Optimal Design of Forest and Capital Income Taxation in an Economy with an Austrian Sector

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Abstract

This paper develops an intertemporal harvesting model under future timber price risk to re-examine the optimal design of forest and capital income taxes in an economy with an Austrian sector. The previous literature sought a tax structure which would give a neutral tax system in the sense of yielding the same discounted net return from investment in both sectors. While tax neutrality is a desirable goal in the first-best situation where government does not face budget constraint, the situation may change in the second-best case when the government tax revenue requirement is taken into account. This paper shows that both introducing uncertainty and allowing for the government tax revenue requirement in the expected value sense changes the results from the optimal design of tax structure. Given a (non-distortionary) site productivity tax it is generally desirable to use both the yield tax on harvesting and the capital income subsidy or tax. The level of the yield tax reflects a trade-off between its social insurance and distortionary properties. Under these circumstances the task of the capital income tax is to alleviate distortion created by the yield tax. This can be done by a capital income subsidy (tax) when the substitution effect of the yield tax is negative (positive) and current harvesting is too small (too large) from the viewpoint of society.

Keywords: Forest taxation, capital income taxation, Austrian sector.

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Introduction

At least since Fairchild's articles (1909 and 1935) there has been concern about the relative effects of various taxes on the return on investments in the forestry and other sectors. Fairchild used the term "deferred yield bias" to describe the phenomenon that any given property tax implies a higher burden on forestry with a long-term production period than on properties that provide an annual income

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cash flow (Fairchild, 1909). Klemperer's term "site burden of taxes" is meant to reflect a similar concern. Klemperer refers to the relative tax-induced reduction in the value of land (e.g., the value of the forest land relative to agricultural land) (Klemperer, 1974, 1976, 1982). According to both authors it is preferable to use a neutral tax system, which does not affect incentives to use land at the margin than to rely on such distortionary taxes (Klemperer, 1982). The term "neutral" refers here to the case where a tax has no effect on relative prices so that it is non-distortionary, while the term "distortionary tax" refers to the case where a tax affects relative prices. In the latter case the relevant distortionary effect depends on the pure relative price effects of the tax, i.e., on the substitution effect (see section three for more details).

These inter- and intra-sectoral tax policy considerations have been dealt with at a more general level as the Austrian sector problem. This term refers to an economy containing a capital asset such as forest or wine which increases in value as it ages. By formulating a model of an economy with an Austrian and an ordinary sector, Kovenock & Rothschild (1983) reformulated and developed further earlier analyses of forest taxation.[1] They assumed that society has to use capital gains taxation and studied its effects on two types of Austrian assets in terms of the intra- and inter-sectoral efficiency of investments.[2] A capital gains tax drives resources from the Austrian sector, leading to a situation in which the social return on the Austrian asset is less than the rate of return in the ordinary sector. Kovenock (1986) then examined the effect of land value and income taxation in an Austrian economy, showing that both inter- and intra-sectoral efficiency is obtained if a properly chosen property tax is levied on the Austrian sector. By assuming a pre-existing distortion caused by capital income tax, Ovaskainen (1992) showed that an ad valorem property tax on the standing timber can be used as a way of restoring the neutrality of taxation in forestry.

[1] For earlier analyses, see e.g. Bentick (1980) and Chisholm (1975).

[2] The intra- and inter-sectoral efficiency within the tree sector is obtained by using time efficiently, i.e., the tree is cut down at a time which maximizes the discounted social value of resources used in this sector. The intersectoral allocation, on the other hand, is efficient when the discounted value of all returns from a dollar investment is the same in both sectors.

These studies pursue a neutral tax system, which is a desirable goal in a certainty situation where the government is free to choose all the tax rates without a budget balance requirement. Unfortunately, this does not usually describe the actual decision problem of the government, which has to finance public spending by collecting taxes and which thus faces a budget balance requirement under uncertainty. The purpose of present paper is to offer some insight into the design of forest taxes in an Austrian sector under these circumstances.[3] We pose and provide an answer to the following question: if the government has to collect a given tax revenue and three types of taxes, site productivity tax, yield tax and capital income tax, are available, how these should be used?[4]

For the purposes of the analysis, we develop a standard two-period model under uncertainty about the future timber price (see e.g. Montgomery & Adams, 1995). The representative forest owner derives utility from current and future consumption and decides on savings and the timing of harvests. Both the values of harvesting and capital income are subject to taxation. The behavioral effects of taxes will be used in determining the optimal taxation through the maximization of the social welfare function subject to a given government tax revenue requirement. It is shown that given the site productivity tax it is optimal to use a yield tax. The level of the yield tax reflects the trade-off between its social insurance and distortionary properties. Under these circumstances the task of the capital subsidy or tax is partly to alleviate distortion created by the yield tax. This can be done by a capital income subsidy (tax), when the substitution effect of the yield tax on harvesting is negative (positive) and current harvesting is too small (too large) from the viewpoint of society.

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[3] In what follows we make a small open economy assumption by treating the pre-tax interest rate *r* as exogenously given. Kovenock's paper (1986) is more general in the sense that the land market equilibrium conditions tie together the Austrian and the ordinary sector. But like us he assumes that the after-tax rate of return is determined outside the Austrian sector.

[4] The optimal design of forest taxes when a government faces budget constraint, has been recently analyzed from various viewpoints in Amacher & Brazee (1997) and in Koskela & Ollikainen (1997a and 1997b). These papers do not, however, explore the potential role of capital income taxes as a part of the optimal design of forest tax policy.

The paper is organized as follows. In section two a two-period model for the analysis of timber selling under price uncertainty and capital income taxation is constructed, and the behavioral effects of forest and capital income taxes and their relevant decompositions are developed. The optimal design of forest and capital income taxes is posed and solved in section three. Finally some brief concluding remarks follow.

