COMPARATIVE STATICS OF CURRENT SITE PRODUCTIVITY TAX

The outcome is in sharp contrast with those derived from exogenous credit rationing models, in which the owner increases his cutting due to the liquidity effect. The difference between this and exogenous credit rationing models lies in the possibility of increasing the size of the bank loan by paying a higher quoted rate. Although a higher loan is associated with a higher interest rate, it pays both parties of the contract to adjust through borrowing, because the marginal conditions of cutting have not been changed. Figure 2 illustrates the results. The original equilibrium is a loan contract $L = L(r', B', x')$. As a result of the increase in the current site productivity tax, the loan demand curve $d(B)$ shifts to the right. A new tangency point of loan supply function and isoutility curve k^1 leads to an increased borrowing and a higher quoted rate. As the site productivity tax is neutral, no change takes place in the timber supply.[5]

[5] The shift of the demand for loans curve and of isoutility curves reflects the fact that, under perfect capital markets, $B_{T_1} > 0$. The new equilibrium under credit rationing is obtained at the tangency point of isoutility curves and zero-profit function. The zero-profit function depends on the second period's variables, whose change shift the zero-profit function.

A change in the future site productivity tax has no effect on current cutting. However, a higher tax causes the owner to decrease the size of the bank loan. The change in the interest rate is given by equation $dr/dT_2 = r_{T_2} + r_B B_{T_1}$ (as $r_x x_{T_2} = 0$). The first term is positive and reflects increased default risk. The second one is negative owing to decreased borrowing. It plausibly dominates so that the quoted rate decreases. This is illustrated in Figure 3 where the initial loan contract is given by the tangency point between the loan supply and isoutility curves. As the site productivity tax increases, the loan supply function shifts upwards, because a higher site productivity tax increases the bank's default risk. The demand function for loans shifts downwards leading to a new credit market equilibrium (r'', B''). As the site productivity tax does not affect the harvesting decision at the margin, timber supply will not change.

$$x_{T_2} = 0$$

$$B_{T_2} = -\beta \int_{\underline{p}_2^*}^{K} I \left[u''(c_2) \left[1 - F\left(\underline{p}_2^* \right) \right] - u'(c_2) z^{-1} f\left(\underline{p}_2^* \right) \right] f\left(p_2^* \right) dp_2^* \left[1 - F\left(\underline{p}_2^* \right) \right]^{-2} \left(P_{BB} \right)^{-1} < 0$$

$$\frac{dr}{dT_2} < 0 \tag{15}$$

The Substitution Effect of Yield Tax

A higher yield tax decreases the marginal return of cutting, shifting some of it to the second period, the wealth effect being zero. The deficit in the income flow is compensated for by increasing the size of the bank loan. To opposing effects will change the quoted rate (as $r_{\tau_1} = 0$). The lower level of cutting means that the default risk de-

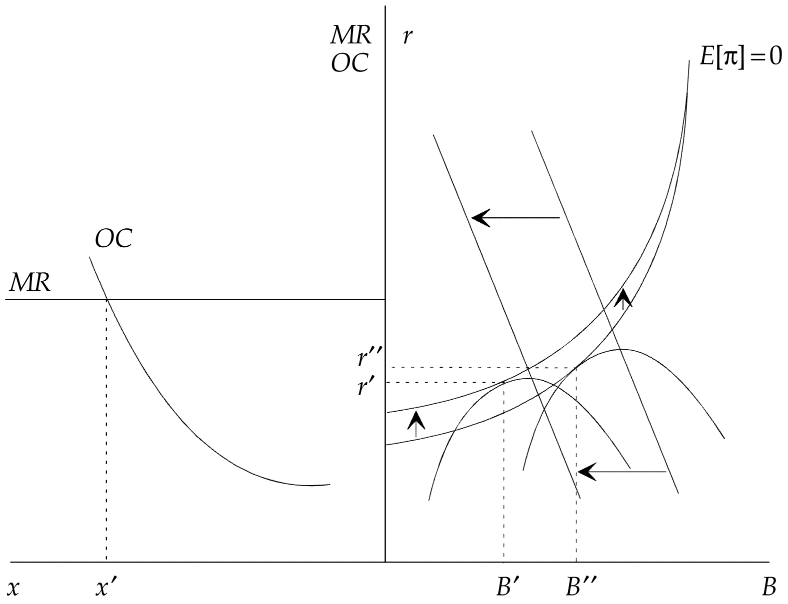

FIGURE 3: COMPARATIVE STATICS OF FUTURE SITE PRODUCTIVITY TAX

creases as a greater quantity of timber is reserved for the second period $(r_x x_{\tau_1} < 0)$. Increased borrowing, however, tends to increase the default risk $(r_B B_{\tau_1} > 0)$. The final sign of the quoted rate depends on the relative magnitudes of $r_x x_{\tau_1}$ and $r_B B_{\tau_1}$.

