OPTIMAL HARVEST POLICY WITH FIRST-ORDER AUTOREGRESSIVE PRICE PROCESS

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ABSTRACT

The optimal harvest decision policy for even-aged stand management when timber price follows a first-order autoregressive process is investigated. It is proved that the expected present value of an even-aged stand at any age is an increasing and convex function of the timber price in the previous year, provided that the maximum age the stand is allowed to grow is sufficiently high. The optimal decision rule at each age depends on the current annual timber growth, the price autocorrelation coefficient, and the discount rate. A critical annual timber growth rate is defined by the timber price autocorrelation coefficient and the discount rate. When stand age is low such that the annual growth rate is higher than this critical rate, it is optimal either to wait independent of the observed price or to harvest the stand when the observed price is relatively low. At higher ages when the annual timber growth rate is lower than the critical rate, there exists an age-dependent reservation price and it is optimal to harvest when the observed price is equal to or greater than the reservation price. The optimal harvest policy when timber price process is random walk has similar properties. A simulation method for determining the optimal decision rules is developed. The effects of price autocorrelation coefficient on the optimal harvest policy and on the expected gain of adaptive decision making are examined using an example.

Keywords: Decision analysis, even-aged stand, reservation price, stochastic optimization, uncertainty.

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INTRODUCTION

Interest in determining adaptive timber harvest policies has increased greatly in the last decade. Traditionally, timber harvest decision analysis aims at determining the optimal harvest schedule or cutting budget based on the assumption that future stand (forest) states and timber prices are known with certainty. On the other hand, it is a well-known fact that stand growth (timber yield) is subject to random variations and timber prices fluctuate unpredictably from year to year. Recognizing that the decision at each time point can be made based on the realized state of nature at

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that time point, stochastic (adaptive) optimization models have been constructed and applied to determine optimal harvest policies at the stand-level (Norstrom, 1975; Lohmander, 1987; Brazee & Mendelsohn, 1988; Haight & Holmes, 1991; Forboseh *et al.,* 1996; Gong, 1998) as well as at the forest-level (Hoganson & Rose, 1987; Gassmann, 1989; Gong, 1994). An adaptive harvest decision model recognizes that the forest (stand) state and/or timber prices in future periods are stochastic, and determines a set of decision rules which specify the optimal harvest levels corresponding to different states of nature. Experience shows that it is much more difficult to formulate and solve an adaptive decision model than a deterministic model. However, the expected gain of implementing the adaptive harvest policy (or the cost of ignoring uncertainty) is substantial.

A relatively well-developed adaptive harvest decision model is the reservation price model which determines the minimum acceptable prices for harvesting an even-aged stand at different ages. Initially, the reservation price model recognizes a single product (timber assortment) and assumes that the decision-maker is risk-neutral (Lohmander, 1987; Brazee & Mendelsohn, 1988). The model has been extended to incorporate multiple products (Forboseh *et al.,* 1996) and risk–aversion (Gong, 1998). The reservation price model assumes that timber prices in different decision periods are independent and identically distributed (iid). Under this assumption, the expected present value (EPV) of the stand, if it is not harvested now, is constant (is independent of the current price). On the other hand, the profit from harvesting the stand now increases when the current price increases. Therefore, it is reasonable to harvest the stand if price is high and wait if price is low.

In cases where the assumption of iid prices is not satisfied, it is not sure that the reservation price strategy is optimal, however. Suppose that prices in successive years are positively correlated, for example. Then, when the current price increases, both the profit from harvesting the stand now and the EPV of the stand in the next year increase. It might be possible that the latter increases faster than the former and thus it might be optimal to wait when current price is high. Clarke & Reed (1989) show that, if timber growth is purely age-dependent and price process is geometric Brown motion, the optimal decision rule under timber growth and price uncertainties is to harvest an even–aged stand at a fixed age. However, they pointed out that this is no longer the case if, for example, fixed costs of postponing harvest were recognized. The analysis by Yin & Newman (1995) indicate that, when price process is geometric Brown motion, it is optimal to harvest if price is low and wait if price is high.

The optimal harvest policy depends on the price process (for a numerical example, see e.g. Haight & Holmes, 1991). It should be emphasized that the future price process is determined by many factors that evolve over time, such as the age-distribution of the forest, timber demand, and forest owners' behavior (harvest policies). The age-distribution of the forest as well as timber demand is likely to change over time, implying that the future price process may differ from the past price process. The interactions between timber price process and the optimal harvest policy make it even more difficult (if possible) to verify which stochastic process best represents the price movements in the future. Anyway, it is widely accepted that future timber prices are uncertain. From a forest owner's point of view, it is reasonable to predict future prices based on the past timber price series.

Previous studies show that timber prices in Sweden can be better described by an AR(1) model (Lohmander, 1987; Hultkrantz, 1995). Several numerical analyses using discrete-state stochastic dynamic programming method indicate that, in cases in which the price process is a stationary AR(1), there exists a reservation price associated to each age such that it is optimal to harvest if and only if the observed price is equal to or higher than the age specific reservation price (Lohmander, 1987; Haight & Holmes, 1991). Yet there is no theoretical proof that such a harvest strategy is optimal in general cases when timber price follows a stationary AR(1) process. Moreover, the method used in previous studies to determine the optimal harvest policy with autoregressive prices recognizes only a finite number of possible price levels.

The purpose of this study is to investigate the properties of the optimal harvest policy for even-aged stand management when timber price follows an AR(1) process, and to develop a method for numerically determining the optimal harvest policy using continuous distributions of the prices at different decision time points.

THE OPTIMAL HARVEST POLICY

We consider the optimal policy for harvesting an existing stand, assuming that the value of bare land is known with certainty and the objective is to maximize the expected present value (EPV) of the stand. Future timber prices follow a stationary AR(1) process,

$$p_t = \alpha + \beta p_{t-1} + \varepsilon(t), \tag{1}$$

where α and β ($\alpha > 0$, $0 < \beta < 1$) are constants, $\varepsilon(t)$, $t = 1, \ldots$, are independent and identically distributed random variables. Assume that $\varepsilon(t)$ has a doubly truncated normal distribution[1] with zero mean and the following probability density function.

$$\phi(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\varepsilon^2}{2\sigma^2}\right)\left[\int_{-\xi}^{\xi}\frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{x^2}{2\sigma^2})dx\right]^{-1},$$
$$-\xi \le \varepsilon \le \xi. \tag{2}$$

Given the price at age $t-1$, the price at age t is also a doubly truncated normal variable, and its probability density function is

$$f\left(p_t \middle| p_{t-1}\right) = \psi(\xi)\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left(p_t - \alpha - \beta p_{t-1}\right)^2}{2\sigma^2}\right),$$
$$A\left(p_{t-1}\right) \le p_t \le B\left(p_{t-1}\right),$$

where $A(p_{t-1}) = \alpha + \beta p_{t-1} - \xi$ and $B(p_{t-1}) = \alpha + \beta p_{t-1} + \xi$ are the lower and upper bounds of p_t conditional on p_{t-1}, and

[1] The doubly truncated distribution is used for generality. The truncation points do not affect the analytical results and thus the analysis that follows applies to untruncated normal distributions.

$$\psi(\xi)=\left[\int_{-\xi}^{\xi}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right)dx\right]^{-1}.$$

Suppose that the permissible harvest age ranges from t^0 to T. Once the stand reaches age T, it will be harvested immediately. At any age $t^0 \le t < T$, one can either accept the prevailing price and harvest the stand or reject the price and postpone the decision until the next year. Like in the case with iid prices, the decision at each age involves a comparison between the revenue of harvesting the stand now and the EPV of the stand if it is harvested later. Given a price at an age t, if the net revenue of harvesting at age t is equal to or larger than the EPV of the stand when harvested later, then the optimal choice is to harvest the stand at age t. Otherwise, it is optimal to wait. Therefore, the optimal decision rule at any age $t^0 \le t < T$ can be described by the set of prices acceptable for harvesting, i.e. the price interval(s) within which the net revenue from harvesting is equal to or greater than the EPV of the stand when harvested later.