TIMBER SUPPLY AND CAPITAL INCOME AND FOREST TAXES UNDER TIMBER PRICE RISK

A Standard Model of Timber Supply

The representative forest owner is assumed to have a preference ordering over present and future consumption (c_1 and c_2). This is represented by a utility function which is assumed to be both additively separable and additive across periods and concave in each argument so that

$$U = u(c_1) + \beta u(c_2)\ , \tag{1}$$

where $\beta = (1+\rho)^{-1}$ describes the time preference factor. Thus U describes the discounted utility from consumption in both periods. In what follows the partial derivatives are denoted by primes for functions with one argument and by subscripts for functions with many arguments, e.g., $u'(c_1) = \partial u(c_1)/\partial c_1$, $A_x(x,y) = \partial A(x,y)/\partial x$ etc. The harvesting possibilities, which determine the biological trade-off between current and future harvesting, are given in equation (2).

$$z = (Q - x) + f(Q - x) \tag{2}$$

The owner can harvest an amount z in the future left from the current harvesting x and the growth of the remaining stock, Q-x, where Q denotes the original volume of timber and f the concave growth rate of forest, $f' > 0$, $f'' < 0$.

The government is assumed to levy two forest taxes on forest owners, namely the site productivity tax T and the yield tax τ. The site productivity tax is a lump-sum tax, independent of harvesting. The yield tax is a proportional tax imposed upon timber revenues.[5] If the timber price is

[5] For a more detailed definition of these taxes, see Klemperer, 1976, p. 113.

denoted by p_i, $i = 1,2$, then the post-tax price is $p_i^* = p_i(1-\tau)$. Harvesting timber entails harvesting costs that are denoted by v per unit. During the first period the forest owner allocates the net revenue from harvesting between consumption (c_1), saving (s) and the site productivity tax (T) so that

$$c_1 = \left(p_1^* - v\right)x - T - s\,, \tag{3}$$

where we have abstracted from other incomes for simplicity. The uncertain future timber price is denoted by a tilde above the timber price, so that the after-tax price is $\tilde{p}_2^*$. As the timber price is uncertain, the future consumption is uncertain as well. The government is assumed to levy a proportional capital income tax on savings t so that the effective after-tax real interest rate is $r^* = (1 - t\,)r$ where the pre-tax real interest rate r is assumed to be exogenously given. Thus uncertain future consumption is defined by the sum of the future net revenue from harvesting and capital income plus savings minus the site productivity tax so that we have

$$\tilde{c}_2 = \left(\tilde{p}_2^* - v\right)z - T + R^* s\,, \tag{4}$$

where $R^* = (1 + r^*)$.

In the spirit of traditional public finance we have assumed that both the site productivity tax T and the yield tax τ, as well as the capital income tax t, are the same now and in future, but their levels are determined by maximizing the social welfare function under the government tax revenue requirement. This means that the policy maker is assumed to credibly commit himself to future policy before any private decisions are made.[6]

The representative forest owner behaves according to the expected utility maximization hypothesis, is risk-averse ($u''(\tilde{c}_2) < 0$) and shows decreasing absolute risk-aversion, $A(c_2) = -u''(c_2)/u'(c_2)$ with $A'(c_2) < 0$ (see e.g. Hirschleifer &

[6] In the terminology of game theory this is a Stackelberg game with the government acting as the leader and forest owners as the follower. If the government cannot enter into binding commitments, but instead reoptimizes at the beginning of each period, then we have the Nash equilibrium without commitment. The analysis of tax policy without commitment, however, lies beyond the scope of this study.

Riley (1992) for the details). The forest owner's decision problem can now be posed as maximizing the expected utility EU with respect to s, x and z subject to (2) – (4). The first-order conditions for the expected utility maximization in terms of saving and harvesting at the interior solution are

$$EU_s = -u'(c_1) + \beta R^* E\left[u'(\tilde{c}_2)\right] = 0\,, \tag{5a}$$

$$EU_x = \left(p_1^* - v\right)u'(c_1) - \beta E\left[u'(\tilde{c}_2)\left(\tilde{p}_2^* - v\right)\left(1 + f'\right)\right] = 0 \tag{5b}$$

Under the assumptions made, the second-order conditions hold and are given by (6).

$$EU_{ss} = u''(c_1) + \beta R^{*^2} E\left[u''(\tilde{c}_2)\right] < 0 \tag{6a}$$

$$\begin{aligned} EU_{xx} = {} & \left(p_1^* - v\right)^2 u''(c_1) + \\ & \beta E\left[u''\left(\tilde{p}_2^* - v\right)^2 \left(1 + f'\right)^2\right] + E\left[u'(\tilde{c}_2)\left(p_2^* - v\right) f''\right] < 0 \end{aligned} \tag{6b}$$

$$\Delta = EU_{ss} EU_{xx} - \left(EU_{sx}\right)^2 > 0 \tag{6c}$$

where Δ is the determinant of the Hessian matrix and the cross-derivative in its formula is given by

$$EU_{sx} = -\left(p_1^* - v\right)u''(c_1) - \beta R^* E\left[u''(\tilde{c}_2)\left(\tilde{p}_2 - v\right)\left(1 + f'\right)\right] > 0\,.$$

According to equation (5a) saving is determined so as to equate the marginal cost in terms of lost marginal utility of current consumption ($-u'(c_1)$) to the expected marginal utility of gained future consumption ($\beta R^* E[u'(\tilde{c}_2)]$). Since we are interested in the capital income taxation, it is assumed that $s > 0$. Equation (5b) can be transformed into a more suitable form. Notice first that $EU_s = 0 \Leftrightarrow u'(c_1) = (\beta R^* E[u'(\tilde{c}_2)])$ and substitute this for $u'(c_1)$ in $EU_x = 0$ to yield the well-known harvesting rule

$$R^* p_1 - (1 + f')\left[\bar{p}_2 + \frac{cov(u'(\tilde{c}_2), \tilde{p}_2)}{Eu'(\tilde{c}_2)}\right] - \frac{(r^* - f')v}{(1-\tau)} = 0 \qquad (7)$$

where $\bar{p}_2 = E[\tilde{p}_2]$ is the expected future timber price and the risk-adjusted future timber price $\bar{p}_2 + \mathrm{cov}(u'(\tilde{c}_2),\tilde{p}_2)/Eu'(\tilde{c}_2)$, with $\mathrm{cov}(u'(\tilde{c}_2),\tilde{p}_2) < 0$ due to risk aversion, is assumed to be positive. One should mention that if the risk-adjusted future timber price were non-positive then one would most likely end up with the corner solution, where all the forest stand is harvested in the current period.[7] According to equation (7) harvesting is carried out so as to equate the marginal return on harvesting and its marginal cost.