$$x_\tau = x_\tau^c = \frac{Ip_1^*}{g''E\left[p_2^*\right]} < 0$$

$$B_{\tau_1} = p_1 x B_{T_1} > 0$$

$$dr / d\tau_1 = r_x x_{\tau_1} + r_B B_{\tau_1} \qquad (16)$$

Equation (16) is illustrated in Figure 4 for the case of $dr/dt_1 > 0$. The loan demand curve shifts upwards and a new equilibrium in the credit market is described by (r'', B''). A higher current yield tax decreases the marginal return

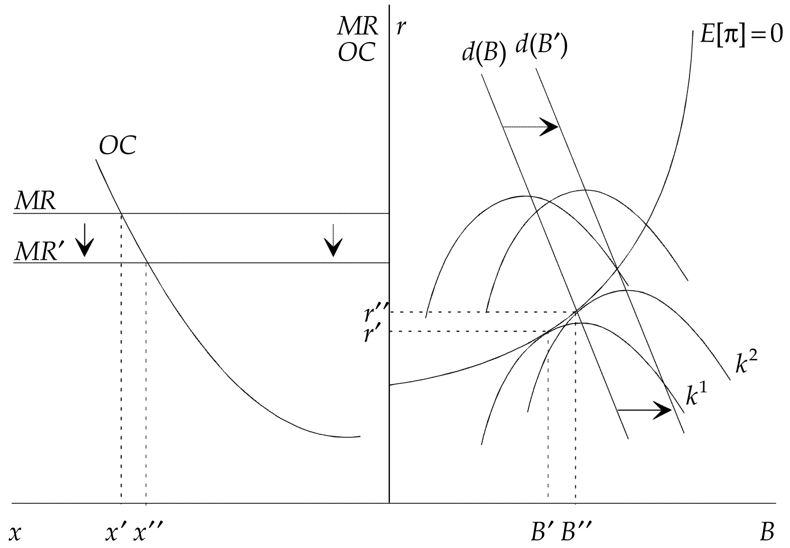

FIGURE 4: COMPARATIVE STATICS OF CURRENT YIELD TAX

of harvesting making it profitable to shift some of the cutting to the future, thus decreasing current harvesting, i.e. $x'' < x'$.

A higher future yield tax boosts current timber supply and decreases the size of the bank loan tending to decrease the quoted rate. On the other hand, a higher timber supply and yield tax level tend to increase it, so that the overall effect is ambiguous. If the loan size effect is dominant, the quoted rate goes down.

$$x_\tau = x_\tau^c = -\frac{(1+g')\left\{\tilde{p}_2 f(\tilde{p}_2) + E\left[p_2^*\right]\right\}}{g'' E\left[p_2^*\right]} > 0$$

$$B_{\tau_2} = E\left[p_2\right] x (1-\tau_2)^{-1} B_{T_2} < 0$$

$$\frac{dr}{d\tau_2} = r_{\tau_2} + r_x x_{\tau_2} + r_B B_{\tau_2} \tag{17}$$

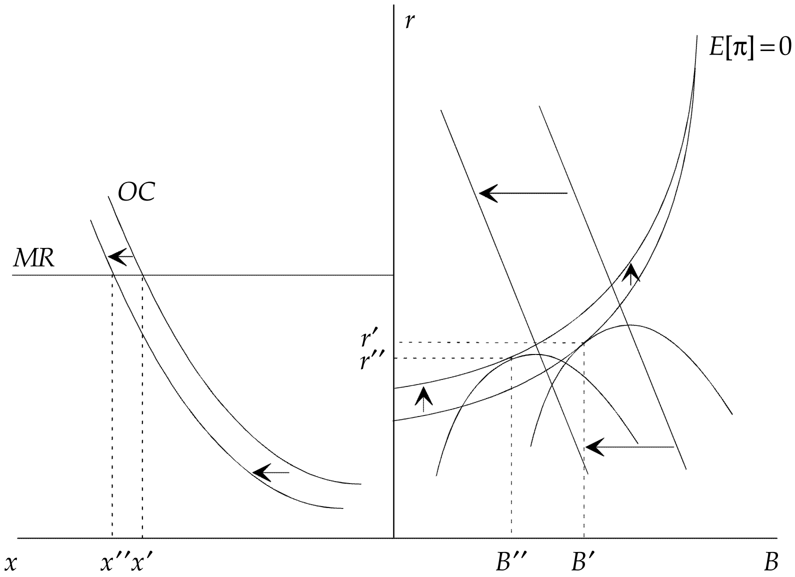

FIGURE 5. COMPARATIVE STATICS OF FUTURE YIELD TAX

Figure 5 illustrates this result for the case where the loan size effect is so great that the quoted rate decreases. The starting point is the original loan contract $L = L(r', B', x')$ and the new credit market equilibrium is (r'', B''). A higher future yield tax decreases the opportunity cost of cutting which tends to increase current timber supply, $x'' > x'$.