Unlike in the case with iid prices, the price model (1) implies that the EPV of the stand when it is harvested later increases as the current price increases. Let $W(t+1,p_t)$ denote the EPV of the stand at age t+1 conditional on the price at age t. The optimal harvest rule at age t depends on whether $W(t+1, p_t)$ is concave or convex in p_t and how fast $W(t+1, p_t)$ increases when p_t increases. Thus, to characterize the optimal harvest rule, we should examine the first and second order derivatives of $W(t+1,p_t)$ with respect to p_t at different ages.

Let us consider the decision at age T–1. The choice is between harvesting the stand at age T–1 or at age T, since the stand is not allowed to grow over age T. Let $V(t)$ denote the per unit area volume of timber at age t, L be the value of bare land, r be the discount rate and $\delta = 1/(1+r)$ the discounting factor. Knowing the price of timber at age T–1, the EPV of the stand at age T is

$$W\left(T,p_{T-1}\right)=\left(\alpha+\beta p_{T-1}\right)V(T)+L. \quad (3)$$

If the stand is harvested at age T–1, the net revenue is

$$\Pi\left(T-1, p_{T-1}\right)=p_{T-1} V(T-1)+L. \tag{4}$$

Given a p_{T-1}, the expected gain from harvesting the stand at age T–1 is

$$\begin{aligned} G\left(T-1, p_{T-1}\right) &= \Pi\left(T-1, p_{T-1}\right)-\delta W\left(T, p_{T-1}\right) \\ &= p_{T-1}\left[V(T-1)-\delta \beta V(T)\right]-\delta \alpha V(T)+L(1-\delta). \end{aligned}$$

Assume that $V(T-1) > \delta\beta V(T)$ (which is satisfied when T is sufficiently large), then the expected gain $G(T-1, p_{T-1}) \geq 0$ if $p_{T-1} \geq p^*_{T-1}$ where

$$p_{T-1}^{*}=\frac{\delta \alpha V(T)-L(1-\delta)}{V(T-1)-\delta \beta V(T)}. \tag{5}$$

Therefore, the optimal price interval for harvesting at age T–1 is $[p^*_{T-1}, +\infty]$. In other words, it is optimal to harvest the stand at age T–1 if the observed price $p_{T-1} \geq p^*_{T-1}$, and to wait one year and harvest the stand at age T if $p_{T-1} < p^*_{T-1}$. The decision rule is similar as in situations in which prices in different periods are independent and identically distributed, although the reservation price may be different.

Based on the decision rule at age T–1, we can determine the EPV of the stand at age T–1 (conditional on the price at age T–2) and the decision rule at age T–2. Given a price at age T–2, the EPV of the stand at age T–1 is

$$\begin{aligned} W\left(T-1, p_{T-2}\right)= & \int_{A\left(p_{T-2}\right)}^{p_{T-1}^{*}} \delta W\left(T, p_{T-1}\right) f\left(p_{T-1} \mid p_{T-2}\right) d p_{T-1} \\ & +\int_{p_{T-1}^{*}}^{B\left(p_{T-2}\right)}\left[p_{T-1} V(T-1)+L\right] f\left(p_{T-1} \mid p_{T-2}\right) d p_{T-1}. \end{aligned}$$

Using Equation (3) and the reservation price function (5), the EPV can be expressed as

$$\begin{aligned} W\left(T-1, p_{T-2}\right)= & \left(\alpha+\beta p_{T-2}\right) V(T-1)+L \\ & +\left[V(T-1)-\delta \beta V(T)\right]\left\{\left[\phi(z)-\phi(\xi)\right] \sigma^{2}+z \Phi(z)\right\}, \end{aligned}$$

where $z = (p^*_{T-1} - \alpha - \beta p_{T-2})$, $\phi(z)$ is the probability density at z (see Equation (2)), and

$$\Phi(z) = \int_{-\xi}^{z} \phi(\varepsilon) d\varepsilon$$

is the accumulative distribution function of the random error term in the price model (1). The first-order derivative of $W(T-1, p_{T-2})$ with respect to p_{T-2} is

$$W'(T-1, p_{T-2}) = \beta V(T-1) - \left[V(T-1) - \delta\beta V(T)\right]\beta\Phi(z). \quad (6)$$

Given that $V(T-1) \geq \delta\beta V(T)$, it can be seen from (6) that

$$\delta\beta^2 V(T) \leq W'(T-1, p_{T-2}) \leq \beta V(T-1). \quad (6')$$

The second-order derivative is

$$W''(T-1, p_{T-2}) = \beta^2 \phi(z)\left[V(T-1) - \delta\beta V(T)\right].$$

It follows from the probability density function (2) that

$$W''(T-1, p_{T-2}) > 0$$
$$\text{if } \frac{\left(p^*_{T-1} - \alpha - \xi\right)}{\beta} \leq p_{T-2} \leq \frac{\left(p^*_{T-1} - \alpha + \xi\right)}{\beta},$$
$$W''(T-1, p_{T-2}) = 0$$
$$\text{if } p_{T-2} < \frac{\left(p^*_{T-1} - \alpha - \xi\right)}{\beta} \quad \text{or} \quad p_{T-2} > \frac{\left(p^*_{T-1} - \alpha + \xi\right)}{\beta}.$$

Thus, $W(T-1, p_{T-2})$ is an increasing and convex function of p_{T-2}.

The expected gain of harvesting the stand at age $T-2$, given a price p_{T-2}, is

$$G(T-2, p_{T-2}) = p_{T-2} V(T-2) + L - \delta W(T-1, p_{T-2}).$$

$G(T-2, p_{T-2})$ is a concave function of p_{T-2} because $W(T-1, p_{T-2})$ is convex in p_{T-2}. This implies that, theoretically, there are five possible cases concerning the set of prices within which $G(T-2, p_{T-2}) \geq 0$ (i.e. when it is optimal to harvest the stand at age $T-2$):

(*i*) It is empty.

(*ii*) It is equal to the set of the possible prices at age $T-2$.

(*iii*) It consists of an upper part of the possible price interval.

(*iv*) It consists of a lower part of the possible price interval.

(*v*) It consists of an interval somewhere between the minimum and maximum possible prices.

