CUTTING RULES FOR FINAL FELLINGS: A MEAN-VARIANCE PORTFOLIO ANALYSIS

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ABSTRACT

The present paper formulates a portfolio model for harvesting problems. The assumption is that each forest stand is an independent asset and consideration is given only to final felling. We proceed to formulate a model for the portfolio value, in which the returns yielded by a forest stand can be seen to comprise three components: timber-price return, physical-growth return and opportunity cost from postponing harvesting. We also discuss the special features of a forest stand as an asset and its effects on the formulation of mean-variance portfolio optimisation. Our observation in the case of boreal coniferous forests is that a good approximation of the forest return can be obtained by ignoring the opportunity cost. A case study using real Finnish forest stands and stock-market data is presented.

Keywords: cutting rules, forest management, portfolio optimisation.

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INTRODUCTION

The present paper sets out to propose a portfolio model for the risk-averse landowner who is in a position to speculate between harvesting forest stands and investing in financial assets. The present model differs from earlier portfolio models in forest economics literature (Mills & Hoover, 1982; Thomson, 1991) in that our main concern is harvesting, not land sales.

Cutting rules for forest stands are traditionally derived by calculating the net present value using a fixed-interest rate level as the discounting factor. In most of the associated literature, the interest is built into long-term rotation models or two period-utility maximisation models (for surveys, see Binkley, 1987 and Ollikainen, 1996) or combinations of these two (Tahvonen & Salo, 1999). During the last ten years, several papers have been presented in which stochastic prices have been used in optimisation models to account for economic uncertainty (e.g., Forboseh *et. al*. 1996;

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Haight, 1991; Haight & Smith, 1991, see also a simulation study by Taylor & Fortson, 1991), while others have applied the Capital Asset Pricing Model (CAPM) to forest investments (e.g., Redmond & Cubbage, 1988; Washburn & Binkley, 1990; Zhang & Binkley, 1993, and Binkley *et. al*. 1996). The authors of these publications have relied on data from the United States or Canada, but several others have also studied the returns and risks of investments in forestry in Finland (Penttinen *et. al*. 1996; Tilli, 1996, and Lausti & Penttinen, 1998).

Using Markowitz's (1952) portfolio-optimisation model, Mills & Hoover (1982) looked into the reasons why investments are made in forest land although the net present values of such investments are lower than those of other assets. Their objective was to study several forest investments and other investments *ex ante* under the assumption that forest land can be freely bought. Thomson (1991) compared single-period and multi-period portfolio models using forest land as an asset. Forest investments are often a multiperiod, and so the use of the multi-period portfolio model is, in many cases, a rational choice. In our case, however, the main concern is in short-term decision making, and so we consider the single-period model to be sufficient. Wagner & Rideout (1991) studied how investments in thinnings affect the return and variance of forest land investments. As in our study, they were interested in harvesting, although they were not interested in optimal asset allocation and final felling.

Our contribution has been to develop a model for portfolio value, which separates the portfolio value into return on financial assets, return on the forest asset, and the end value of the bare land. The return on forest asset has three components: timber-price return, physical-growth return, and opportunity cost from postponing harvesting. We observed that in the case of boreal coniferous forests a good approximation for forest return can be calculated by ignoring the opportunity cost. Our contribution derives also from slightly modifying the traditional Markowitz mean-variance optimisation model by setting extra constraints, which attend to the special features of growing timber stock as an asset. We also applied the optimisation model to a case, which uses real Finnish forest stands and stocks as investment alternatives.

RETURN ON PORTFOLIO

In practical forestry, the forest holding is divided into stands, which are managed independently. Cutting is usually implemented by forest stand, not by timber assortment or tree species. Each stand has its own mixture of timber species and assortments and physical growth. In practice, all harvestable stands are mixed stands that include at least sawlogs and pulpwood. In this study, we considered only final fellings, not thinnings. We assumed that when a forest stand is harvestable (in the sense of final felling), it could be considered an asset. Stands are not assumed to have identical species compositions. We further assumed that the land is used exclusively for forestry. The managerial decision of the landowner is whether he is going to harvest or postpone the harvesting decision. We did not take into consideration land sales and so the liquid forest asset is the growing stock.