The Permanent Effects of Forest Taxes

Since a credible tax policy should not be based on changing tax rates, one must analyze the comparative statics of the model under constant tax rates. Under this assumption, the optimal conditions of credit market equilibrium defined by equations (11) – (13) still hold with the proviso that one has to substitute the transitory tax rates for constant ones in the arguments of the utility function (10) and the bank's expected profit function (8). The effects of constant forest taxes on endogenous variables can be expressed as a sum of transitory effects. For the site productivity tax one obtains

$$x_T = 0$$

$$B_T = B_{T_1} + B_{T_2}$$

$$dr/dT = r_T + r_B B_T \tag{18}$$

The site productivity tax is neutral. The sign of borrowing, however, is ambiguous a priori. Its sign depends on the

$$\operatorname{sgn} B_T = \operatorname{sgn}(u''(c_1) - \beta\left[1 - F(\underset{-}{p_2^*})\right]^{-2} I \int_{p_2^*}^{K} \left[u''(c_2) - u'(c_2)(1 - F(\underset{-}{p_2^*}))^{-1} z^{-1}\right] f(p_2^*)dp_2^*).$$

This expression reveals that the greater the amount of timber left to the second period and the smaller the probability of default, the more probable it is that the owner will ask for larger loan. If $B_T > 0$ the quoted rate goes up, because both the direct effects r_T and the indirect effect $r_B B_T$ are positive. A higher yield tax boosts current timber supply, because it affects the bank's expected profits negatively, as the upper level of the expected timber price goes down. Had the banks the right to take their money before taxes in the case of default, the yield tax would be neutral. Now the case for decreasing borrowing is stronger than in the case of the site productivity tax, as there is an additional negative term in B_τ. The effect on the interest rate is ambiguous a priori, because the direct default risk r_τ and increased cutting tend to increase the quoted interest rate but decreased borrowing tends to lower it.

$$x_\tau = x_\tau^c = -\frac{(1+g')\tilde{p}_2 f(\tilde{p}_2)}{g'' E\left[p_2^*\right]} > 0$$

$$B_\tau = \underset{+}{p_1 x B_{T_1}} + \underset{-}{E[p_2] z B_{T_2}} + \underset{-}{\frac{\beta\left(1 + F\left(\underset{-}{p_2^*}\right)\right) u'(c_2) I f\left(\tilde{p}_2^*\right)}{P_{BB}}}$$

$$dr/d\tau = r_\tau + r_x x_\tau + r_B B_\tau \tag{19}$$

Proposition 2 sums up the findings under steady-state forest taxes.

PROPOSITION 2: *Under endogenous credit rationing with nonlinear interest rate, (a) the site productivity tax is neutral in terms of timber supply, (b) the effect of the yield tax depends on the institutional arrangements; if the government (bank) first collects the taxes (its money), then in the case of default, the yield tax is distortionary (neutral) and increases (does not change) timber supply.*

The Implications of the Endogenous Credit Rationing Model

Equations (18)–(19) show that the site productivity tax is neutral and the yield tax distortionary. How does this relate to the corresponding results derived in the Faustmann model? Chang (1982) observed that the site productivity tax is neutral while the yield tax is distortionary in terms of the rotation period. Given that the rotation period implicitly determines the steady-state timber supply, the immediate conclusion from Chang's analysis is that the site productivity tax does not affect steady-state timber supply. In the two-period model the overall timber supply is given by the sum of current and future cutting according to equation (1). As the site productivity tax is neutral, it does not affect current timber supply and future cutting will not change either $(z_t = -(1 + g')x_T = 0)$. Consequently, the site productivity tax does not change steady-state timber supply in a two period model. The results are, thus, qualitatively the same. The yield tax increases the rotation period, but how is a longer rotation period and steady-state timber supply related in the Faustmann model?[6] As shown, in Jackson (1980), a longer rotation period increases steady-state timber supply. In the two-period credit rationing model, the yield tax increases cur-

