TIMBER SUPPLY, AMENITY VALUES AND BIOLOGICAL RISK

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ABSTRACT

This paper uses the Kreps-Porteus-Selden non-expected utility approach to study the effects of biological risk on harvesting behavior when forest owners have a quasi-linear utility function, linear in harvest revenue and concave in amenity services. Biological risk is assumed to be associated with either forest growth or the initial forest stock, and it may show up either in multiplicative or additive form. It is shown that a rise in multiplicative forest growth risk increases current but decreases future harvesting, which can be interpreted as a precautionary motive due to riskier return on future harvesting. But higher timber stock risk decreases current timber supply, because it lessens the certainty equivalent value of random-forest stocks and thereby increases the marginal utility of amenity services. Moreover, the source and type of biological risk also matters for the marginal propensity to harvest out of stock. It is between zero and unity for multiplicative stock risk and unity in the other cases. In the former case the marginal propensity to harvest is not usually constant thus suggesting that the size distribution of forest stock affects aggregate timber supply.

Keywords: amenity valuation, biological risk, non-expected utility.

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INTRODUCTION

In his seminal essay Samuelson (1976) pointed out many possible sources of biological risk in forestry management. Besides measurement errors in forest-stand invention and forest-growth estimations, exogenous environmental changes may introduce stochasticity to forestry. These con-

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siderations are more acute now than two decades ago. Even though forestry-planning systems have much developed, measurement error problems still exist. Moreover, external environmental threats have increased. Climate change and acid rain are two major future threats for forestry, and they may also reinforce the traditional damages caused by various forest diseases and insects, or wind and fire. Clearly, forestry is subject to many kinds of biological stochasticities which may show up either in the forest growth or the volume of standing timber stock.

The existence of biological risk raises many questions. First, how does uncertainty affect the allocation of forests into harvesting and amenity service purposes? Second, are there differences in harvesting behavior depending on how the stochasticity impacts? Third, what is the role of the size distribution of forest stocks for aggregate harvesting? Can one expect that the greater the initial volume of timber, the smaller the propensity to harvest out-of-timber stock under biological risk?

While the effects of price risk on timber supply have been studied in many papers (e.g. Brazee & Mendelsohn (1988) in the rotation framework and Koskela (1989) in the two period model), the increasingly important questions of biological risk have been subject only to relatively few studies. Within the Faustmann rotation framework, Reed (1984) and Clarke & Reed (1989, 1990) analyzed how optimal rotation is affected by natural catastrophes and other uncertainties, which may take the form of age-dependent or size-dependent stochastic growth. They, however, considered neither the problem in the amenity valuation setting, nor modeled a distinction between stock and growth uncertainties. [1]

The purpose of this paper is to study the consequences of biological risk within the two period harvesting model for multiple-use forestry where uncertainty enters either via forest growth or via the volume of forest stock. The representative nonindustrial forest owner is assumed to de-

[1] Amacher & Brazee (1997), and Koskela & Ollikainen (1997) analyzed optimal taxation with amenities under perfect foresight and timber price risk respectively in the framework of a two-period model without biological risk.

rive utility from harvest revenue and amenities provided by the forest stocks. To make the analysis more tractable, we will assume that biological risk is normally distributed. This is a natural assumption, for instance, when measurement errors or acidification are concerned.

Depending on whether the forest owner uses harvesting or forest stock as the decision variable, biological stochasticity enters his objective function differently. If he decides upon harvest volume, his consumption is certain but current and future forest stocks will be stochastic. Alternatively, if he decides upon the size of forest stocks, his harvest revenue and, thereby consumption, become stochastic. To make this distinction understandable, think, for instance, about forest stock uncertainty caused by decay, which is typical for spruce forests. Deciding upon a given harvest volume, the remaining forest stock will be random because of uncertainty associated with decay. But if the forest owner decides to conserve a given number of trees and harvests the rest, harvest will be stochastic.

The rest of the paper is organized as follows. We start by presenting the basic framework and assumptions to analyze timber supply and multiple-use under both types of risk. Comparative statics of timber supply as well as the relationship between the marginal propensities to harvest and the size distribution of forest stocks are shown thereafter. Finally, there is a brief concluding section.

TIMBER SUPPLY AND MULTIPLE USE OF FORESTS UNDER BIOLOGICAL RISK: FRAMEWORK

Background

The representative forest owner maximizes his utility from consumption and amenities provided by forest stocks. If the exogenous initial income is denoted by Y, the intertemporal budget constraint can be expressed as

$$c = c_1 + R^{-1}c_2 = p_1 x + R^{-1} p_2 z + Y \,,$$

where $c_1(c_2)$ is current (future) consumption, R=1+r is the interest rate factor and $x(z)$ is current (future) harvesting and p_1 (p_2) is current (future) timber price.