Concentrating on harvesting enabled us to hold one of the assumptions needed for a portfolio optimisation model: namely, that all investments are assumed to be perfectly divisible. Redmond & Cubbage (1988) argue that "timberland investments would certainly be less divisible than investments in stocks and are substantially less liquid as well." In our case, timber could be thought of as being perfectly divisible, because the landowner could sell any amount of timber from a forest stand. Timber markets are not as liquid as stock markets, but certainly are at least as liquid as the markets for forest land. For example, during the period 1990–93, the annual numbers of forest property sales transactions in Finland have varied between 1250 and 1650 compared to the 135,000 timber sales transactions (Aarne, 1994). But although the landowner can sell any amount of timber, buying can be a problem. Finland, and many other countries, lack markets for the growing stock (There are exceptions, e.g., the southern United States). However, if the optimal portfolio is such that the landowner needs more of the same kind of forest stand that he already has, then the problem might be to find such forest: of identical in species composition and assortments, growth and with suitable stand values. A similar problem was faced by Hazell (1971) in farm planning, and he assumed that identical farmland was not available. Collins & Barry (1986), however, argued that the constraints in agricultural business can be ignored by leasing more land. In section three of this paper, we go on to discuss how the non-availability of the same kind of growing stock affects portfolio optimisation. In section four we then calculate the optimal portfolio in both cases.

Assuming that the initial capital at the disposal of the landowner consists of financial assets and forest assets, let W_{0a} be the initial wealth represented by financial risky assets. The initial wealth W_{0fi} from a forest stand i consists of the value of the timber stock and the land. The value of the timber stock is $P_{0i}V_{0i}$, where P_{0i} is the stumpage price and V_{0_i} is the initial timber stock. Let the value of bare land of stand i at the beginning of period be P_{0i}^0, where the superscript denotes the number of growing periods the new tree generation has grown. The initial wealth W_0 is then

$$W_0 = W_{0a} + \sum_i P_{0i}V_{0i} + \sum_i \pi_{0i}^0, \tag{1}$$

where the sums are over stands owned by the forest owner. The speculative (liquid) part of wealth is defined as

$$W_{00} = W_{0a} + \sum_i P_{0i}V_{0i}, \tag{2}$$

because it is assumed that the forest land is not sold. This is the initial capital for the portfolio optimisation problem, i.e., the assets whose weights can be changed in the portfolio. We also define s_i as the share of liquid forest asset i in wealth, i.e.,

$$s_i = P_{0i}W_{0i}/W_{00}. \tag{3}$$

The share of the liquid forest asset i in the final portfolio is then

$$w_{fi} = x_i s_i, \tag{4}$$

where x_i is the proportion of the timber stock retained and $1-x_i$ is the proportion harvested.

Let us next consider the total wealth at the end of period. The variables topped by ~ denote random variables. Using the exponential growth function e^g (g=growth) for a forest stand, the wealth $\tilde{W}_{1fi}$ from the forest stand i at the end of the period is then the sum of non-harvested and harvested land as follows

$$\tilde{W}_{1f_i} = x_i\left[\tilde{P}_{1i}e^{g_i}V_{0i} + \tilde{\pi}_{1i}^0\right] + (1-x_i)\tilde{\pi}_{1i}^1, \tag{5}$$

where $\tilde{P}_1$ is the stumpage price at the end of period, $\tilde{\pi}_{1i}^0$ is the end-of-period value of the bare land, and $\tilde{\pi}_{1i}^1$ is the end-of-period value of the land harvested at the beginning. Using (3) and (4), we obtain

$$x_i = \frac{w_{fi}}{s_i} = \frac{w_{fi}W_{00}}{P_{0i}V_{0i}}.$$

Now (5) may be rewritten as

$$\tilde{W}_{1fi} = W_{00}w_{fi}\left[\frac{\tilde{P}_{1i}e^{g_i}}{P_{0i}} + \frac{\tilde{\pi}_{1i}^0}{P_{0i}V_{0i}} - \frac{\tilde{\pi}_{1i}^1}{P_{0i}V_{0i}}\right] + \tilde{\pi}_{1i}^1. \tag{6}$$