Lohmander (1987, Appendix M1) depicted and explained the first four cases[2]. Case (*v*) comes from the fact that the EPV of the stand when it is harvested later, $W(T-1, p_{T-2})$, is convex in the price at age $T-2$. Let $\Pi(T-2, p_{T-2}) = p_{T-2}V(T-2) + L$ denote the net revenue from harvesting now. Both $\Pi(T-2, p_{T-2})$ and $\delta W(T-1, p_{T-2})$ increase when the current price p_{T-2} increases. However, it is possible that $\Pi(T-2, p_{T-2})$ increases faster than $\delta W(T-1, p_{T-2})$ does when p_{T-2} is low, but $\delta W(T-1, p_{T-2})$ increases faster than $\Pi(T-2, p_{T-2})$ does when p_{T-2} is high. Hence, it might be possible that the expected gain of harvesting now is greater than zero only if the current price is not too low and not too high (see Figure 1). If the price is very high, the expected price next year is also high and it would be better to harvest next year, provided that the stand is growing sufficiently fast. If the current price is very low, the stumpage value and thus the opportunity cost of waiting are low. Under certain circumstances, it is optimal to wait one or two years.

Recognizing the five possible cases, the optimal decision rule at age $T-2$ can be generally described as to harvest the stand if and only if the observed price $p_{T-2} \in \left[p^1_{T-2}, p^2_{T-2}\right]$, where p^1_{T-2} and p^2_{T-2} denote the minimum and maximum prices acceptable for harvesting (see Table 1). In what follows, I will first prove that this general decision rule is optimal at the other decision time points, and then discuss which of the five cases is more likely to be true at different ages.

[2] Lohmander described Cases (*i*), (*ii*) and (*iv*) as the results of nonstationary price process. However, each of these cases may be valid when price process is a stationary AR(1).

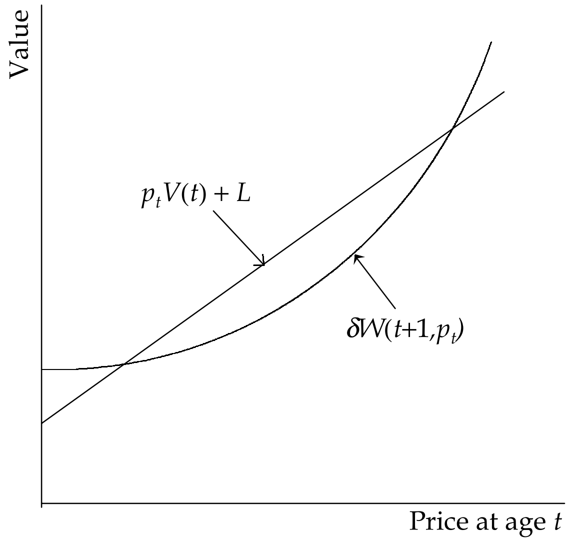

FIGURE 1. THE EXISTENCE OF AN OPTIMAL RESERVATION PRICE INTERVAL.

Assume that $W(t+1,p_t)$, $t^0<t\leq T-2$, increases when p_t increases and is convex in p_t. Then, the expected gain of harvesting at age t, $G(t,p_t) = p_t V(t) + L - \delta W(t+1, p_t)$, is concave in p_t. Like the decision rule at age T−2, the optimal decision rule at age t can be expressed as to harvest the stand if and only if the observed price $p_t \in [\, p_t^1, p_t^2 \,]$. The EPV of the stand at age t conditional on the price at age t−1 is

TABLE 1. FIVE POSSIBLE CASES OF THE OPTIMAL DECISION RULE.

Case	Reservation prices ($p_{T-2}^1 \leq p_{T-2}^2$)	Decision
i	$p_{T-2}^1 \geq B$ or $p_{T-2}^2 \leq A$	Wait
ii	$p_{T-2}^1 \leq A$ and $p_{T-2}^2 \geq B$	Harvest
iii	$p_{T-2}^1 \in (A,B)$ and $p_{T-2}^2 \geq B$	Harvest if $p_{T-2} \geq p_{T-2}^1$, wait if $p_{T-2} < p_{T-2}^1$
iv	$p_{T-2}^1 \leq A$ and $p_{T-2}^2 \in (A,B)$	Harvest if $p_{T-2} \leq p_{T-2}^2$, wait if $p_{T-2} > p_{T-2}^2$
v	$p_{T-2}^1 \in (A,B)$ and $p_{T-2}^2 \in (A,B)$	Harvest if $p_{T-2}^1 \leq p_{T-2} \leq p_{T-2}^2$, wait otherwise

A and *B* are the lower and upper limits of the possible price, respectively.

$$W(t,p_{t-1}) = \int_{A(p_{t-1})}^{p_t^1} \delta W(t+1,p_t) f(p_t|p_{t-1})\, dp_t + \int_{p_t^1}^{p_t^2} [p_t V(t) + L] f(p_t|p_{t-1})\, dp_t + \int_{p_t^2}^{B(p_{t-1})} \delta W(t+1,p_t) f(p_t|p_{t-1})\, dp_t.$$

Consider the case when $A(p_{t-1}) < p_t^1 < p_t^2 < B(p_{t-1})$, that is, harvesting at age t is optimal if the observed price is not very low and not very high. In this case, we have $G(t, p_t^1) = G(t, p_t^2) = 0$, and because $G(t, p_t)$ is concave in p_t, $G'(t, p_t^1) > 0$ and $G'(t, p_t^2) < 0$. Let $z^1 = (p_t^1 - \alpha - \beta p_{t-1})$ and $z^2 = (p_t^2 - \alpha - \beta p_{t-1})$. As p_t is normally distributed with mean $(\alpha + \beta p_{t-1})$, $W(t,p_{t-1})$ can equivalently be expressed as

$$W(t,p_{t-1}) = \int_{-\xi}^{z^1} \delta W(t+1,\alpha + \beta p_{t-1} + z)\phi(z)\, dz + \int_{z^1}^{z^2} [(\alpha + \beta p_{t-1} + z)V(t) + L]\phi(z)\, dz + \int_{z^2}^{\xi} \delta W(t+1,\alpha + \beta p_{t-1} + z)\phi(z)\, dz.$$

The first order derivative of $W(t, p_{t-1})$ with respect to p_{t-1} is

$$\frac{dW(t,p_{t-1})}{dp_{t-1}} = \frac{\partial W(t,p_{t-1})}{\partial p_{t-1}} + \frac{\partial W(t,p_{t-1})}{\partial z^1}\frac{dz^1}{dp_{t-1}} + \frac{\partial W(t,p_{t-1})}{\partial z^2}\frac{dz^2}{dp_{t-1}}.$$

Note that

$$\frac{\partial W(t,p_{t-1})}{\partial z^1} = \left[\delta W(t+1,p_t^1) - p_t^1 V(t) - L\right]\phi(z^1) = -G(t,p_t^1)\phi(z^1) = 0.$$

Similarly, we have

$$\frac{\partial W(t,p_{t-1})}{\partial z^2} = \left[-\delta W(t+1,p_t^2)+p_t^2 V(t)+L\right]\phi(z^2)$$
$$= G(t,p_t^2)\phi(z^2) = 0.$$

Therefore,

$$\frac{dW(t,p_{t-1})}{dp_{t-1}} = \frac{\partial W(t,p_{t-1})}{\partial p_{t-1}}$$
$$= \int_{-\xi}^{z^1} \delta \frac{dW(t+1,\alpha+\beta p_{t-1}+z)}{d(\alpha+\beta p_{t-1}+z)} \beta\phi(z)dz$$
$$+ \int_{z^1}^{z^2} \beta V(t)\phi(z)dz$$
$$+ \int_{z^2}^{\xi} \delta \frac{dW(t+1,\alpha+\beta p_{t-1}+z)}{d(\alpha+\beta p_{t-1}+z)} \beta\phi(z)dz. \tag{7}$$

where $dW(t+1, \alpha+\beta p_{t-1}+z)/d(\alpha+\beta p_{t-1}+z) = dW(t+1, p_t)/dp_t > 0$. Thus, $dW(t, p_{t-1})/dp_{t-1} > 0$.