The following features of the harvesting rule merit to note. First, in the case of certainty, the covariance is zero so that the harvesting decision is separable from the preferences of the forest owner. In addition, if unit harvesting costs were zero, harvesting would be carried out to the point, where the after-tax marginal returns on harvesting $R^* p_1$would be equal to the after-tax marginal costs of harvesting $p_2(1+f')$ so that the yield tax would be neutral. This corresponds the case of zero regeneration costs in the Faustmann model (see e.g. Gamponia & Mendelsohn, 1987). Even though the yield is neutral, in this case the capital income tax is distortionary (see e.g. Kovenock, 1986). Under certainty and positive harvesting costs $R^* p_1 - p_2(1+f') \geq (<) 0$ as $r^* \geq (<) f'$. Second, allowing for timber price risk affects current harvesting positively and harvesting is no longer separable from the preferences of the forest owner (see e.g. Koskela, 1989).

Third, to determine the effects of taxes on harvesting at the margin under uncertainty, it is useful to distinguish between the following three cases in terms of taxes in use.[8]

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7 For instance, in the case of zero-harvesting costs, $v = 0$, there would be no interior solution with non-positive risk-adjusted timber price.

8 The complete set of behavioral effects of taxes is presented in section two and Appendix 1.

Case 1: $t = 0$ and $\tau = 0$.

If there are no other taxes levied on the forest owner than a lump-sum type site productivity tax, the harvesting rule (7) reduces to the benchmark case under uncertainty,

$$Rp_1 - (1+f')\left[\bar{p}_2 + \frac{\mathrm{cov}(u'(\tilde{c}_2), \tilde{p}_2)}{Eu'(\tilde{c}_2)}\right] - (r - f')v = 0 \qquad (7')$$

where the site productivity tax has no effect on the incentives at the margin, i.e. it is neutral.

Case 2: $t = 0$ and $\tau > 0$.

If society levies a yield tax on the harvest revenue but abstains from using the capital income tax, then the harvesting rule reduces to

$$Rp_1 - (1+f')\left[\bar{p}_2 + \frac{\mathrm{cov}(u'(\tilde{c}_2), \tilde{p}_2)}{Eu'(\tilde{c}_2)}\right] - \frac{(r - f')v}{(1-\tau)} = 0\,. \qquad (7'')$$

As equation (7′′) suggests, the yield tax causes no distortion at the margin if $r=f'$, but becomes distortionary if $r\neq f'$.

Case 3: $t > 0$ and $\tau = 0$.

Under a positive capital income tax and a zero yield tax one obtains

$$Rp_1 - (1+f')\left[\bar{p}_2 + \frac{\mathrm{cov}(u'(\tilde{c}_2), \tilde{p}_2)}{Eu'(\tilde{c}_2)}\right] - (r - f')v - rt(p_1 - v) = 0\,. \qquad (7''')$$

The capital income tax is always distortionary at the margin.[9]

[9] This finding has been established in the rotation framework by Kovenock (1986) and for an ad valorem property tax in the two-period model by Ovaskainen (1992).

COMPARATIVE STATICS OF TIMBER SUPPLY UNDER UNCERTAINTY WITH FOREST AND CAPITAL INCOME TAXATION

Some Preliminaries

To express the comparative statics results of timber supply in terms of forest and capital income taxes, the Slutsky decompositions are first developed. Given that the second-order conditions hold, the first-order conditions implicitly define the optimal consumption and harvesting in terms of taxes, so that $s = s(T,\tau,t,...)$ and $x = x(T,\tau,t,...)$. Substituting these for the corresponding variables in the target function (1) gives the expected indirect utility in terms of taxes. Utilizing the envelope theorem to the expected indirect utility function $EU^*(T,\tau,t,...) = u^0$ and the fact that $u'(c_1) = \beta R^* E[u'(\tilde{c}_2)]$, yields

$$EU_T^* = -\left(1+R^*\right)\beta E\left[u'\left(\tilde{c}_2\right)\right] < 0 \tag{9a}$$

$$EU_t^* = -rs\beta E\left[u'\left(\tilde{c}_2\right)\right] = \left(1+R^*\right)^{-1} rsEU_T^* < 0 \tag{9b}$$

$$\begin{aligned} EU_\tau^* &= -\beta E\left[u'\left(\tilde{c}_2\right)\right]y \\ &= \left(1+R^*\right)^{-1} nEU_T^* - \beta z\operatorname{cov}\left(\tilde{u}'\left(c_2\right)\right) < 0 \end{aligned} \tag{9c}$$

where $n = R^* p_1 x + \bar{p}_2 z > 0$ and

$$y = R^* p_1 + z\left[\bar{p}_2 + \frac{\operatorname{cov}\left(u'\left(\tilde{c}_2\right),\tilde{p}_2\right)}{Eu'\left(\tilde{c}_2\right)}\right] > 0\,,$$

as the risk-adjusted future timber price is positive. According to (9a–9c), forest owners become worse-off when tax rates increase, ceteris paribus.