The total wealth $\tilde{W}_1$ at the end of the period is

$$\tilde{W}_1 = W_{00}\left[\sum_a w_a\tilde{z}_a + \sum_i w_i\left[\tilde{r}_{fi}e^{g_i} - \left(\frac{\tilde{\pi}_{1i}^1 - \tilde{\pi}_{1i}^0}{P_{0i}V_{0i}}\right)\right]\right] + \sum_i \tilde{\pi}_{1i}^1, \tag{7}$$

where

$$\frac{\tilde{\pi}_{1i}^1 - \tilde{\pi}_{1i}^0}{P_{0i}V_{0i}} = \tilde{c}_i, \tag{8}$$

is the opportunity cost of forest land i from postponing harvesting by one period, $\tilde{z}_a$ is the return on financial asset a, and $\tilde{r}_{fi} = \tilde{P}_{1i}/P_{0i}$ is the price return on timber stock, and $\sum_i \tilde{\pi}_{1i}^1$ is the land value of all the stands, if they were to be harvested one period earlier. Note also that $\sum_i \tilde{\pi}_{1i}^1$ is independent of any harvesting decisions. As we can see from (7), the return on the growing stock can be divided between price return, physical-growth return, and opportunity cost. The return on growing stock (i.e., on liquid forest asset i) is then

$$\tilde{z}_{fi} = \tilde{r}_i e^{g_i} - \tilde{c}_i. \tag{9}$$

The land value affects portfolio weights through opportunity cost. Later we will consider the effect of opportunity cost on portfolio returns with numerical examples.

Asset Allocation

The portfolio-optimisation model is defined so that all rates of return are pre-tax and no transaction costs are assumed. Investors are assumed to prefer more wealth to less wealth. The investor's problem is to maximise the utility of return on forest and financial assets as distinct from his consumption decision. Let us denote $\tilde{\mathbf{z}} = (\tilde{\mathbf{z}}_a, \tilde{\mathbf{z}}_f)$ as an n-vector of the stochastic returns on wealth, financial $\tilde{\mathbf{z}}_a$ and forest assets $\tilde{\mathbf{z}}_f$, and $\mathbf{w}$ is an n-vector of weights. We assume that the investor is risk averse; That is, the investor has a concave utility function.

The expected utility over portfolio returns $\mathbf{w}'\tilde{\mathbf{z}}$ can be defined in terms of means and variances if (*i*) asset returns are normally distributed or (*ii*) utility function is quadratic. Although the mean-variance principle is commonly used, both arguments include shortcomings, which should be taken into account. First, in many cases the empirically observed financial returns are not normally distributed. There is less knowledge about forest returns. Second, quadratic utility can represent an investor who prefers more to less over a restricted range of wealth only. Mean-variance analysis is robust in that it frequently holds approximately even when the above assumptions are violated. (Elton & Gruber, 1995).