[6] Note that the long run timber supply function defined by the Faustmann model is not a supply function in the usual meaning of the word. Owing to the assumption of one price independent of time, it does not tell us what happens to the quantity supplied when the price increases in a particular period. This fact explains the curious supply results of the Faustmann model — e.g. a higher timber price lowers long-term timber supply (see Johansson& Löfgren, 1985, 110–111).

rent timber supply and decreases future supply according to equation $z_\tau = -(1 + g')x_\tau = 0$. Thus the discounted sum of changes in the value of current and future supply is given by $x_\tau - I^{-1}(1 + g')x_\tau = (i - g')I^{-1}x_\tau$. It is positive if $i > g'$, which is found to be a necessary condition for dynamic efficiency in OLG models applied to forestry (see Löfgren, 1991). Summing up, endogenous credit rationing — at least in this model type — does not change the taxation results derived in the Faustmann model. Thus, the challenge posed by exogenous credit rationing models is not as demanding as it might have seemed at first glance.

THE CHOICE OF THE TAX BASE: INCENTIVE AND WELFARE ANALYSIS

The above analysis of credit market equilibrium and its comparative static properties provide a basis for analyzing further forest taxation issues: the incentive effects of forest taxation in terms of timber supply, and the properties of optimal forest taxation from the viewpoint of society. Solving incentive effects supplies the government with an answer to the question of which forest tax base will produce a greater or a smaller timber supply? The most interesting is, of course, to discover the properties of the optimal forest taxation under credit rationing. Before going into the detailed analysis three assumptions are needed. First, in line with optimal taxation literature, it is assumed that the tax rates are chosen subject to an exogenously given forest tax revenue requirement. Second, it is assumed that before any private decisions are made, the government announces a tax policy and commits itself to it. Third, it is assumed that the risk is idiosyncratic, i.e., that it is identically and independently distributed among individual forest owners so that the government can be regarded as risk-neutral and the tax revenue requirement is deterministic (see, e.g. Varian, 1980). This implies that the government uses the competitive interest rate i, which has an important impact on the optimal forest taxation.[7] The government budget constraint is given

[7] According to Arrow & Lind (1970), the use of perfect capital market's interest rate in government decisions is also legitimate under aggregate risk, if the risk is small.

in equation (20) where $\bar{p} = E[p_2]$, i.e., the expected pre-tax timber price.

$$G = T_1 + I^{-1}T_2 + \tau_1 p_1 x + I^{-1}\tau_2 \bar{p}_2 z \tag{20}$$

Incentive Effects of Forest Taxes

As the comparative static results reveal, changes in forest taxes only cause substitution effects on cutting. Therefore, the incentive effects of the switches in the forest tax base in terms of timber supply can be quickly derived. Assume first, that government decreases the yield tax and increases the site productivity tax during the first period, while keeping the tax revenue constant. Differentiating (20) with respect to T_1, τ_1 and x produces $0 = dT_1 + xd\tau_1 + mdx$, where $m = [\tau_1 - (1+i)^{-1}(1+g')\tau_2]$. The resulting change in timber supply can be defined as $dx = x_{T_1}dT_1 + x_{\tau_1}d\tau_1$. Solving the government budget constraint for $d\tau_1$ and substituting in the second equation produces

$$\frac{dx}{dT_1} = \frac{x_{T_1} - x^{-1}x_{\tau_1}}{1 + x^{-1}mx_{\tau_1}} > 0. \tag{21}$$

Comparative static results (14) and (16) indicate that $x_{T_1} = 0$ and $x_{\tau_1} < 0$; thus the denominator is positive and the numerator can also be shown to be positive if we assume that tax revenue increases when tax rates increase, i.e., under a positive Laffer-curve. Shifting the tax base towards the site productivity taxation during the first period thus increases current timber supply.

Assume next that the same switch takes place during the second period. Differentiating the budget constraint with respect to T_2, τ_2 and x produces $0 = I^{-1}dT_2 + I^{-1}\tilde{p}_2 z d\tau_2 + mdx$, with m is as above. The change in timber supply is given by $dx = x_{T_2}dT_2 + x_{\tau_2}d\tau_2$. Solving the two equations for dT_2 and dx produces (22). Because $x_{T_2} = 0$ and $x_{\tau_2} > 0$, timber supply will decrease as a result of the tax switch.

$$\frac{dx}{dT_2} = \frac{x_{T_2} - \tilde{p}_2 z x_{\tau_2}}{1 + z^{-1}mx_{\tau_2}} < 0 \tag{22}$$

Finally, by applying the same procedure one can show that a tax switch from yield tax towards site productivity tax under constant tax rates decreases current timber supply, as equation (23) suggests.

$$\frac{dx}{dT} = \frac{x_T - \bar{p}_2 z x_\tau}{1 + z^{-1} m x_\tau} < 0 \tag{23}$$

To sum up, results (21) – (23) have established the following proposition.