Given that $EU_T^* < 0$, $EU^*(T,\tau,t,...) = u^0$ can be inverted for T in terms of the yield tax, capital income tax and maximum utility so that $T = g(t,\tau,u^0)$. Substituting this expression for T in EU^* gives the compensated indirect utility function $EU^*[g(t,\tau,u^0),\tau,t] = u^0$.[10] The expected compensated in-

[10] See e.g. Diamond & Yaari (1972).

direct utility function answers the following question: if the yield tax rate τ (or capital income tax rate t) is increased, how much the site productivity tax T has to be changed so as to keep the expected utility of the forest owner unchanged? Differentiating with respect to τ and t gives $EU_T^* g_\tau + EU_\tau^* = 0$ and $EU_T^* g_t + EU_t^* = 0$ so that

$$g_\tau = -EU_\tau^* {EU_T^*}^{-1} = -\left(1+R^*\right)^{-1} y < 0 \tag{10a}$$

$$g_t = -EU_t^* {EU_T^*}^{-1} = -\left(1+R^*\right)^{-1} rs < 0 \ . \tag{10b}$$

These expressions indicate the compensation necessary to keep the level of expected utility unchanged as the yield tax or the capital income tax changes.

It is known that at the expected utility maximization point

$$x(T,\tau,t) = x^c\left(\tau,t,u^0\right) \tag{11a}$$

$$s(T,\tau,t) = s^c\left(\tau,t,u^0\right), \tag{11b}$$

where x is the uncompensated timber supply and x^c is the compensated timber supply. The compensated the timber supply is obtained when the yield tax or the capital income tax is changed and the forest owner is compensated by a change in site productivity tax so as to keep the expected utility unchanged. This timber supply concept describes the distortionary effects of taxes, while their total effect is described by the uncompensated timber supply. Analogous interpretations apply for s and s^c. Next we utilize the relationships (11a) and (11b) between uncompensated and compensated timber supply and saving to develop the Slutsky decompositions and the qualitative properties of timber supply and saving functions (11a, 11b).

Taxes and Current Harvesting

Substituting the g-function for T in the uncompensated timber function x and differentiating the equation (11a) with respect to τ and t gives $x_\tau + x_T g_\tau = x_\tau^c$ for yield tax and $x_t + x_T g_t = x_t^c$ for capital income tax. Utilizing the previously

TABLE 1. TAXES AND CURRENT HARVESTING.

The total effects of taxes on current harvesting decomposed into the substitution and income effects.

THE EFFECT ON HARVESTING	YIELD TAX	CAPITAL INCOME TAX
Substitution Effect	$\leq (>) 0$ as $r^* \geq (<) f'$	< 0
Income Effect	> 0	> 0
Total Effect	$= ?$ as $r^* > f'$	$= ?$
	> 0 as $r^* \leq f'$	

solved expressions for g_t and g_τ one gets the following Slutsky decompositions for current harvesting

$$x_\tau = x_\tau^c + \left(1+R^*\right)^{-1} y x_T \tag{12a}$$

$$x_t = x_t^c + \left(1+R^*\right)^{-1} rsx_T \ , \tag{12b}$$

The total effect of taxes (x_t,x_τ) has been decomposed into substitution effects (x_t^c,x_τ^c) and income effects ($(1+R^*)^{-1}yx_T$, $(1+R^*)^{-1}rsx_T$), respectively. The substitution effects reflect distortionary effects of taxes, while the income effects describe behavioral changes due to the fact that forest owners become worse-off when the tax rates increase.

The harvesting effects of taxes are shown in Table 1 (see Appendix 1 for the details).

A rise in the site productivity tax T makes the forest owner worse-off. He will react to the decreased consumption possibilities by increasing current timber supply under decreasing absolute risk-aversion.[11] A rise in the yield tax also makes the forest owner worse-off so that the income effect of the yield tax is of the same sign as the effect of the site productivity tax. On the other hand, the owner's incentives at the margin are affected. The sign of the substitution effect of the yield tax depends on the relationship between the after-tax interest rate and the growth rate of the forest.[12] A rise in τ decreases both the marginal return and the

[11] Under constant absolute risk-aversion, the income effect vanishes so that $x_T = 0$. Thus the total effects of taxes are given by the substitution effects only.

[12] The harvesting rule (7) allows all cases $r^* \geq (<) f'$ at the interior solution.

opportunity cost of current harvesting. Timber supply tends to decrease due to the first effect and increase due to the second effect. The first dominates if the interest rate is greater than the rate of the growth rate of the forest and vice versa. In the special case of $r^* = f'$ the substitution effect is zero, i.e., the yield tax is neutral at the margin. Hence, the total effect of the yield tax is a priori ambiguous (positive) when $r^* > f'$ ($r^* \leq f'$).

As for the capital income tax, the income effect is positive under decreasing absolute risk-aversion. A rise in the capital income tax makes the forest owner worse-off so that timber supply increases. The substitution effect is then negative, because higher capital income tax decreases the marginal return on current harvesting. Hence the total effect of a change of the capital income tax is a priori ambiguous.

Taxes and Saving

Turning to the saving effects of taxes, the Slutsky equations for saving from (11b) can be developed to yield

$$s_\tau = s_\tau^c + \left(1 + R^*\right)^{-1} y s_T \tag{13a}$$

$$s_t = s_t^c + \left(1 + R^*\right)^{-1} r s s_T \,. \tag{13b}$$

It can be shown that the harvesting effects of taxes are as follows (see Appendix 1 for details).

TABLE 2. TAXES AND SAVING.

The total effects of taxes on saving decomposed into the substitution and income effects.

THE EFFECT ON SAVING	YIELD TAX	CAPITAL INCOME TAX
Substitution Effect	$\leq (>)\ 0$ as $r^* \geq (<) f'$	< 0
Income Effect	> 0	> 0
Total Effect	$= ?$ as $r^* > f'$	$= ?$
	> 0 as $r^* \leq f'$	

A rise in T increases current harvesting, which shows up partly as an increase in saving, $s_T > 0$.[13] A rise in the yield tax makes the forest owner worse-off so that the income effect of the yield tax is positive. Incentives at the margin are also affected. The substitution effect of τ depends on the relationship between the interest rate and the growth rate of the forest as in the case of current harvesting with a similar interpretation. The substitution effect is negative (positive) if $r^* > (<) f'$. In the former (latter) case, the total effect of the yield tax on saving is a priori ambiguous (positive).