From Equation (7), we get the second-order derivative of $W(t, p_{t-1})$ with respect to p_{t-1}.

$$\frac{d^2W(t,p_{t-1})}{d(p_{t-1})^2} = \int_{-\xi}^{z^1} \delta \frac{d^2W(t+1,\alpha+\beta p_{t-1}+z)}{d(\alpha+\beta p_{t-1}+z)^2} \beta^2\phi(z)dz$$
$$+\beta^2\left[G'(t,p_t^1)\phi(z^1) - G'(t,p_t^2)\phi(z^2)\right]$$
$$+ \int_{z^2}^{\xi} \delta \frac{d^2W(t+1,\alpha+\beta p_{t-1}+z)}{d(\alpha+\beta p_{t-1}+z)^2} \beta^2\phi(z)dz$$

Since $W(t+1, p_t)$ is convex in p_t, $G'(t, p_t^1) > 0$ and $G'(t, p_t^2) < 0$, we have $d^2W(t, p_{t-1})/d(p_{t-1})^2 > 0$. Therefore, $W(t, p_{t-1})$ is an increasing and convex function of p_{t-1}.

In a similar way, one can prove that $W(t, p_{t-1})$ is an increasing and convex function of p_{t-1} with each of the other possible cases of the optimal decision rule at age t. Therefore, if $W(t+1, p_t)$, $t^0 < t \leq T-2$, increases when p_t

increases and is convex in p_t, then the EPV of the stand at age t, $W(t, p_{t-1})$, is an increasing and convex function of the price at age t–1. We have proved that $W(T-1, p_{T-2})$ is an increasing and convex function of p_{T-2}. Thus, one can conclude that the EPV of the stand at any age $t \in [t^0+1, T-1]$, $W(t, p_{t-1})$, is an increasing and convex function of the price in the previous year p_{t-1}.

Therefore, the expected gain of harvesting the stand at any age $t \in [t^0, T-2]$ is concave in p_t. This means that the optimal decision rule at any age $t \in [t^0, T-2]$ is, in general, to harvest the stand if and only if the observed price $p_t \in [p_t^1, p_t^2]$, where p_t^1 and p_t^2 are the lower and upper reservation prices associated with age t.

It was pointed out that the reservation price interval is a general expression of five decision rules (see Table 1). Which of the decision rules is optimal depends on timber growth, the discount rate, the bare land value, the price model coefficients (α and β), and the possible range of random price variations. However, it is possible to show under what conditions each of the decision rules may be optimal. In fact these conditions can be described as whether $V(t)$ is greater, equal to, or smaller than $\delta\beta V(t+1)$, or equivalently whether the annual timber growth rate at age t is smaller, equal to, or greater than $(1 + r - \beta)/\beta$.

Let $g(t)$ denote the annual timber growth rate at age t. Assume that the timber yield function $V(t)$ is concave. $g(t)$ decreases as the stand age increases. Let $\bar{t}$ denote the stand age at which the annual timber growth rate equals $(1 + r - \beta)/\beta$. Then, $g(t) > (1 + r - \beta)/\beta$ and $V(t) < \delta\beta V(t+1)$ for $t < \bar{t}$; and $g(t) < (1 + r - \beta)/\beta$ and $V(t) > \delta\beta V(t+1)$ at any age $t > \bar{t}$.

First, let us consider the optimal decision rule at age $t > \bar{t}$. From the inequalities (6′) and Equation (7) it can be shown that, $dW(t+1, p_t)/dp_t \le \beta V(t+1)$ and hence

$$\frac{dG(t,p_t)}{dp_t} = V(t) - \delta\frac{dW(t+1,p_t)}{dp_t} \ge V(t) - \delta\beta V(t+1) > 0.$$

The expected gain of harvesting at age t increases when p_t increases. Note that

$$G(t,p_t)=p_tV(t)+L-\delta W(t+1,p_t)$$
$$\leq p_tV(t)+L-\delta\left[(\alpha+\beta p_t)V(t+1)+L\right]. \qquad (8)$$

Obviously, $G(t, p_t) < 0$ when $p_t < [\delta\alpha V(t+1) - L(1-\delta)]/(V(t) - \delta\beta V(t+1))$. Thus, at any age $t > \bar{t}$ the equation $G(t, p_t) = 0$ has only one solution p_t^1, and the optimal decision rule is to harvest the stand at age t if and only if the observed price $p_t \geq p_t^1$ (Case *iii*). However, it is possible that the optimal reservation price p_t^1 is smaller than the minimum possible price, then the stand should be harvested with probability 1 (Case *ii*). Also, it may be possible that p_t^1 is greater than the maximum possible price, and then the stand should never be harvested at this age (Case *i*).

If $\bar{t}$ is an integer and $\bar{t} \geq t^0$, a decision should also be made at age $\bar{t}$. The fact that $g(\bar{t}) = (1+r-\beta)/\beta$ implies $V(\bar{t}) = \delta\beta V(\bar{t}+1)$, $V(\bar{t}+1) > \delta\beta V(\bar{t}+2)$, and consequently $V(\bar{t}) > (\delta\beta)^2 V(\bar{t}+2)$. Note that

$$W(\bar{t}+1,p_{\bar{t}})\geq\delta\left[\left(\alpha+\beta\left(\alpha+\beta p_{\bar{t}}\right)\right)V(\bar{t}+2)+L\right],$$

where the right-hand-side is the expected revenue (discounted to age $\bar{t}+1$) if the stand is harvested at age $\bar{t}+2$. Therefore,

$$\begin{aligned}G(\bar{t},p_{\bar{t}})&\leq p_{\bar{t}}V(\bar{t})+L\\&\quad-\delta^2\left[\left(\alpha+\beta\left(\alpha+\beta p_{\bar{t}}\right)\right)V(\bar{t}+2)+L\right]\\&=p_{\bar{t}}\left[V(\bar{t})-(\delta\beta)^2V(\bar{t}+2)\right]\\&\quad-\delta^2(\alpha+\alpha\beta)V(\bar{t}+2)+L\left(1-\delta^2\right).\end{aligned}$$

Since $V(\bar{t}) > (\delta\beta)^2 V(\bar{t}+2)$, we have $G(\bar{t},p_{\bar{t}}) < 0$ if

$$p_{\bar{t}}<p^l(\bar{t})=\frac{\delta^2(\alpha+\alpha\beta)V(\bar{t}+2)-L\left(1-\delta^2\right)}{V(\bar{t})-(\delta\beta)^2V(\bar{t}+2)}.$$

In words, when the stand age equals $\bar{t}$, it is optimal to wait if the observed price is low. Because $V(\bar{t}+1) >$