The covariance matrix of the asset returns $\tilde{\mathbf{z}}$ is defined as a nxn-matrix $\mathbf{\Sigma}$. The risk is assumed to be the variance of return on invested wealth. The risk of catastrophic events such as fire or tree diseases is ignored. We further assume that there are no short sales and no riskless lending and borrowing. Following Ingersoll (1987) the minimum-variance portfolio with expected return μ is the solution $\mathbf{w}(\mu)$

$$\text{Min} \quad \mathbf{w}'\sum\mathbf{w} \tag{10}$$

$$\text{s.t.} \quad \mathbf{l}'\mathbf{w} = 1 \tag{11a}$$

$$\mathbf{w}'\tilde{\mathbf{z}} = \mu \tag{11b}$$

$$\mathbf{w} \geq 0 \tag{11c}$$

$$\mathbf{w}_f \leq \mathbf{s}. \tag{11d}$$

(11a) is the equivalent of a budget constraint, i.e., the sum of the investment proportions is one. Because no short sales are allowed, we have the constraint (11c). The buying constraint for growing timber stock is defined in (11d). In case timber can also be bought, only the constraints (11a–c) remain in the model. Now, the vector $\mathbf{w}_f$ defines how much timber should be kept in a portfolio, and so ($\mathbf{s}$–$\mathbf{w}_f$) is the amount, which should be sold, i.e., it defines the cutting rule resulting from portfolio optimisation.

It is clear that the constraint (11d) leads to solutions, which are suboptimal in markets where growing stock can also be bought freely. Later we will compare the numerical results of the portfolio optimisation with and without the buying constraint.

EMPIRICAL RESULTS

Evaluation of The Opportunity Cost

Next we consider how significant the opportunity cost $\tilde{c}_i$ is for the portfolio return. The problem now is how land value is defined. As a benchmark evaluation, we apply the land value model by Faustmann (1849) under the assumption of constant prices. Then

$$\pi^0 = \frac{P_0 V_0 e^{-rT} - R}{1 - e^{-rT}} \tag{12a}$$

and

$$\pi^1 = \frac{P_0 V_0 e^{-rT} - R}{1 - e^{-rT}} e^r \tag{12b}$$

where T is the optimal length of rotation in forest, r is the interest rate, and R stands for regeneration costs.

From (8) and (9), we observe that the land value affects the return of the liquid forest asset through parameters r, R and T. The Faustmann land value model then gives the following for the opportunity cost

$$c = \frac{\pi^1 - \pi^0}{P_0 V_0} = \left(e^r - 1\right)\left(\frac{e^{-rT}}{1 - e^{-rT}}\right)\left(1 - \frac{R}{e^{-rT} P_0 V_0}\right) \tag{13}$$

where $e^r-1\approx r$. For fairly long rotation times T, as is the case in boreal coniferous forest, $(e^{-rT})/(1-e^{-rT})$ is small. Therefore, for small values of r and large values of T, the product of the first two terms is small compared to r. For example, if r=0.04 and T=80, it is 0.17 % and if r=0.05 it is 0.09 %. The third term will decrease c even further. If we use a one hectare stand as an example, with P_0=FIM 200, V_0=300m^3 R=FIM 2000/hectare, r=0.04 and T=80, we get 0.03 %. Thus, the opportunity cost does not markedly affect the value of portfolio in long-rotation forests and correspondingly the portfolio model can be defined without the opportunity cost, i.e.,

$$\tilde{W}_1 = W_{00}\left[\sum w_a \tilde{z}_a + \sum_i w_{fi}\left[\tilde{r}_{fi}e^{g_i}\right]\right] + \sum_i \pi_{1i} \tag{14}$$

and return on growing stock as

$$\tilde{z}_{fi} = \tilde{r}_i e^{g_i}. \tag{15}$$

Our conclusion is that the opportunity cost from postponing harvesting is negligible if the rotation time is long enough and the interest rate is not too small. However, if r is very small, say 0.005, c is relevant. Then the return on the growing stock should be calculated using Equation (9) as in the case of short-rotation forests.

In the following case study, we defined the expected returns as logarithms

$$E\left[\tilde{z}_{fi}\right] = E\left[\ln \tilde{P}_{1,i} - \ln P_{0,i} + g_i\right] = E\left[\tilde{r}_i + g_i\right], \tag{16}$$

where E is the expectation operator. Physical growth is assumed to be deterministic, although there are small variations in practice. The variance of growth is assumed to be insignificant relative to the variance of price changes. The expected return is the sum of price changes and growth, but growth does not affect variance.