Finally, a rise in the capital income tax tends to make the forest owner worse off so that he tends to increase timber supply via a positive income effect. However, the net return on saving decreases so that the substitution effect is negative. Thus the total effect is a priori ambiguous.

Optimal Forest and Capital Iincome Taxation under Timber Price Risk

The above analysis of cutting and saving behavior and their comparative statics provides a basis to consider the issue of optimal forest taxation in an economy with an Austrian sector from the viewpoint of society. Before doing this one has to clear up a few things. First, in the line with the optimal taxation literature it is assumed that forest taxes are chosen so as to keep the government tax revenue given.[14] Second, we treat the government tax revenue requirement as deterministic. This assumption can be justified along several independent lines: (a) if government is risk-neutral, then it is interested in the expected value of tax revenue and the stochasticity of the timber price need not be taken into account in the design of tax policies; (b) if risk is private, i.e., independent across forest owners, then government revenue at the aggregate can be regarded as de-

[13] If absolute risk-aversion is constant, saving is not affected by the site productivity tax so that $s_T = 0$. Thus the total effects of taxes are given by the substitution effects only. Notice, however, that under constant absolute risk-aversion also the substitution effect of the yield tax is zero, $s_\tau^c = 0$.

[14] See e.g. Atkinson & Stiglitz (1980).

terministic.[15] Third, we assume that the government uses the pre-tax discount rate in determining the present value of tax revenues (see e.g. King, 1980, pp. 118, and Kovenock, 1986). The expected present value of government forest and capital income tax revenues can then be written as

$$G = \left(1 + R^{-1}\right)T + \tau\left[p_1 x + R^{-1}\overline{p}_2 z\right] + tsR^{-1} \quad , \tag{14}$$

where the first two RHS terms describe forest taxes and the last component capital income taxes. According to (14) the government is indifferent between the timing of tax payments and is concerned only to extract a given present value of tax revenue G.

The social planner's problem — acting as a "benevolent dictator" — is to choose the site productivity tax T and yield and capital income tax rates τ and t so as to maximize the social welfare function subject to both the government budget constraint (14) and to the behavioral responses of taxes. The welfare function is the indirect utility function of the representative forest owner.

$$W = EU^*(T, \tau, t) \tag{15}$$

As mentioned earlier, before any private decisions are made the government is assumed to announce a tax policy and to commit itself to it so that we study a Stackelberg equilibrium with government as the dominant player.[16]

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[15] As demonstrated in an empirical study from Finland by Tilli & Uusivuori (1994), timber price risk may be partly private in at least two ways. First, regional timber prices have varied considerably in a given year independently of their volatility over time. Second, in a given year there have been differences in the prices of various timber assortments which, together with different species distribution on a particular plot, causes private risk. The case of aggregate risk has been analyzed in a slightly different context in Koskela & Ollikainen (1997b).

[16] One should mention that here we abstract from the issue of credibility. An example of this "time inconsistency" is the so-called capital-levy problem, which has been analyzed in the context of capital income taxation by Fischer (1980). The issue is the following: If taxes distort economic decisions ex ante, the optimal policy should equalize the marginal distortion on the last unit of revenue across all time periods and all tax bases in the traditional Ramsey fashion. Notice, however, that once saving has been accumulated, its supply elasticity becomes zero, and the tax on it is not distortionary anymore. This means that the constraints on the government's tax problem look different ex post and ex ante. Therefore the promise to tax future capital at the ex ante optimal rate is not credible or "time consistent". In an equilibrium with rational expectations and no credible commitment private agents will save little because they recognize this incentive for high taxes on capital ex post. This leads to the analysis of Nash equilibrium without commitment, which lies beyond the scope of this paper (see, also Persson & Tabellini (1990) for a survey on these issues).

The first-order conditions for the social welfare maximization under the tax revenue requirement can be solved by setting the partial derivatives of the Lagrangian function $L = W + \lambda G$ with respect to T, τ and t equal to zero so that

$$L_T = EU_T^* + \lambda G_T = 0 \tag{16}$$

$$L_\tau = EU_\tau^* + \lambda G_\tau = 0 \tag{17}$$

$$L_t = EU_t^* + \lambda G_t = 0\,. \tag{18}$$

It is well-known that the optimal design of tax structure depends on the availability of instruments. In what follows we start by analyzing two special cases, but assume that the site productivity tax can always be used. These provide some background and intuition for the exploration of the optimal design when all taxes can be used.

The Optimal Yield Tax in the Absence of the Capital Income Tax

According to (16) the optimal site productivity tax is obtained by equating the loss of the marginal social utility due to the site productivity tax ($W_T = EU_T^* < 0$) with the increase in the tax revenues λG_T, which is equal to $\lambda(1 + R^{*-1}) + \lambda\tau x_T$. Given that the site productivity tax has been set optimally ($T = T^*$) and $t = 0$ the optimal yield tax rate is implicitly defined by equation (19) (see Appendix 2 for the details).

$$\begin{aligned} L_{\tau|T=T^*} &= -\mathrm{cov}\big(u'(\tilde{c}_2), \tilde{p}_2\big) \\ &+ \lambda R^{-1}\left\{\tau\big(Rp_1 - \bar{p}_2(1+f')\big)\left(x_\tau^c + \frac{z\,\mathrm{cov}\big(u'(\tilde{c}_2), \tilde{p}_2\big)}{(1+R)E\big[u'(\tilde{c}_2)\big]}x_T\right)\right\} = 0 \end{aligned} \tag{19}$$

To see whether the yield tax is needed at all when ($T = T^*$) and ($t = 0$), one has to evaluate (19) at $\tau = 0$. This gives

$$L_{\tau|T=T^*} = -\mathrm{cov}\big(u'(\tilde{c}_2), \tilde{p}_2\big) > 0 \tag{20}$$