The preferences of the forest owner are represented by an additively separable function of the present value of consumption (c) and current and future amenity services, which depend on the current and future forest stocks (k_1,k_2).

$$V = u(c) + v(k_1) + R^{-1}v(k_2), \tag{1}$$

where $u'(c), v'(k_1), v'(k_2) > 0$ and $u''(c), v''(k_1), v''(k_2) < 0$.[2]

The joint production of timber and amenities is given by

$$k_1 = Q - x \tag{2a}$$

$$k_2 = Q - x + g(Q - x) - z, \tag{2b}$$

where the forest growth function is assumed to be concave, i.e., $g' > 0$ but $g'' < 0$. A typical representative for concave forest growth function is the logistic Lotka-Volterra growth, for which there are two values of k_1, k_1=0 and $k_1 = k_1^0$, at which $g(0) = g(k_1^0) = 0$ so that there is a unique value k_1^* at which $g'(k_1^*) = 0$. In what follows we use this to ensure that $g''' = 0$. This corresponds with the plausible notion that eventually the growth of forest stock ceases, and the decay sets in (see e.g., Clark, 1990, p. 268).

Forestry is subject to biological uncertainty, which may concern either the forest growth or standing timber stock. The following two specifications of risk have been used extensively in theoretical work elsewhere (see Newbery & Stiglitz, 1981, p. 65–66).

1. For any given growth function, we have either *multiplicative risk*

$$\tilde{\theta} g(Q - x), \tag{3a}$$

or *additive risk*

$$\tilde{\theta} + g(Q - x), \tag{3b}$$

where the random variables are assumed to be normally distributed with $N(1, \sigma_\theta^2)$ and $N(0, \sigma_\theta^2)$, respectively.

[2] In what follows the partial derivatives are denoted by primes for functions with one argument and by subscripts for functions with many arguments. Hence, e.g., $u'(c) = \partial u / \partial c$ while $A_x(x, y) = \partial A / \partial x$ etc.

2. Analogously, for a given forest stock we have either *multiplicative risk*

$$\tilde{\varepsilon}Q \tag{3c}$$

or *additive risk*

$$\tilde{\varepsilon}+Q \tag{3d}$$

with $N(1,\sigma_{\varepsilon}^{2})$ and $N(0,\sigma_{\varepsilon}^{2})$, respectively.

Both formulations of risk may be relevant for forest management. Under growth uncertainty, multiplicative risk, while preserving the expected maximum-sustained-yield-point, shifts the growth function vertically up- and downwards as a constant fraction of the stock, and additive risk back- and forthwards in a horisontal axis irrespective of the stock (see Figures 1A. and 1B., where the solid lines denote for the expected growth and dashed lines the

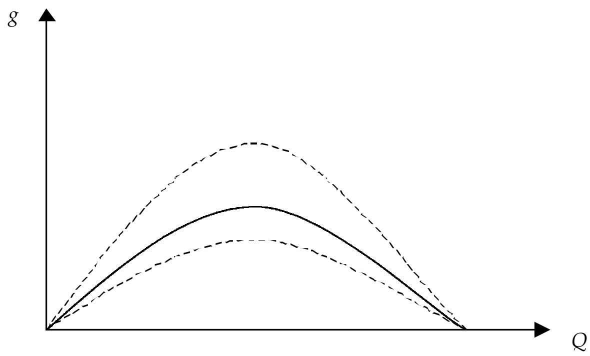

FIGURE 1A. MULTIPLICATIVE GROWTH RISK.

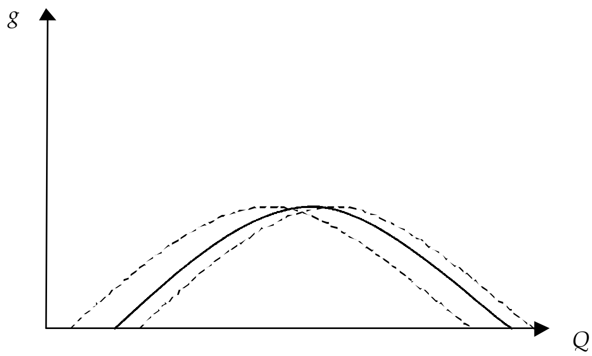

FIGURE 1B. ADDITIVE GROWTH RISK.

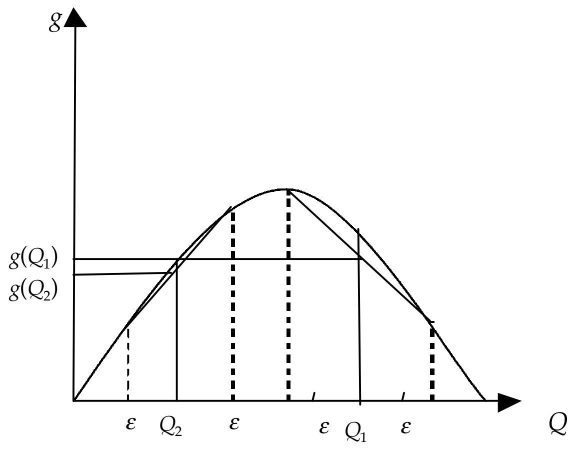

FIGURE 2. ADDITIVE AND MULTIPLICATIVE STOCK RISK.

stochastic variations of the growth). Under uncertain forest stock multiplicative risk means that, for instance, forest decays, insects or acid rain destroy a constant fraction of the forest stock regardless of its size, while additive risk means that they destroy forest stock only in a limited area independently of the total stock (see Figure 2, where the expected values for stock are the solid lines Q_1 and Q_2 and dashed ones describe stochastic variations of the stock. For additive risk, variations are independent of Q but for multiplicative cases risk increases with Q).