Case Study

First, we studied historical risks and returns using four forest stands from eastern Finland as examples of typical coniferous stands, and the HEX stock market index as an investment alternative. Our example of a real forest holding

TABLE 1. CHARACTERISTICS OF THE FOUR EXAMPLE STANDS.
Table 1 presents descriptive data about timber species and assortments, growth, value, weight in the original portfolio, and forest area of the four example stands.

	Stand #162	Stand #163	Stand #165	Stand #173	Total	Prices FIM/m^3 1996:12
Pine sawlogs		16 m^3	17 m^3			248.1
Spruce sawlogs	67 m^3	1 m^3		267 m^3		206.1
Birch sawlogs						98.1
Pine pulpwood		9 m^3	16 m^3			123.8
Spruce pulpwood	25 m^3	2 m^3		191 m^3		246.9
Birch pulpwood		3 m^3	14 m^3			94.2
Total	92 m^3	31 m^3	47 m^3	458 m^3	628 m^3	
Growth (%)	3.2	4.1	4.1	3.7		
Value (FIM)	16904	5589	7106	78675	108273	
Weight	0.16	0.05	0.07	0.72		
Area (ha)	0.7	0.5	0.6	2.2	4	

lies in eastern Finland, and its total forest land area is 22.6 hectares. The holding included fourteen stands as recognised in the forestry plan. Finnish forestry legislation limits the landowner's freedom to make final fellings in young stands. Regional Forestry Centres define which forest stands are harvestable. In this case, final felling was recommended for four stands, and so the timber stock on those four forest stands were determined in this study as being liquid forest. These stands are described in Table 1. Two of the stands consisted primarily of spruce sawlogs and pulpwood, and the other stands primarily of pine sawlogs and pulpwood. The ages of the stands varied between 70 and 90 years, and the annual growth was between 3.2 and 4.1 percent. The total timber volume of the stands was 628 m^3 and using the 1996:12 prices, the value of the stands was FIM 108,273. In our case, we assumed that the weight of financial assets in the landowner's initial portfolio is zero. Thus, the landowner had at his disposal FIM 108,273 for investments.

In this case, we are living in a world where the alternative investment possibility for forests is equities. We used the general stock market index HEX as describing equities in general. The monthly arithmetic average of HEX's daily closing values were calculated to make the data comparable to stumpage prices, which are calculated as monthly averages (Washburn & Binkley, 1990). Because each forest stand is a unique asset, the return data have to be calculated separately for each stand. These data are subsequently used for computing the expected return and the covariance matrix of assets. For example, if we have a fifty-fifty stand of pine sawlogs and pulpwood, we have to calculate the return data using these weights.

During the period 1985:10–1996:12, stocks offered a higher return with higher risk than the forest stands used as the example (Table 2). Stands containing mainly pine sawlogs and pulpwood involved a higher risk than spruce stands, but the return has not necessarily been higher. In fact, the returns on different forest stands have been quite similar. The returns on stocks were about 16% a year and on forest stands 6%. The physical growth return has throughout the period been approximately two-thirds of the total returns. Because we feel that the significance of the returns from growth has been essential, it is also easy to draw the conclusion that low interest rates are favourable to the competitiveness of forest investments, when compared with bonds and short-term interest papers as returns from physical growth is independent of the price changes of the other assets. The correlation between timber assortments and the HEX index was low. The correlation between timber assortments varied between 0.7–0.9, and it appeared that timber is quite a good hedge against equities.

The original weights of the forest stands 162,163,165,173 and the HEX stock market index in the initial budget were 0.16, 0.05, 0.07, 0.72 and 0.00, respectively. Assuming that the landowner decides not to spend the above FIM 108,273, how should he then allocate his money among these five assets? The expected returns and variances of the individual assets on a yearly basis are presented in Table 2, and the covariance matrix of asset returns on a monthly basis in Table 3. We used the data covering 1985:10–1996:12 to calculate the expected returns and risks for each of the assets.