It is welfare-increasing to introduce the yield tax at the margin since yield tax reduces the risk caused by the future timber price uncertainty and is thus beneficial for risk-averse forest owners. The beneficial effect of social insurance outweighs its distortionary effect. But how far should one go in increasing the yield tax rate? Evaluating (20) at $t = 0$, $\tau = 0$ yields

$$L_{\tau}\Big|_{T=T^*,t=0,\tau=1} = \lambda R^{-1}\left(Rp_1 - \overline{p}_2\left(1+f'\right)\right)x_{\tau}^{c} < 0 \quad \text{as } r \neq f'.^{17} \tag{21}$$

Thus unless $r \neq f'$ in which case the yield tax is non-distortionary, it is welfare-increasing to decrease it from the 100% level. One can, now, solve the optimal tax rate $0 < \tau^* < 1$ with $r \neq f'$ from equation (19) to give

$$\tau^*\Big|_{T=T^*,t=0} = \frac{Rz\mathrm{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)}{\lambda\left(Rp_1 - \overline{p}_2\left(1+f'\right)\right)\left(x_{\tau}^{c} + \dfrac{z\mathrm{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)}{(1+R)E\left[u'(\tilde{c}_2)\right]}x_T\right)} > 0, \tag{22}$$

as $r \neq f'$.

Thus we have

RESULT 1: *If the government tax revenue requirement is regarded as deterministic and the site productivity tax has been set at the optimal level, (a) it is desirable to introduce the distortionary yield tax at the margin, (b) the optimal yield tax depends on the trade-off between its social insurance and distortionary effects, (c) the optimal yield tax is zero if there is no uncertainty or forest owners are risk-neutral.*

The yield tax has both distortionary and social insurance effects. The former effect has to do with the question of how timber supply reacts to changes in the yield tax at the

[17] Recall from Appendix 1 that $\hat{c} = 1$ at the covariance term vanishes so that the substitution effect of the yield tax is simply

$$x_{\tau}^{c} = \Delta^{-1}\left\{\beta E\left[u'(\tilde{c}_2)\left(Rp_1 - \overline{p}_2\left(1+f'\right)\right)\right]EU_{ss}\right\} \leq (>) 0 \quad \text{as } r \geq (<) f'.$$

In addition one gets from (7) at $\tau = 1$ that $Rp_1 - \overline{p}_2\left(1+f'\right) \geq (<)\, 0$ as $r \geq (<) f'$ so that equation (21) is negative regardless of the sign of x_{τ}^{c}.

margin and the latter reflects the fact that the yield tax affect after-tax timber price risk. The higher the yield tax, the higher the distortion indicated by x_τ^c but the better the social insurance indicated by $\mathrm{cov}(u'(\tilde{c}_2,)\tilde{p}_2)$. The optimal level of τ reflects the trade-off between these two aspects. The distortionary yield tax should not be used if it has no insurance role, i.e., if forest owners do not care about uncertainty or if there is no uncertainty. In this case forest taxation is neutral because of the availability of lump-sum tax T.

The Optimal Capital Income Tax in the Absence of the Yield Tax

Let us now turn to the analysis of the optimal capital income tax by assuming that $T = T^*$ and $\tau = 0$ so that yield tax cannot be used. In its explicit form, equation (18) can be written as follows:

$$L_{t|\tau=0} = EU_t^* + \lambda R^{-1}\{rs + rts_t\} = 0. \tag{18'}$$

Using the Slutsky decompositions, equation (9b) and the fact that the site productivity tax has been set to optimum, one can express the partial derivative of the Lagrangian with respect to t as

$$L_{t|T=T^*,\tau=0} = \lambda R^{-1} rts_t^c. \tag{23}$$

The derivative of the Lagrangian at the margin with $t = 0$, when $T = T^*$ and $\tau = 0$ gives

$$L_{t|T=T^*,\tau=0} = 0 \tag{24}$$

Thus we have

Result 2: *If the government tax revenue requirement is regarded as deterministic and the site productivity tax has been set at the optimal level, the distortionary capital income tax is not needed.*

This result is quite natural. If the lump-sum type site productivity tax can be used, the distortionary capital income tax is not needed. This reflects the notion that nondistortionary tax dominates the distortionary one from the efficiency viewpoint. Result 2 gives support to the requirement for a neutral overall tax system in forestry advocated, e.g., in Klemperer (1982) and Gamponia & Mendelsohn (1987).

The Optimal Yield Tax and Capital Income Subsidy/Tax

After these special cases, we now turn to the more general case analyzed in section three. To see whether the use of capital income tax is needed, when both T and τ are used, one has to evaluate (23) by assuming that $T = T^*$, $\tau = \tau^*$. By setting $t = 0$, one derives from (23)

$$L_{t}\big|_{T=T^*,\tau=\tau^*,t=0} = \lambda R^{-1}\tau^*\left(Rp_1 - \overline{p}_2\left(1+f'\right)\right)x_t^c \tag{25}$$

where $0 < \tau^* < 1$ and $x_t^c < 0$.

We have shown in Appendix 3 that

$$L_{t}\big|_{T=T^*,\tau=\tau^*,t=0} \leq (>) 0 \text{ as } x_\tau^c \leq (>) 0. \tag{26}$$

Hence, it is desirable to introduce a subsidy for the capital income of the forest owner at the margin to increase harvesting, which has become too small as a result of positive yield tax, when $x_t^c < 0$.[18] In the case of $x_t^c > 0$, it is optimal to decrease harvesting, which has become too large as a result of a positive yield tax. This can be done by taxing capital income.

Thus we have our main result:

PROPOSITION: *If the government tax revenue requirement is regarded as deterministic and the site productivity and yield taxes have been set optimally, it is desirable to introduce (a) a capital income subsidy at the margin, when the substitution effect of the yield tax is negative, (b) a capital income tax at the margin, when the substitution effect of the yield tax is positive.*

[18] It can be shown that the optimal capital income subsidy (tax) is less than 100%.