$\delta\beta V(\bar{t}+2)$, we have $dW(\bar{t}+1, p_{\bar{t}})/dp_{\bar{t}} \leq \beta V(\bar{t}+1)$ and thus

$$\frac{dG(\bar{t}, p_{\bar{t}})}{dp_{\bar{t}}} \geq V(\bar{t}) - \delta\beta V(\bar{t}+1) = 0.$$

Since $G(\bar{t}, p_{\bar{t}})$ is concave in $p_{\bar{t}}$, $dG(\bar{t}, p_{\bar{t}})/dp_{\bar{t}} > 0$ when $p_{\bar{t}}$ is low and it approaches to zero when $p_{\bar{t}}$ is extremely high. Therefore, $G(\bar{t}, p_{\bar{t}})$ increases as $p_{\bar{t}}$ increases from $p^l(\bar{t})$. If $G(\bar{t}, p_{\bar{t}})$ reaches 0 at a $p^1_{\bar{t}}$, then it is optimal to harvest the stand when the observed price is equal to or greater than $p^1_{\bar{t}}$ (Case *iii* or *ii*, depending on whether $p^1_{\bar{t}}$ is greater or smaller than the minimum possible price at age $\bar{t}$). If the reservation price $p^1_{\bar{t}}$ is greater than the maximum possible price, or if the reservation price does not exist (i.e. there is no $p^1_{\bar{t}}$ such that $G(\bar{t}, p^1_{\bar{t}}) = 0$), then it is optimal to wait and harvest the stand at a higher age (case *i*).

At any age $t < \bar{t}$, $g(t) > (1 + r - \beta)/\beta$ and $V(t) < \delta\beta V(t+1)$. From the inequality (8) we have $G(t, p_t) < 0$ if $p_t > p^l(t) = [\delta\alpha V(t+1) - L(1-\delta)]/(V(t) - \delta\beta V(t+1))$, which implies that, when the stand age $t < \bar{t}$, the optimal decision is to wait if the observed price is high. Depending on the timber yield function, the price model, bare land value and the discount rate, $p^l(t)$ may lie outside the range of possible prices. Then, the optimal decision is to wait independent of what the observed price is (Case *i*). Suppose that $p^l(t)$ is larger than the minimum possible price. Whether it is optimal to harvest the stand when the observed price is lower than $p^l(t)$ depends on the bare land value and the maximum number of years one can wait. The optimal decision rule might be to harvest the stand when the observed price is relatively low (Case *iv* or *v*), or to wait independent of what the observed price is (Case *i*).

Results from the foregoing analysis are summarized as the follow. With a stationary AR(1) price process and a given range of permissible harvest age $[t^0, T]$, the EPV of an even-aged stand at any age $t^0 < t < T$ conditional on the price at age $t-1$, p_{t-1}, is an increasing and convex function of p_{t-1}, provided that the maximum age T the stand is allowed to grow is sufficiently high. The optimal decision rule depends on the age of the stand. When stand age is high such that the current annual growth rate $g(t) < (1 + r -$

β)/β, it is optimal to harvest the stand if and only if the observed price is equal to or higher than the optimal reservation price. An optimal reservation price may or may not exist for the age t at which $g(t) = (1 + r - \beta)/\beta$. If the optimal reservation price exists, it is optimal to harvest when the observed price is equal to or higher than the reservation price. Otherwise, the stand should not be harvest at this age. At lower ages at which the annual timber growth rate is high (i.e., when $g(t) > (1+r - \beta)/\beta$), the optimal decision rule is either to harvest the stand when price is relatively low or to wait no matter what the observed price is.

It is worth pointing out that the analysis presented above is valid even in situations in which $\alpha = 0$ and $\beta = 1$ (i.e. when timber price process is random walk). In this case, the EPV of the stand at any age $t^0 < t < T$ conditional on the price at age $t - 1$, p_{t-1}, is an increasing and convex function of p_{t-1}, given that the maximum age T the stand is allowed to grow is sufficiently high. The reservation price rule is optimal at high ages when the annual timber growth rate is lower than the discount rate. When the stand age is low and the annual timber growth rate is equal to or higher than the discount rate, the optimal decision rule may vary from case to case.

The analysis with a stationary AR(1) price process can also be generalized to the case when $\beta = 0$ (i.e. when timber prices in different years are independent and identically distributed). In this case it can be seen from Equations (6) and (7) that $dW(t+1, p_t)/dp_t = 0$ for $t = t^0, \ldots, T-2$. Thus, the expected gain of harvesting at any age $t^0 \leq t < T$, $Q(t, p_t) = p_t V(t) + L - \delta W(t+1, p_t)$, increases monotonously as price p_t increases, and the optimal decision rule is to harvest if and only if the observed price is equal to or greater than the reservation price.

NUMERICAL METHODS FOR DETERMINING THE OPTIMAL HARVEST POLICY

The optimal decision rule at each age t, and thus the optimal harvest policy is defined in terms of solution(s) to the equation $p_t V(t) + L - \delta W(t+1, p_t) = 0$. In situations in which prices in different periods are independent and identically distributed, one can calculate $W(t+1, p_t)$ analytically or using numerical integration methods and solve this equation recursively for $t = T-1, \ldots, t^0$ to determine the optimal reservation prices at different ages (see, e.g., Brazee & Mendelsohn, 1988; Lohmander, 1987). When timber prices are autocorrelated, $W(t+1, p_t)$ depends on p_t and is a multidimensional integral (except when $t = T-1$). It is impossible to solve the multidimensional integral to calculate $W(t+1, p_t)$ and determine the price interval within which harvesting is optimal for each age $t \in [t^0, T-1]$. In this case, the optimal harvest policy can be determined using discrete-state stochastic dynamic programming. Alternatively, one can use a simulation method to estimate $W(t+1, p_t)$ based on the optimal decision rules at ages $t+1$ to T and determine the optimal price interval for each age backwards from age $T-1$.

Stochastic Dynamic Programming

If the possible timber price at each decision point in time is described by a finite number of price states, then discrete-state stochastic dynamic programming can be used to determine the optimal decision for each possible price state at each age (Norstrom, 1975; Lohmander, 1987; Haight & Holmes, 1991). Let $p(k)$, $k = 1, \ldots, S$, be the average price in price state k. Define $prob(k, j)$ as the price state transition probability, i.e., the probability of being in price state j at age $t+1$ given that the price state at age t is k. Let $Z_t(k)$ denote the EPV of the stand at age t in price state k. Given a price state k at age t, if the stand is harvested at age t, the net revenue obtained from harvesting is $\Pi\left[t, p(k)\right] = p(k)V(t) + L$.

If the stand is not harvested, the EPV of the stand at age $t+1$ is

$$W\left[t+1, p(k)\right] = \sum_{j=1}^{S} prob(k,j) Z_{t+1}(j). \tag{9a}$$

If $\Pi[t, p(k)] \geq \delta W[t+1, p(k)]$, then it is optimal to harvest the stand at age t, otherwise it is optimal to wait. Thus, the EPV of the stand at age t in price state k is

$$Z_t(k) = \max\left\{\Pi\left[t, p(k)\right], \delta W\left[t+1, p(k)\right]\right\}. \tag{9b}$$

Since the maximum rotation length is T, the EPV at age T given price state k at age $T-1$ is

$$W\left[T, p(k)\right] = \sum_{j=1}^{S} prob(k,j)\left[p(j)V(T) + L\right]. \tag{10}$$

Using the boundary condition [Equation (10)], one can solve Equations (9b) and (9a) from age T–1 backward to t^0. The optimal solutions give the EPV, $Z_t(k)$, and the optimal decision associated to each possible price state at each age (i.e. the optimal harvest policy).