PROPOSITION 3: *Under endogenous credit rationing and an upward-sloping Laffer-curve, changing the tax base while keeping government tax revenue constant will a) under differing tax rates, increase current timber supply in a switch from yield tax to site productivity tax during the first period, and decrease current timber supply in a switch from yield tax to site productivity tax during the second period, b) under constant tax rates decrease current timber supply in a tax switch from yield taxation to site productivity taxation.*

Optimal Forest Taxation

Optimal forest taxation under endogenous credit rationing from the viewpoint of society is to be obtained by assuming a social planner who maximizes the social welfare function by choosing the yield and the site productivity tax subject to both the government budget constraint and the behavioral and credit market constraints analyzed in the previous sections. The social welfare function consists of the sum of the expected indirect utility function of the forest owner $EU^*(T,\tau)$ and of the expected indirect profit function of the bank $E\pi^*(T,\tau)$. These functions define the maximum expected utility and the maximum expected profits given by the exogenous parameters, the constant forest taxes. However, as the competitive banking sector operates under zero-profit condition, $E\pi^*(T,\tau)$ will always be zero. The social welfare function can therefore be written only in terms of the indirect utility function of the owner, which includes the endogenous determination of cutting, borrowing and the quoted rate.

$$W = EU^*(T, \tau) \tag{24}$$

To solve the social welfare maximization problem, form the Lagrangian function $L = W + \lambda G$, where G is written in terms of constant tax rates, i.e., $G = T(1+I^{-1}) + \tau(p_1 x + I^{-1}\overline{p}_2 z)$. The first-order conditions for the social welfare maximi-zation under a given tax revenue requirement can be obtained by setting the partial derivatives of the Lagrangian with respect to T and τ equal to zero. Equation (25) gives the optimal condition of the site productivity tax.

$$L_T = EU_T^* + \lambda\left(1 + I^{-1}\right) = 0, \tag{25}$$

where

$$EU_T^* = -u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2) f\left(p_2^*\right) dp_2^*$$
$$= -\left(1 + I\left[1 - F\left(\underline{p}_2\right)\right] - 1\right) \beta \int_{\underline{p}_2^*}^{K} u'(c_2) f\left(p_2^*\right) dp_2^* < 0.$$

As (25) shows the optimal site productivity tax has to be chosen so that the present value of the marginal utility of consumption is equal to the cost of tax, $-(1+I)EU_T^* = \lambda$. Solving the optimal yield tax rate is more complicated, as the timber supply effects of the yield tax on the government tax revenue requirement have to be taken into account.

$$L_\tau = EU_\tau^* + \lambda\left[\left(p_1 x + \overline{p}_2 z I^{-1}\right) + \left(I p_1 - (1 + g')\overline{p}_2\right) I^{-1} \tau x_\tau^c\right] = 0, \tag{26}$$

with

$$EU_\tau^* = \alpha EU_T^* - \beta z \, cov\left(u'(c_2), p_2\right) - \beta \widetilde{p}_2 u(\cdot) f\left(\widetilde{p}_2^*\right) p_1 x u'(c_1)$$
$$- \int_{p_2^*}^{K} p_2 z f\left(p_2^*\right)) dp_2^* \beta \int_{p_2^*}^{K} u'(c_c) f\left(p_2^*\right) dp_2^*,$$

where the rule $E(ab) = E(a)E(b) + \mathrm{cov}(a,b)$has been used and α is defined as follows:

$$\alpha = \left(1 + I\left[1 - F\left(\underline{p}_2\right)\right]^{-1}\right)^{-1} \times$$
$$\left(p_1 x I\left[1 - F\left(\underline{p}_2\right)\right]^{-1} + \int_{\underline{p}_2^*}^{K} p_2 z f\left(p_2^*\right) dp_2^*\right) \beta \int_{\underline{p}_2^*}^{K} u'(c_2) f\left(p_2^*\right) dp_2^*.^{8}$$