This result can be interpreted as follows. Under timber price risk, there are three concerns in designing the tax system; tax revenue requirement, social insurance and the distortionary effects of taxes. The site productivity tax is nondistortionary and is assigned the task of collecting tax revenue. The yield tax provides social insurance and is distortionary. Its optimum reflects the trade-off between these two properties of the yield tax. The capital income tax under these circumstances should alleviate tax distortions. This can be done by introducing a capital income subsidy (tax) if the substitution effect of the yield tax is negative (positive). When the substitution effect is negative harvesting is too small from the viewpoint of society so that tax policy should encourage harvesting, and the other way round in the case of positive substitution effect, when harvesting is too large from the viewpoint of society.

As we indicated earlier, the harvesting rule (7) is consistent with $r^* \geq (<) f'$. In the steady-state situation , $p_1 = \bar{p}_2 = p$, however, equation (7) reduces to

$$r^* - f' = \frac{(1+f')}{p - v/(1-\tau)} \frac{\operatorname{cov}(u'(c_2), \tilde{p}_2)}{Eu'(c_2)} < 0 \tag{7*}$$

Thus we have

COROLLARY: *If the government tax revenue requirement is regarded as deterministic and the site productivity and yield taxes have been set optimally in the steady-state situation, it is desirable to introduce the capital income tax at the margin.*

This results simply from the positivity of the substitution effect of the yield tax in the steady-state.

It is interesting to contrast this proposition and its corollary to the findings in Kovenock (1986), Gamponia & Mendelsohn (1987) and Ovaskainen (1992). All these analyses seek a combination of taxes which is able to neutralize the distortions caused by each of the taxes. Gamponia and Mendelsohn propose the combination of yield tax (lengthening the rotation period) and property tax (shortening the rotation period). Kovenock proposes a combination of income tax (lengthening the rotation period) and property tax (shortening rotation period). Thus both combinations can in principle be used to produce a neutral tax system. Finally, in a slightly different model, Ovaskainen (1992) demonstrates that a combination of ad valorem property tax and capital income tax is also able to achieve neutrality. These results depend on the assumptions of certainty and on the absence of a government budget constraint so that taxes are either neutral or distortionary and government is in the first-best situation, i.e., free to choose any combination of taxes regardless of the need to finance government spending. The difference between these results and our proposition comes primarily from timber price risk. Under timber price risk the yield tax is not only distortionary but also beneficial in providing social insurance. Since τ^* is set so as to reflect the trade-off between its social insurance and distortionary effects, the capital income subsidy (tax) alleviates the distortionary effect by increasing (decreasing) timber supply, which has become too low (high) as a result of the yield tax.

CONCLUDING REMARKS

This paper has developed an intertemporal harvesting model under future timber price risk to re-examine the optimal design of forest and capital income taxes in an economy with an Austrian and an ordinary sector. The previous literature has sought a tax structure which would produce a neutral tax system under certainty in the sense of yielding the same discounted net return from investment in both sectors. While tax neutrality is often a desirable goal in the first best situation where government does not face a budget constraint, the situation may change in the second best context where the government tax revenue requirement is taken into account.

Our analysis shows that both introducing uncertainty and allowing for the government tax revenue requirement changes the optimal design of tax structure. The main result of this study is to show that given the (non-distortionary) site productivity tax it is desirable to use both the yield tax and the capital income subsidy, when the substitution effect of the yield tax is negative, but the capital income tax when the substution effect of the yield tax is positive. The level of the yield tax reflects a trade-off between its social insurance and distortionary properties. The task of the capital income tax is to partly alleviate distortion created by the yield tax. This can be done by a capital income subsidy (tax) when current harvesting is too small (too large) from the viewpoint of society.

There are several interesting possibilities for extending the analysis. First, one could allow for the amenity values of forest stands on the part of the forest owners and society. To the extent that the amenity services of forest stands have public goods characteristics, the externalities are present which may change the desing of optimal taxation (see Koskela & Ollikainen 1997a for an analysis of this case without capital income taxation). Second, the tax analysis under aggregate risk when the government tax revenue is regarded as stochastic would also be also a worthwhile extension.

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APPENDIX 1:
Comparative Statics of Taxes

a) Harvesting

By applying Cramer's rule, one obtains the income effect of the site productivity tax

$$x_T = \Delta^{-1}\left\{\left(1+R^*\right)u''(c_1)\beta E\left[u''\left(\tilde{c}_2\right)e\right]\right\} > 0, \text{ as } A'(c_2) < 0 \tag{1}$$

where $e = R^* p_1^* - \tilde{p}_2^*(1+f') - \left(r^* - f\right)v$. [19]

As for the Slutsky decomposition (12a) for the yield tax, the income effect can be expressed in terms of T, $(1+R^*)^{-1}yx_T$ and is positive under the assumptions made. The substitution effect of τ is given by

$$x_\tau^c = \Delta^{-1}\left[R^* p_1 - (1+f')\left(\bar{p}_2 + \frac{\operatorname{cov}\left(u'\left(\tilde{c}_2\right),\tilde{p}_2\right)}{E\left[u'\left(\tilde{c}_2\right)\right]}\right)\right]\beta E\left[u'\left(\tilde{c}_2\right)\right]EU_{ss} \le (>) 0 \tag{2}$$

as $r^* \ge (<) f'$, where the rule for stochastic variables a and b, $E(ab)$ = $E(a)E(b)$ + cov(a,b), has been used.[20]

The income effect of the capital income tax in (12b) is $(1+R^*)^{-1}rsx_T >$ 0. The substitution effect of the capital income tax is given in (3).

$$x_t^c = -\Delta^{-1}\left\{r\beta E\left[u'\left(\tilde{c}_2\right)\right]EU_{sx}\right\} < 0 \tag{3}$$

b) Saving

The effect of the site productivity tax on saving is

$$s_T = mx_T > 0, \tag{4}$$

where $m = \left(1+R^*\right)^{-1}\left[\left(p_1^* - v\right) + \left(\tilde{p}_2^* - v\right)(1+f')\right] > 0.$ [21]

The substitution effect can be shown to be

[19] For a proof, see Koskela 1989.

[20] Harvesting rule (7) in the text (p. 113) indicates that the term in braces is positive (negative) if $r^* > (<) f'$ and zero if $r^* = f'$.