The advantage of stochastic dynamic programming is that it is straightforward. For each price state at age t, the net revenue from harvesting and the EPV of the stand if it is not harvested at age t are calculated. If the former is equal to or greater than the latter, then it is optimal to harvest the stand. Otherwise, the optimal decision is to wait one more year. The disadvantage is that only a finite number of discrete price levels can be recognized. This would lead to biased estimation of the EPV associated to very low and very high prices, especially when the range of possible timber price and the number of decision periods are large. At low stand ages, the bias in the EPV could be large enough such that it leads to erroneous decisions for the extreme price states.

Simulation Method

The simulation method estimates the EPV of the stand at each age t conditional on the price at age t–1, $W(t, p_{t-1})$, for different p_{t-1}, using simulated price scenarios, and then determines the price interval within which it is optimal to harvest the stand at age t–1 by finding the solution(s) to the equation $p_{t-1}V(t-1)+L-\delta W(t,p_{t-1})=0$.

Let $P_t^1=\left[p_t^1,p_{t+1}^1,\ldots,p_T^1\right]$ and $P_t^2=\left[p_t^2,p_{t+1}^2,\ldots,p_T^2\right]$ be two vectors representing the minimum and the maximum prices acceptable for harvesting at different ages from t to T. Let $P_t^k(p_{t-1})=\left[p_t^k(p_{t-1}),p_{t+1}^k(p_{t-1}),\ldots,p_T^k(p_{t-1})\right]$ be a random timber price series (price scenario) from age t to T, simulated using the price model (1) with an initial price p_{t-1} at age t–1. Suppose that P_t^1 and P_t^2 are known, then given a price scenario $P_t^k(p_{t-1})$, the age at which the stand should be harvested is

$$t_k=\min\left\{a:p_a^1\le p_a^k(p_{t-1})\le p_a^2 \text{ and } t\le a\le T\right\},$$

and the present value at age t associated with price scenario $P_t^k(p_{t-1})$ is

$$R\left[P_t^1,P_t^2,P_t^k(p_{t-1})\right]=\left\{p_{t_k}^k(p_{t-1})V(t_k)+L\right\}\delta^{(t_k-t)}.$$

Given a price p_{t-1} at age t–1, the EPV of the stand at age t can be estimated by taking the average of the present values associated with a larger number of random price scenarios.

$$W(t,p_{t-1}) = \frac{1}{N}\sum_{k=1}^{N} R\left[P_t^1, P_t^2, P_t^k(p_{t-1})\right]. \tag{11}$$

Knowing the optimal harvest policy from age t to T, the optimal price interval for harvesting the stand at age t–1, [p_{t-1}^1, p_{t-1}^2], can be determined based on the solution(s) to the following optimization problem.

$$\underset{p_{t-1}}{\text{Min}}\, D(p_{t-1}) = \left| p_{t-1}V(t-1) + L - \delta W(t,p_{t-1}) \right|. \tag{12}$$

Let A and B denote the lower and upper limits of the possible price at age t–1, respectively. According to the results from the theoretical analysis, we know that problem (12) may have two, one, or no solution such that the objective function $D(p_{t-1})$ is approximately zero. In the first case, p_{t-1}^1 is equal to the smaller one and p_{t-1}^2 the larger one of the two solutions. In case there exists only one price (denoted by p_{t-1}^*) such that $D(p_{t-1}^*) \approx 0$, then $p_{t-1}^1 = p_{t-1}^*$, $p_{t-1}^2 = B$ if $V(t-1) \geq \delta\beta V(t)$, and $p_{t-1}^1 = A$, $p_{t-1}^2 = p_{t-1}^*$ if $V(t-1) < \delta\beta V(t)$. Finally, if the optimal objective function value $D(p_{t-1})$ is significantly larger than zero, then $p_{t-1}^1 = A$, $p_{t-1}^2 = B$ if $AV(t-1) + L - \delta W(t,A) \geq 0$, and $p_{t-1}^1 = B$, $p_{t-1}^2 = B$ if $BV(t-1) + L - \delta W(t,B) < 0$.

The assumption on the maximum rotation length implies that one should accept all possible prices and harvest the stand once it reaches age T, i.e., $p_T^1 = A$ and $p_T^2 = B$. Knowing p_T^1 and p_T^2, problem (12) can be solved backwards from age T–1 to t^0 to determine the optimal harvest policy within the range of the permissible harvest ages.

It should be pointed out that the EPV estimated using Equation (11) and consequently the solution to problem (12) depend on the set of price scenarios used in the calculation. The EPV may be over- or underestimated, especially when the number of price scenarios N is small, and thus the price interval for harvesting determined by solving problem (12) might be narrower or wider than the optimal price interval. In other words, different solutions may be obtained when problem (12) is solved repeatedly. Given that the price scenarios are generated randomly each time the problem is solved, the average of the solutions obtained by solving the problem for a number of times would better approximate the optimal harvest policy than a single solution.

Compared with discrete-state stochastic dynamic programming, the simulation method requires more calculations to estimate the EPV associated to different possible prices. On the other hand, the simulation method is applicable whether the range of possible timber price is small or large. Another advantage of the simulation method is that the variance of the present value of the stand at each age can be estimated, and therefore the analysis can be extended to situations in which the forest owner is not risk-neutral (see, Gong, 1998).

Optimization Results

For demonstration purpose, the optimal harvest policy for a *Pinus contorta* stand is determined using stochastic dynamic programming and the simulation method described above. Timber yields are estimated with the following function (Fridh & Nilsson, 1980)

$$V(t) = 630.3744 \times \left(1 - 6.3582^{-t/60}\right)^{2.8967}.$$

It is assumed that only pulpwood is produced. The price process is estimated using the annual real price of pulpwood of pine species during 1970–96 (prices are inflated to 1996's price level using the Swedish consumer price index). The mean real price over this time period is 356.65 SEK/m^3 (Swedish kronor per cubic meter) and the standard deviation is 63.34 SEK/m^3. The estimated price model is

$$p_t = 118.59 + 0.661 p_{t-1} + \varepsilon(t). \qquad \text{(p0)}$$

The standard deviation of the random term is σ = 50.24, and it is assumed that the upper and lower limits of the random term are $\pm 2\sigma$, respectively.

In the calculations, a harvesting cost of 92.3 SEK/m^3 was deducted from timber price. Bare land value is 1500 SEK/ha and the discount rate is 3%. The optimal rotation age under deterministic assumptions is 45 years and the EPV of the stand (at the age of 25 years) is 39931 SEK/ha. In the stochastic analyses, the minimum and maximum rotations were taken as 25 and 65 years, respectively. Moreover, it is assumed that when the stand age is 65 years it will be harvested only if the net revenue is greater than zero.