Table 4a-b shows the results of both constrained and the usual unconstrained optimisation. The buying constraint moves the efficient frontier downward (Fig. 1). The difference between the constrained and unconstrained optimum is observed only in the case of low returns and risks, be-

TABLE 2. RETURNS AND RISKS OF THE FOREST STANDS.
Table 2 shows the returns and risks of the forest stands #162, #163, #165 and #173, and of the HEX stock market index on a yearly basis during the period 1985:10–96:12.

	#162	#163	#165	#173	HEX
Return, r_t	0.02	0.01	0.02	0.02	0.16
Growth return, g_t	0.03	0.04	0.04	0.04	0.00
Total return, z_t	0.06	0.05	0.06	0.06	0.16
Standard deviation	0.07	0.09	0.10	0.07	0.21

cause all the forest stands had low returns and risks compared with the stock market index. With the return level at 6%, a half a percent decrease in risk could be achieved if similar growing stock could be bought. The optimal asset mix in such a case is very different between the constrained solution and the unconstrained solution. While in the unconstrained case over half of stand #173 is cut down, in the constrained case it is left almost untouched. In the unconstrained case, more stands like #162 and #163 should be bought, but parts of stands #165 and #173 should be cut. It should be noted that even if the landowner prefers low return and risk, the optimal asset mix includes some equities. It is interesting to observe that even the returns and risks are the same in the constrained optimum and the unconstrained optimum, the optimal asset mix could be different. The same risks and returns can be achieved by two different kinds of portfolios.

TABLE 3. THE COVARIANCE MATRIX.
Table 3 presents the covariance matrix of the forest stands #162, #163, #165 and #173, and of the HEX stock market index on a monthly basis during the period 1985:10–96:12 (x100).

	#HEX	#162	#163	#165	#173
HEX	0.363	0.000	-0.001	-0.006	0.002
#162		0.038	0.041	0.045	0.040
#163			0.060	0.068	0.044
#165				0.081	0.048
#173					0.042

TABLE 4A. RESULTS — CONSTRAINED CASE.

Table 4a gives the portfolio return, standard deviation, and investment composition (%) in the real-life case based on the returns for the period 1985:10-1996:12: Constrained case.

Return	Risk	Stand #162	Stand #163	Stand #165	Stand #173	Stocks
%	%	%	%	%	%	%
6	1.8	15.6	5.2	6.6	70.6	2.0
9	2.3	15.6	5.2	6.6	40.3	32.3
12	3.8	0.0	0.0	6.6	31.7	61.7
15	5.5	0.0	0.0	0.0	8.1	91.9
Original weights		15.6	5.2	6.6	72.6	0.0

The cutting rules on constrained optimisation expressed in cubic meters for different levels of expected returns are as follows: the amount of timber to be cut at the 6% level of return from stand #173 equals ((0.726–0.706)/0.726)∗458=13 m^3 and at the 9% level it is ((0.726-0.403)/0.726)∗458=204 m^3. At the 12% level, stands #162 and #165 have to clear-cut, and the amount of timber to be cut from stand #173 would be ((0.726–0.317)/0.726)∗458=258 m^3. At the 15% level, stands #162, #163 and #165 would have to be clear-cut and the amount of timber to be cut from stand #173 would be ((0.726–0.081)/0.726)∗458=407 m^3.

TABLE 4B. RESULTS — UNCONSTRAINED CASE.

Table 4b gives the portfolio return, standard deviation, and investment composition (%) in the real-life case based on the returns for the period 1985:10-1996:12: Unconstrained case.