[21] The expression for s_T is actually

$$s_T = mx_T - \Delta^{-1}\left\{\beta E\left[u'\left(\tilde{c}_2\right)\tilde{p}_2 f''\right]\left(u''\left(c_1\right) - \beta R^* E\left[u''\left(\tilde{c}_2\right)\right]\right)\right\}.$$

The last term $\left(u''\left(c_1\right) - \beta R^* E\left[u''\left(\tilde{c}_2\right)\right]\right)$ is, however zero at the optimum where $u'\left(c_1\right) = \beta R^* E\left[u'\left(\tilde{c}_2\right)\right]$.

$$s_\tau^c = -\Delta^{-1}\left\{R^* p_1 - (1+f')\left(\overline{p}_2 + \frac{\text{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)}{E\left[u'(\tilde{c}_2)\right]}\right)\right\}\beta E\left[u'(\tilde{c}_2)\right]EU_{sx}$$
$$= bx_\tau^c \leq (>) 0, \text{ as } r^* \geq (<) f', \tag{5}$$

where $b = -EU_{sx}(EU_{ss})^{-1} > 0.$

The income effect of t on saving is $(1+R^*)^{-1}rss_T > 0$ and the substitution effect is given by (6).

$$s_t^c = \Delta^{-1}\left\{r\beta E\left[u'(\tilde{c}_2)\right]EU_{xx}\right\} = dx_t^c < 0, \tag{6}$$

where $d = -(EU_{sx})^{-1} EU_{xx} > 0.$

APPENDIX 2
Formula for the Optimal Yield Tax

Here we show how one obtains equation (19) in the text from equations (16) and (17).

Recall that $EU_\tau^* = \left(1+R^*\right)^{-1} nEU_T^* - \beta z\text{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right).$ Applying this to (17) yields

$$L_\tau = (1+R)^{-1}EU_T^* - \text{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)$$
$$+ \lambda R^{-1}\left\{\left(Rp_1 x + \overline{p}_2 z\right) + \left(Rp_1 - \overline{p}_2(1+f')\right)x_\tau + rts_\tau\right\} = 0. \tag{1}$$

Utilizing the Slutsky decompositions for x_τ and s_τ produces

$$L_\tau = (1+R)^{-1} yL_T - \text{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right) - (1+R)^{-1} y\lambda\left(1+R^{-1}\right)$$
$$+\lambda R^{-1}\left\{\left(Rp_1 x + \overline{p}_2 z\right) + \left(Rp_1 - \overline{p}_2(1+f')\right)\left(x_\tau^c + \frac{z\text{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)}{(1+R)E\left[u'(\tilde{c}_2)\right]}x_T\right) + trs_\tau^c\right\} = 0. \tag{2}$$

Assuming means that $T = T^*$ means that $L_T = 0$, so that we have

$$L_{\tau|T=T^*} = -\text{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)$$
$$+ \lambda R^{-1}\left\{\tau\left(Rp_1 - \overline{p}_2(1+f')\right)\left(x_\tau^c + \frac{z\text{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)}{(1+R)E\left[u'(\tilde{c}_2)\right]}x_T\right) + trs_\tau^c\right\} = 0. \tag{3}$$

Setting $t = 0$, finally, yields

$$L_{\tau|T=T^*} = -cov\left(u'(\tilde{c}_2),\tilde{p}_2\right)$$
$$+ \lambda R^{-1}\left\{\tau\left(Rp_1 - \overline{p}_2(1+f')\right)\left(x_\tau^c + \frac{z\text{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)}{(1+R)E\left[u'(\tilde{c}_2)\right]}x_T\right)\right\} = 0 \tag{4}$$

which was given as equation (19) in the text.

APPENDIX 3

The Sign of Equation (25)

Equation (25) of the text is reproduced here for convenience.

$$L_{t|T=T^*,\tau=\tau^*,t=0} = \lambda R^{-1}\tau^*\left(Rp_1 - \bar{p}_2(1+f')\right)x_t^c \tag{1}$$

where $0<\tau^*<1$ and $x_t^c<0.$

The sign of (1) depends on the sign of the term $\left(Rp_1 - \bar{p}_2(1+f')\right)$. Its sign can be determined by examining equation (19) in the text (p.17). Assuming that $T = T^*$, $\tau = \tau^*$ and $t = 0$, equation (19) of the text is

$$\begin{aligned} L_\tau &= -\mathrm{cov}\left(u'(\tilde{c}_2), \tilde{p}_2\right) \\ &+ \lambda R^{-1}\left\{\tau^*\left(Rp_1 - \bar{p}_2(1+f')\right)\left(x_\tau^c + \frac{z\mathrm{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)}{(1+R)E\left[u'(\tilde{c}_2)\right]}x_T\right)\right\} = 0 \end{aligned} \tag{2}$$

Solving this for $Rp_1 - \bar{p}_2(1+f')$ yields

$$Rp_1 - \bar{p}_2(1+f') = \frac{\mathrm{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)R}{\lambda\tau^*\left(x_\tau^c + \frac{z\mathrm{cov}\left(u'(\tilde{c}_2),\tilde{p}_2\right)}{(1+R)E\left[u'(\tilde{c}_2)\right]}x_T\right)} > 0$$

as $x_\tau^c \le 0$ as $r \ge f'$. Thus we know that

$$L_{t|T=T^*,\tau=\tau^*,t=0} < 0, \text{ as } x_\tau^c < 0. \tag{3a}$$

On the other hand the harvesting rule (7) can be written as

$$Rp_1 - \bar{p}_2(1+f') = (1+f')\frac{\mathrm{cov}\left(u'(c_2),\tilde{p}_2\right)}{\beta E\left[u'(\tilde{c}_2)\right]} + \frac{(r-f')}{(1-\tau)}v < 0 \text{ as } r < f'.$$

This together with $x_t^c < 0$ yields

$$L_{t|T=T^*,\tau=\tau^*,t=0} > 0, \text{ as } x_\tau^c > 0. \tag{3b}$$