Equation (26) implicitly defines the optimal yield tax rate. (Notice that the third term in equation (26) is not the cutting rule (13) because the distribution of the future timber price is given here in terms of the pre-tax price.) If the site productivity tax has been set optimally, (26) can be written as

$$L_\tau = \alpha EU_T^* - cov\left(u'(c_2), p_2\right) - \beta \tilde{p}_2 u(\cdot) f\left(\tilde{p}_2^*\right) - \alpha\lambda\left(1 + I^{-1}\right)$$
$$+ \lambda\left[\left(p_1 x + \overline{p}_2 z I^{-1}\right) + \left(I p_1 - \overline{p}_2(1 + g')\right) I^{-1} \tau x_\tau^c\right] = 0$$

which reduces to

$$L_{\tau|T=T^*} = -cov\left(u'(c_2), p_2\right) - \beta u(\cdot) \tilde{p}_2 f\left(\tilde{p}_2^*\right) + \lambda\phi\tau x_\tau^c + \Psi = 0 \quad (27)$$

where

$$\phi = \left(I p_1 - (1 + g')\overline{p}_2\right) I^{-1} \quad \text{and}$$
$$\Psi = \lambda\left(p_1 x + I^{-1}\overline{p}_2 z\right) - \alpha\lambda(1 + I) > 0.$$

$\Psi > 0$ reflects the special features of this endogenous credit rationing model. It emerges because the government uses the pre-tax expectation value of the future timber price

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[8] Notice that the derivative of EU_τ^* with respect to the lower intergration factor, the critical timber price, is zero.

but the owner (and the banks) use just the truncated price distribution (in the perfect capital market case the term would be zero as can be seen in Appendix 2).

In order to see whether the yield tax is needed at all under endogenous credit rationing, it is necessary to look at the corner solutions, $\tau = 0$ and $\tau = 1$. If the yield tax rate approaches zero, it no longer affects the integration limits of the utility function, so that the second and third terms in (27) vanish yielding

$$L_{\tau=0\big|T=T^*} = -cov\big(u'(c_2), p_2\big) + \Psi > 0. \tag{28}$$

The positivity of equation (28) reveals that the zero yield tax rate cannot be optimal. Thus given the optimal site productivity tax, it is welfare-increasing to introduce the yield tax at the margin. The yield tax reduces the volatility of timber prices by limiting the price distribution from below and above. Even though it raises the critical price, it increases cutting and decreases borrowing, working as a risk decreasing devise. But how far should one go in increasing the yield tax rate? Not to 100%, because $\tau = 1$ contradicts the interior solution. The reason for this is as follows. If $\tau = 1$ and the government collects all timber-selling revenues as taxes, the optimal solution for the banks is to stop giving loans to forest owners. This, however, does not reflect the optimum conditions (11a) and (11b) which indicate a positive bank loan size. Therefore, it is not optimal to tax away all the uncertainty. Tax rate can be solved from equation (27) to yield equation (29), where $\Phi = -\beta u(\cdot)\tilde{p}_2 f(\tilde{p}_2^*) + \Psi$.

$$\tau^* = \frac{cov\big(u'(c_2), p_2\big) + \Phi}{\lambda I^{-1}\big[Ip_1 - (1+g')\bar{p}_2\big]x_\tau^c} > 0 \tag{29}$$

It is interesting to contrast these findings with the optimal forest taxation under perfect capital market conditions. The optimal forest tax formulas are derived in Appendix 2. Under perfect capital markets (27) can be expressed as follows (the upper index denoting perfect capital markets).

$$L_{\tau}^{0}\Big|_{T=T^*} = -\beta z\, cov\big(u'(c_2), p_2\big) - \frac{\beta z\, cov\big(u'(c_2), p_2\big)}{(1+I)\beta E\big[u'(c_2)\big]} x_T + \lambda\tau\phi x_{\tau}^{c} = 0 \tag{30}$$

Evaluating (30) at the corners, $\tau = 0$ and $\tau = 1$ (and, noting that at $\tau = 1$, the covariance terms vanish and $Ip_1 - \overline{p}_2(1+g') = 0$) one obtains

$$L_{\tau=0}^{0}\Big|_{T=T^*} = -\beta z\, cov\big(u'(c_2), p_2\big) - \frac{\beta z\, cov\big(u'(c_2), p_2\big)}{(1+I)E\big[u'(c_2)\big]} x_T > 0 \tag{31a}$$

$$L_{\tau=1}^{0}\Big|_{T=T^*} = 0 \tag{31b}$$

It is thus immediately clear that, under perfect capital markets, it is beneficial to introduce the yield tax after the site productivity tax has been chosen optimally. The optimal tax rate is 100%. The reason for this is the fact that the yield tax functions as an insurance devise against timber price uncertainty. The government simply taxes away uncertainty caused by stochastic future timber price — the covariance term in (27) — and redistributes the money as lump sum subsidies. This seemingly differs from solution (28) for credit rationing, under which it is not optimal to tax away all the uncertainty because of the banking sector. Notice, however, that this result is mainly of theoretical importance and without practical relevance. It is unrealistic to imagine that the 100% yield tax scheme would be implemented in practice given all the imperfections in the economy and the possible problems of moral hazard that the scheme with lump-sum transfers would cause.