Return	Risk	Stand #162	Stand #163	Stand #165	Stand #173	Stocks
%	%	%	%	%	%	%
6	1.6	43.5	16.1	5.2	31.6	3.7
9	2.3	22.1	3.7	7.7	34.0	32.5
12	3.8	0.0	0.0	6.8	31.5	61.8
15	5.5	0.0	0.0	0.0	8.1	91.9
Original weights		15.6	5.2	6.6	72.6	0.0

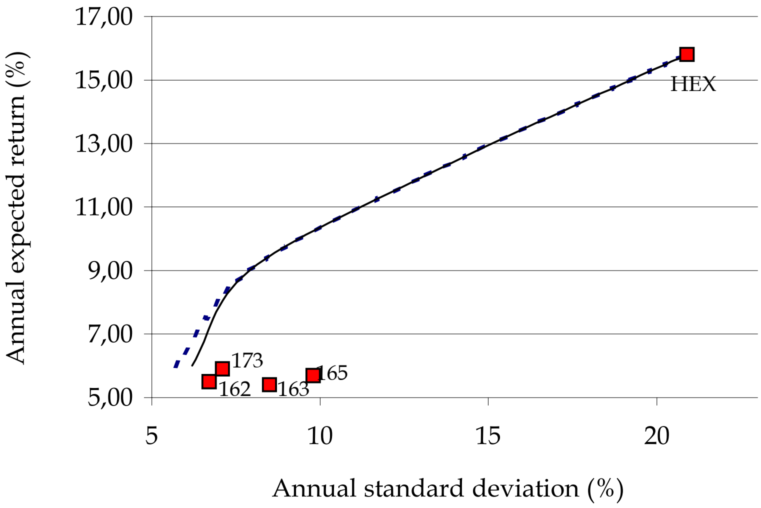

FIGURE 1. THE EFFICIENT FRONTIER.
The efficient frontier of portfolio optimisation in the unconstrained case (dotted line), and the case with a budget constraint for timber, and the returns and standard deviations for individual forest stands and the general stock market index HEX. Data covering the period 1985:10 – 1996:12.

CONCLUSIONS

In this paper we propose a portfolio model for a landowner willing to speculate between harvesting now and investing in financial assets or retaining the timber stock. First we studied how the value of a forest stand and the portfolio as a whole has to be calculated. We used the exponential growth assumption as a way of incorporating growth in the portfolio model. We found that the value of the final portfolio consisted of the following parts: return on financial asset, the price return on the growing stock, the physical-growth return, the opportunity cost of postponing harvesting by one period, and the land value of total land area. Using Faustmann's model, we found that the opportunity cost of postponing harvesting was very small when the rotation period was long and the interest rate was not extremely low. This can be thought to be the case at least in boreal coniferous forests (e.g., in Finland). In this case, the return on the timber stock can be approximated simply as the sum of the price return and physical-growth return. Otherwise, the opportunity cost should be taken into account.

Our portfolio optimisation model was otherwise the conventional Markowitz model, but we suggested that the buying constraint has to be set on the forest stands. This was due to the fact that it is not usually possible to buy exact duplicate of the kind of growing stock that one already has, because for this to be possible, the species composition, growth and volume would all have to be the same. Moreover, many countries lack a market for growing timber stocks. However, we did compare the solutions obtained both with and without the buying constraint. We applied our model to a case study of four real Finnish forest stands and the HEX stock market index. The cutting rules changed considerably if the buying constraint for timber was used. One explanation can be that the returns and risks of forest stands were very close to one another when compared with the alternative asset, HEX stock market index. The difference of the cutting rules could be smaller if there were several other investment alternatives such as bonds and interest-rate instruments.

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REFERENCES

Aarne, M.(ed.), 1994. *Yearbook of Forest Statistics 1993–94*. The Finnish Forest Research Institute.

Binkley, C.S., 1987. Economic Models of Timber Supply. In M. Kallio, D.P. Dykstra & C.S.Binkley (eds.). *The Global Forest Sector: An Analytical Perspective* (John Wiley & Sons).

Binkley, C.S., Raper C.F. & Washburn C.L., 1996. Institutional Ownership of US Timberland. History, Rationale and Implications for Forest Management. *Journal of Forestry,* 94(9): 21–28.