To test the effects of price autocorrelation, optimal harvest policies are determined using each of the following hypothetical price models, in addition to the base case model (p0).

$$p_t = 356.65 + \varepsilon(t), \quad \sigma = 63.34; \tag{p1}$$

$$p_t = 209.89 + 0.4p_{t-1} + \varepsilon(t), \quad \sigma = 61.36; \tag{p2}$$

$$p_t = 52.47 + 0.85p_{t-1} + \varepsilon(t), \quad \sigma = 35.27; \tag{p3}$$

$$p_t = 34.98 + 0.9p_{t-1} + \varepsilon(t), \quad \sigma = 29.18; \tag{p4}$$

$$p_t = 17.49 + 0.95p_{t-1} + \varepsilon(t), \quad \sigma = 20.91; \tag{p5}$$

$$p_t = 7.0 + 0.98p_{t-1} + \varepsilon(t), \quad \sigma = 13.32; \tag{p6}$$

$$p_t = p_{t-1} + \varepsilon(t), \quad \sigma = 51.45; \tag{p7}$$

For price models (p1) and (p7), the standard deviation of the random term were estimated using the past price series. Price models (p2)–(p6) were constructed in such a way that they are identical with the base case price model with respect to the mean and variance of price in the long-run.

The stochastic dynamic programming model was solved using a price range between 50 and 655 SEK/m^3 in 5 SEK/m^3 intervals. Price state transition probabilities were estimated using the base case price model (p0). The solution shows that at each age it is optimal to harvest the stand when the observed price is equal to or higher than a reservation price associated to that age. This is consistent with the numerical results from previous studies (see, e.g., Lohmander, 1987; Haight & Holmes, 1991). It is also consistent with the analytical results from this study. Given the numerical assumptions, the condition $V(t) > \delta\beta V(t+1)$ is satisfied at all ages $25 \leq t < 65$, and therefore, the optimal decision policy is to harvest the stand when the observed price is equal to or higher than the reservation price at each age. The stochastic dynamic programming solution shows that the reservation price decreases as stand age increases.

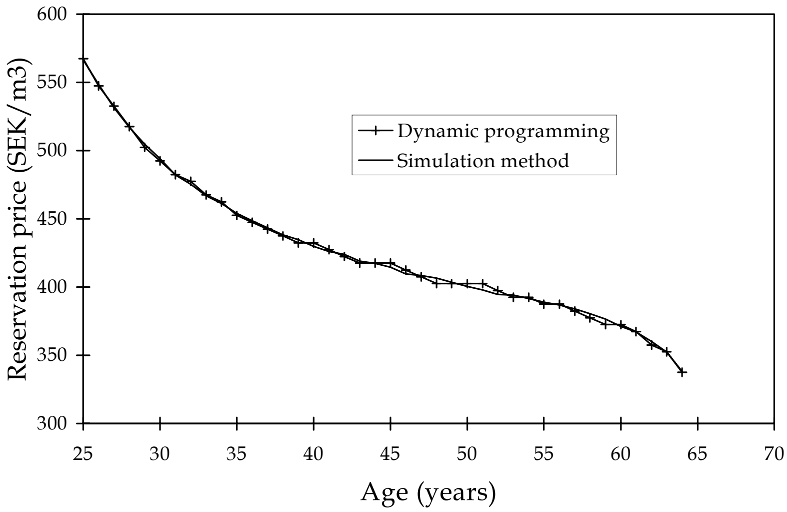

FIGURE 2. OPTIMAL RESERVATION PRICES DETERMINED USING STOCHASTIC DYNAMIC PROGRAMMING AND THE SIMULATION METHOD.

The expected gain of waiting at a lower age is greater partly because the stand is growing faster and partly because one can wait for a longer time to get a high price. Therefore, the optimal reservation price at a lower age is higher.

To examine the performance of the simulation method, problem (12) was solved using 6 different numbers of price scenarios (100, 500, 1000, 2000, 5000, and 10000) to estimate the EPV of the stand. The results show that the solution is robust when the number of price scenarios is 500 or greater. The optimal reservation prices determined using the simulation method (the average of five solutions to problem (12) with 1000 price scenarios) are similar to those obtained using stochastic dynamic programming method, but decrease more smoothly with age (Figure 2). This indicates that the simulation method works very well with a relatively small number of price scenarios.

Figures 3A and 3B presents the optimal reservation prices determined using different price models. With price models (p1)–(p5) and a discount rate of 3%, $V(t) > \delta\beta V(t+1)$ at all permissible harvest ages. This implies that, at each age t, the net revenue from harvesting increases more quickly than the EPV of the stand at age t+1 when the observed price at age t increases. Therefore, an optimal reservation

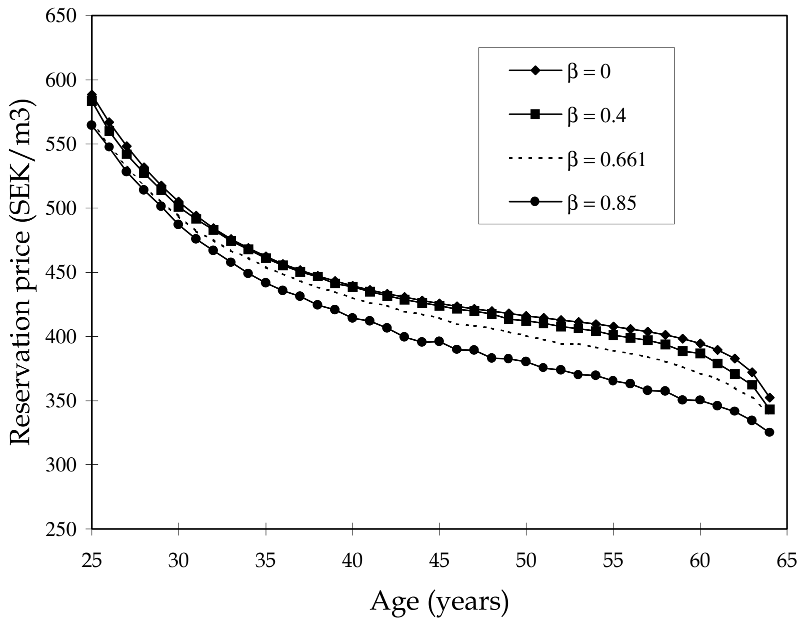

Figure 3A. Effects of Price Autocorrelation on The Optimal Reservation Prices.

price exists for each age t and the optimal decision rule is to harvest the stand if and only if the observed price is equal to or greater than the reservation price. It should pointed out that when the price autocorrelation coefficient $\beta = 0.95$, i.e. with price model (p5), the optimal reservation prices at ages 25–27 years are higher than the maximum possible prices[3]. With price model (p6), the inequality $V(t) > \delta\beta V(t+1)$ holds at ages $t \geq 33$ years. The reservation price strategy (i.e., to harvest when observed price is equal to or higher than the reservation price) is optimal at ages $t \geq 33$. However, the optimal reservation prices at ages 33–35 years (not shown in Figures 3A and 3B) are higher than the maximum possible prices. At ages lower than 33 years, the expected gain of harvesting is always negative. Therefore, the stand should not be harvested before it has reached 36 years. When timber price follows a random walk process (p7), the reservation price strategy is optimal at ages t

[3] It is assumed that the price at age 0 is 349.82 SEK/m^3, which is the long-run average price.