The properties of optimal forest taxation under credit rationing and perfect capital markets are collected as the Proposition 4.

PROPOSITION 4: *Under endogenous credit rationing, the optimal forest tax structure consists of a combination of site productivity and yield taxes. Given the optimal site productivity tax, it is desirable to introduce the yield tax at the margin as an insurance devise. The optimal yield tax rate is less than*

100% under credit rationing indicating that it is not optimal to eliminate uncertainty altogether. Instead, under perfect capital markets it is optimal to set the yield tax rate to 100% and redistribute the tax revenue as lump-sum subsidies to forest owners so that the uncertainty caused by future timber price is eliminated.

DISCUSSION

This paper has presented an initial application of the equilibrium credit rationing hypothesis to the analysis of timber supply and forest taxation. The credit rationing in the model was caused by timber price uncertainty and the forest owner's borrowing, which generate a default risk for the risk-neutral banks. The purpose of the paper was to study cutting behavior under credit rationing and the effects of forest taxation in terms of incentives and optimality especially. The central finding was that endogenous credit rationing implies separability and thus does not invalidate the rotation results, as was suggested by exogenous credit rationing models. This result also has clear consequences for the comparative statics. The site productivity tax turned out to be neutral and the yield tax distortionary, similarly to the rotation models. Thus the challenge of exogenous credit rationing models to traditional analysis was not as demanding as was proposed. Why was this so? The difference between endogenous and exogenous credit rationing models lies in the renegotiation of the loan contract. This makes timber supply a function of loan availability, implying that the dependence of the timber supply function on loan availability should be explicitly taken into account in empirical analyses of imperfect capital markets. This is absent in the analyses undertaken so far.

The result of separability under endogenous credit rationing derived in this paper is not just an occasion, but an important and, in many cases, a robust result. A similar outcome has been derived in the debate about Ricardian Equivalence by Toshiki Yotsuzuka (1987). In his famous article, Barro demonstrated that whether government spending is financed by bonds or taxes has no effects on the economy under some assumptions, which include perfect capital markets (Barro, 1974). The Keynesian critics of Ricardian Equivalence argue that under imperfect capital markets, liquidity constraints prevent the agent's optimal intertemporal adjustment and Barro's neutrality result will not hold. Yotsuzuka proved that endogenous credit rationing — at least in certain types of models — leads to optimal intertemporal adjustments and results in the neutrality of money, as was stated by Barro.

The final conclusion of this paper is that the separability of the harvesting decision from preferences is a more general feature than has been thought so far. It is even more general than Paul Samuelson assumed in his article. Credit rationing does not then necessarily lead to the rejection of rotation frameworks and the conventional results of forest taxation. The theoretical analysis of credit rationing is, however, important if we want to understand its relationship with harvesting behavior and to find a plausible hypothesis for empirical research concerning harvesting behavior under credit rationing.

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APPENDIX 1: ISOUTILITY AND ISOPROFIT CURVES

A. To prove the Remark 1, fix the utility function at some level $k = u(c_1) + \beta \int_{\underline{p}_2^*}^{K} u(c_2) f(p_2^*) dp_2^*$. Differentiating it with respect to B and r produces

$$\frac{dr}{dB} = \frac{u'(c_1) - \beta \int_{\underline{p}_2^*}^{K} u'(c_2) R f(p_2^*) dp_2^*}{\beta \int_{\underline{p}_2^*}^{K} u'(c_2) B f(p_2^*) dp_2^*} .$$

If the loan size chosen is optimal B^*, then $dr/dB = 0$ due to the first-order conditions. For any value of $B < B^*$, $dr/dB > 0$ and for any value of $B > B^*$, $dr/dB < 0$. Thus, around the optimum, the isoutility curves are first increasing and then decreasing. To prove the second part of the proposition, define the owner's expected indirect utility function EU^* by substituting the optimum values $x^* = x^*(B,r,...)$ and $B^* = B^*(B,r,...)$ implicitly defined by the first-order conditions (6) into the direct utility function. Applying the envelope theorem to EU^* and differentiating it with respect to r produces

$$EU_r^* = -\beta \int_{\underline{p}_2^*}^{K} u'(c_2) B f(p_2^*) dp_2^* ,$$

which is negative, indicating that a lower interest rate leads to higher utility and showing that lower isoutility curves in $\{r,B\}$ -space are associated with higher utility.