Collins, R.A. & Barry P.J., 1986. Risk Analysis with Single-index Portfolio Models: An Application to Farm Planning. *American Journal of Agricultural Economics,* 68:152–161.

Elton, E.J. & Gruber, M.J., 1995. *Modern Portfolio Theory and Investment Analysis.* Fifth edition (John Wiley & Sons, Inc).

Faustmann, M., 1849. Calculation of the Value which Forest Land and Immature Stands Possess for Forestry. Republished with permission from Commonwealth Forestry Association in *Journal of Forest Economics,* 1(1):7–44.

Forboseh, P.F., Brazee, R.J. & Pickens, J.B., 1996. A Strategy for Multiperiod Stand Management with Uncertain Future Prices. *Forest Science,* 42(1):58–66.

Haight, R.G., 1991. Stochastic Log Price, Land Value and Adaptive Stand Management: Numerical Results for California White Fir. *Forest Science,* 37(5):1224–1238.

Haight, R.G. & Smith, W.D., 1991. Harvesting Loblolly Pine Plantations with Hardwood Competition and Stochastic Prices. *Forest Science,* 37(5):1266–1282.

Hazell, P.B.R., 1971. A Linear Alternative to Quadratic and Semivariance Programming for Farm Planning under Uncertainty. *American Journal of Agricultural Economics,* 53:53–62.

Ingersoll, J., 1987. *Theory of Financial Decision Making* (Rowman & Littlefield).

Lausti, A. & Penttinen, M., 1998. The Analysis of Return and Its Components of Non-industrial Private Forest Ownership by Forestry Board Districts in Finland. *Silva Fennica*, 32(1):75–94.

Markowitz, H., 1952. Portfolio Selection. *The Journal of Finance,* 7:77–91.

Mills, W.L. & Hoover, W.L., 1982. Investment in Forest Land: Aspects of Risk and Diversification. *Land Economics*, 58(1): 33–51.

Ollikainen, M., 1996. *Essays on Timber Supply and Forest Taxation.* Government Institute for Economic Research. Helsinki.

Penttinen, M., Lausti A., Kasanen E. & Puttonen V., 1996. Risk and Returns in Forest Investments in Finland. *Liiketaloudellinen aikakauskirja,* 1996(1):111–124.

Redmond, C.H. & Cubbage, F.W., 1988. Portfolio Risk and Returns from Timber Asset Investment. *Land Economics,* 64(4):325–337.

Tahvonen, O. & Salo, S., 1999. Optimal Forest Rotation with In Situ Preferences. *Journal of Environmental Economics and Management,* 37:106–128.

Taylor, R.G. & Fortson J.C., 1991. Optimum Plantation Planting Density and Rotation Age Based on Financial Risk and Return. *Forest Science,* 37(3):886-902.

Thomson, T.A., 1991. Efficient combinations of timber and financial market investments in single-period and multiperiod portfolios. *Forest Science,* 37(2):461–480.

Tilli, T., 1996. *Valtakunnallisen puun hintasopimusjärjestelmän purkautumisen vaikutus metsäpääomaan sisältyvään riskiin: Puun ennustamaton hintavaihtelu riskin mittarina Suomessa vuosina 1985–91 ja 1991-93.* (In Finnish). Licenciate thesis, University of Helsinki.

Wagner, J.E. & Rideout, D.B., 1991. Evaluating Forest Management Investments: The Capital Asset Pricing and The Income Growth Model. *Forest Science*, 37(6):1591–1604.

Washburn C.L. & Binkley C.S., 1990. On The Use of Period-average Stumpage Prices to Estimate Forest Asset Pricing Models. *Land economics*, 66(4): 379–393.

Zhang, D. & Binkley, C.S., 1993.The Informational Efficiency of The Vancouver Logmarket and The Financial Risk of Holding Logs in Storage. *Canadian Journal of Forest Research,* 24: 550–557.