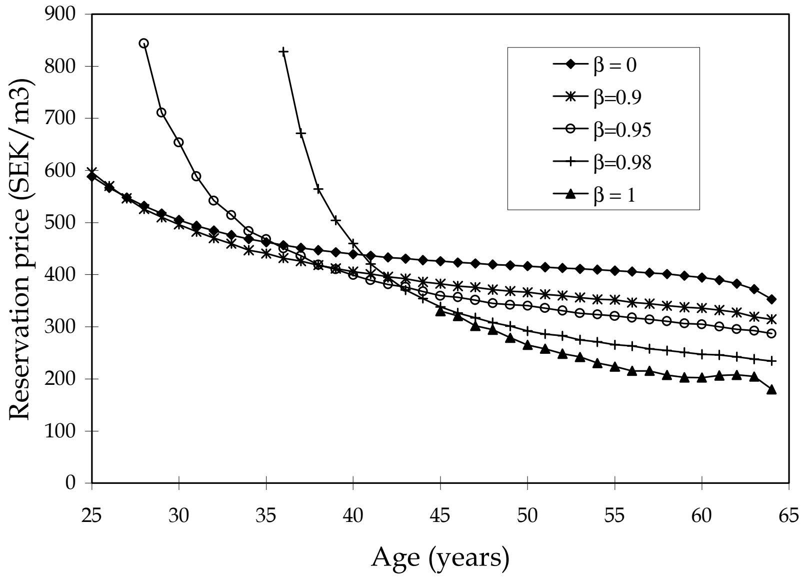

FIGURE 3B. EFFECTS OF PRICE AUTOCORRELATION ON THE OPTIMAL RESERVATION PRICES.

≥ 45 and the stand should not be harvested before it has reached 45 years[4].

The results show that price autocorrelation has significant impacts on the optimal harvest policy. When price autocorrelation coefficient β is relatively small, the reservation price strategy remains optimal over the entire range of permissible harvest ages, but the optimal reservation prices are lower compared with the case in which prices in different periods are independent and identically distributed (Figure 3A). When β is large, the optimal reservation prices are higher at low ages and much lower at high ages than in the case in which prices in different periods are independent and identically distributed (Figure 3B). With

[4] Haight & Holmes (1991) found that with a random walk price model the optimal policy is to harvest when the observed price is less than the age-dependent reservation price. It should be noted that the random walk model used by Haight and Holmes was for logarithm prices. In the original unit ($/Mbf), their price model is $p_t = p_{t-1}\varepsilon_t$, which differs from the price model used in this study ($p_t = p_{t-1} + \varepsilon_t$).

the way the alternative price models were constructed, a larger β value implies less unpredicted variations of timber price. Meanwhile, a large β value implies that if the current price is high (low), then the expected prices in the future will be high (low). Therefore, with autocorrelated prices the optimal reservation price may be higher (lower) than with iid prices when stand age is low (high). When the price autocorrelation coefficient is sufficiently large, it is no longer optimal to follow the reservation price strategy at very low ages. In general, a larger price autocor-relation coefficient implies that the optimal reservation price decreases more quickly when stand age increases.

The EPV of the stand at age 25 years (assuming that price at age 0 equals the long-run average price) as well as the expected gain of the adaptive harvest policy over the fixed rotation age decreases when price autocorrelation coefficient increases (Table 2). A larger price autocorrelation coefficient implies a lower degree of price uncertainty in the short-run, which in turn implies a smaller probability of deviating from the optimal rotation age under the deterministic assumption. Therefore, the expected gain of using the adaptive harvest policy is smaller. Note that the expected gain is larger for the random walk model than for the AR(1) model when β equals 0.98, due to the different degrees of price uncertainty in the two models. In all the cases, the price-adaptive harvest policy is superior to the fixed rotation age. This is consistent with the results of Haight & Holmes (1991).

Washburn & Binkley (1990, 1993) argued that the non-stationarity of "suitably deflated" stumpage prices is a sufficient condition for weak-form market efficiency, which implies no net gain from price-responsive harvesting strategies. According to their definition of weak-form efficiency of stumpage market, the optimal harvest age is independent of price if the price process is random walk. However, these authors neglected the fixed costs (e.g. land rent) of postponing harvesting. If the fixed costs were recognized, then the optimal harvest age would depend on the realized price (Yin & Newman, 1995), and thus the price-responsive harvest policy would be superior to harvesting at a fixed age. Therefore, the non-stationarity of stumpage prices may not be a sufficient condition for an informationally efficient stumpage market. Even if the past

TABLE 2. THE EXPECTED NET PRESENT VALUE AT THE AGE OF 25 YEARS WITH DIFFERENT PRICE MODELS (PRICE AT AGE 0 EQUALS 349.82 SEK/M^3).

Price model	β	EPV (SEK/ha)	Expected gain (%)	Expected rotation age (years)
p0	0.661	50463	26.4	46
p1	0	53231	33.3	45
p2	0.4	52592	31.7	46
p3	0.85	46435	16.3	47
p4	0.9	44467	11.4	47
p5	0.95	41708	4.5	47
p6	0.98	40153	0.6	46
p7	1	41091	2.9	45

stumpage (timber) prices are non-stationary, it does not necessarily mean that the stumpage (timber) market is efficient and one should ignore future price uncertainty in harvest decisions.

On the other hand, it was mentioned in the Introduction section that timber price process interacts with the optimal harvest policy. When every forest owner chooses a price-adaptive harvest policy, the aggregate timber supply becomes more elastic and future price uncertainty may be smaller than the past price variations. As a result, the expected gain from adaptive harvest decision making may be smaller than what is estimated under the assumption that the price model is constant over time.

CONCLUSIONS

This paper examines the optimal adaptive harvest policy when timber price follows an AR(1) process and presents a method for determining the optimal harvest policy using continuous distributions of prices at different ages. The analysis shows that the optimal decision rule depends on the annual timber growth rate (which in turn depends on tree species, site quality and stand age), price autocorrelation coefficient and discount rate. The reservation price strategy is optimal when timber growth rate is lower than a critical level determined by the price autocorrelation coefficient and discount rate. If timber growth rate is higher than this critical level, the optimal decision is either to wait one more period or to harvest when the observed price falls between two reservation prices. Thus, given a price model and a discount rate, the optimal harvest strategy might change as stand age increases. With respect to the harvest decision rule, a random walk price model can be viewed as a special case of an AR(1) model. The optimal harvest policy for a random walk price model has similar properties as the optimal policy for an AR(1) price model, although the optimal harvest strategy changes at different ages. For a random walk price model, the reservation price strategy is optimal when stand age is sufficiently high so that the annual timber growth rate is lower than the discount rate.

Like in situations in which prices in different periods are independent and identically distributed, the optimal harvest policy for an AR(1) or random walk price model can be determined using continuous distributions of prices at different ages. A major advantage of the simulation method demonstrated in this paper is that the variance of the present value of the stand at each age can be estimated, and thus the method can be applied to situations in which the forest owner is not risk-neutral. Numerical results show that price autocorrelation affects the optimal harvest policy and the expected present value of the stand. A larger price autocorrelation coefficient leads to lower optimal reservation prices when stand age is high. When stand age is low, the optimal reservation price (if exists) first decrease then increase as price autocorrelation coefficient increases. The expected present value of the stand and thus the expected gain of using the adaptive harvest policy decreases when price autocorrelation coefficient increases.

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