B. To prove Remark 2 set (8) equal to zero. Recalling that $\underline{p}_2^* = [RB + T_2]/z$, and $\partial \underline{p}_2^* / \partial B = R/z$ and $\partial \underline{p}_2^* / \partial r = B/z$ produces

$$dr/dB = -\left[R\left(1 - F\left(\underline{p}_2^*\right)\right) - I\right] / B\left[1 - F\left(\underline{p}_2^*\right)\right] > 0.$$

The denominator is always positive. The numerator can also be shown to be positive as follows: dividing (8) by $-B$ reveals that

$$-\left[R\left(1 - F\left(\underline{p}_2^*\right)\right) - I\right] = B^{-1}\int_0^{\underline{p}_2^*}\left[p_2^* z - T_2\right] f\left(p_2^*\right) dp_2^* > 0.$$

Further, d^2r/dB^2 is also positive by

$$\frac{d^2r}{dB^2} = \frac{\left[R^2 + B^2(dr/dB)\right] f\left(\underline{p}_2^*\right)}{zB\left[1 - F\left(\underline{p}_2^*\right)\right]} > 0$$

indicating that a zero-profit curve is convex. The curve emanates from the r-axis. The loan supply is zero, when $r < i$. Under certainty, the quoted rate $r = i$ and the loan supply is infinite. Under default risk, the bank charges a higher quoted rate for larger loans. For the second part of remark 2, notice that the derivative of the bank's indirect profit function is

$$E[\pi]_r r = B\left[1 - F\left(\underline{p}_2^*\right)\right] > 0,$$

indicating that a higher expected profit is associated with a higher interest rate.

APPENDIX 2: OPTIMAL FOREST TAXATION UNDER PERFECT CAPITAL MARKETS

Under perfect capital markets, the forest owner can borrow freely at a constant interest rate i. Therefore his maximization problem can be expressed in the way familiar from the standard version of the two-period model.

$$\text{Max } EU = u(c_1) + \beta E\left[u(c_2)\right], \tag{1}$$

where

$$\int_0^{\overline{p}_2} u(c_2) f(p_2) dp_2 = E\left[u(c_2)\right]$$

and future consumption

$$c_2 = p_2^* z - T - I\left(c_1 - p_1^* x + T\right).$$

Drawing on results from previous literature, it can be shown that the site productivity tax is not neutral, but causes a positive wealth effect denoted by $x_T > 0$ and the influence of yield tax is given by the sum of wealth and substitution effects as follows

$$x_\tau = (p_1 x + p_2 z)(1+I)^{-1} x_T - \frac{\beta z \, cov\left(u'(c_2), p_2\right)}{(1+I)\beta E\left[u'(c_2)\right]} x_T + x_\tau^c,$$

where the second term refers to the risk effect and the third term to the substitution effect, (see, Koskela 1989b).

The planner chooses T and τ so as to maximize the Lagrangian function $L = W + \lambda G$, where $W = EU^*$ and G denotes the tax revenue requirement given in the text, yielding

$$L_T = EU_T^* + \lambda\left(1 + I^{-1}\right) + \lambda\tau\left(Ip_1 - (1+g')\overline{p}_2\right) I^{-1} x_T = 0 \tag{1a}$$

$$L_\tau = EU_\tau^* + \lambda\left[\left(p_1 x + \overline{p}_2 z I^{-1}\right) + \left(Ip_1 - (1+g')\overline{p}_2\right) I^{-1} \tau x_\tau\right] = 0. \tag{1b}$$

For the envelopes it holds that

$$EU_\tau^* = (1+I)^{-1}\left(p_1 x + I^{-1}\overline{p}_2 z\right)EU_T^* - \beta z\, cov\left(u'(c_2)p_2\right),$$

where the covariance term, cov(u¢$(c_2)p_2$) is negative. Utilizing this and the Slutsky equation of x_t leads to

$$L_\tau = -\beta z\, cov\left(u'(c_2)p_2\right) - \frac{\beta z\, cov\left(u'(c_2)p_2\right)}{(1+I)E\left[u'(c_2)\right]}x_T$$
$$+\lambda\left(Ip_1 x - (1+g')\widetilde{p}_2\right)I^{-1}\tau x_\tau^c = 0, \qquad (2)$$

where the first two terms reflect the effects of an uncertain future timber price and the third term is the substitution effect. Evaluating (2) at the corners (τ = 0, τ =1) yields equation (3).

$$L_{\tau=0\,|\,T=T^*} = -cov\left(u'(c_2), p_2\right) > 0 \qquad (3a)$$

$$L_{\tau=1\,|\,T=T^*} = 0. \qquad (3b)$$