As for the description of preferences of forest owners, we adopt the Kreps-Porteus-Selden non-expected utility approach which distinguishes preferences over deterministic consumption and amenity services from preferences towards risk aversion (Kreps & Porteus, 1978, and Selden, 1978). Drawing from Weil (1993) we specify an intertemporal model based on non-expected utility preferences, which is exponential in the risk dimension. This has the advantage that, together with the normality assumption about random variables, a simple and easily interpretable expression for risk attitudes is obtained. To illustrate, consider the case when x and z are decision variables. If the forest growth is uncertain, then the future forest stock k_2 is stochastic $\tilde{k}_2$. If the risk preferences of the forest owner are described by an exponential utility function $W(k_2) = -\exp(-Ak_2)$, where $A = -W''(k_2)/W'(k_2)$ is the Arrow-Pratt measure of constant absolute risk-aversion and k_2 is normally distributed with a mean $\bar{k}_2$ and variance $\sigma^2_{k_2}$, then maximizing $EW(k_2)$ is equivalent to maximizing $\hat{k}_2 = \bar{k}_2 - \frac{1}{2}A\sigma^2_{k_2}$.[3] Now $\hat{k}_2$ is the certainty equivalent value of random future forest stock $\tilde{k}_2$ for a risk-averse agent, i.e., what sure future forest stock k_2 is equivalent to $\tilde{k}_2$ in terms of utility that it yields.

Harvesting versus Forest Stock as the Decision Variable

This section describes the forest owner's decision problem, when he alternatively decides upon harvesting or forest stocks, and faces forest growth and stock risk either of the multiplicative or the additive type.

A. The Forest Owner Chooses Harvesting

When the forest owner decides upon harvesting, the amenity services will be stochastic. Under multiplicative forest growth risk, future forest stock is stochastic and given by $\tilde{k}_2 = Q - x + \tilde{\theta}g(Q-x) - z$. The decision problem can be described as choosing x and z so as to

$$\max_{\{x,z\}} V = u(c) + v(k_1) + R^{-1}v\left(\hat{k}_2\right), \tag{4a}$$

subject to

$$k_1 = Q - x \tag{4b}$$

$$\hat{k}_2 = \bar{k}_2 - \tfrac{1}{2}Ag^2\sigma^2_\theta, \tag{4c}$$

where $c = c_1 + R^{-1}c_2$ is the present value of consumption, $\hat{k}_2$ the certainty equivalent of random future forest stock in the expected utility sense, and $\bar{k}_2 = (Q-x) + g(Q-x) - z$.The Kreps-Porteus-Selden representation (4a) distinguishes the attributes towards risk (the certainty equivalent of amenity services) from intertemporal preferences (the utilities from the present value of consumption c and from amenity services k_1 and $\hat{k}_2$). If the forest growth risk is additive, then the constraint (4c) is replaced by $\hat{k}_2 = \bar{k}_2 - \frac{1}{2}A\sigma^2_\theta$, but the maximization problem remains otherwise the same.

If uncertainty is associated with the initial forest stock, both k_1 and k_2 are stochastic since $\tilde{k}_1 = \tilde{\varepsilon}Q - x$ and

[3] See e.g. Newbery & Stiglitz (1981, pp. 85–92).

$\tilde{k}_2 = \tilde{\varepsilon}Q - x + g(\tilde{\varepsilon}Q - x) - z$ with multiplicative risk and $\tilde{k}_1 = \tilde{\varepsilon} + Q - x$ and $\tilde{k}_2 = \tilde{\varepsilon} + Q - x + g(\tilde{\varepsilon} + Q - x) - z$ with additive risk. In the former case, the problem of the forest owner is to choose x and z so as to

$$\max_{\{x,z\}} V = u(c) + v\left(\hat{k}_1\right) + R^{-1} v\left(\hat{k}_2\right) \tag{5a}$$

subject to

$$\hat{k}_1 = \bar{k}_1 - \tfrac{1}{2} A Q^2 \sigma_\varepsilon^2 \tag{5b}$$

$$\hat{k}_2 = \bar{k}_2 - \tfrac{1}{2} A g'^2 Q^2 \sigma_\varepsilon^2 \tag{5c}$$

where $\hat{k}_1$ and $\hat{k}_2$ are the certainty equivalents of random stocks, and $\bar{k}_1 = Q - x$ and $\bar{k}_2 = (Q - x) + g(Q - x) + \frac{1}{2} g'' Q^2 \sigma_\varepsilon^2 - z$ are the expected values so that $\hat{k}_2 = (Q - x) + g(Q - x) - z + \frac{1}{2}(g'' - A g'^2) Q^2 \sigma_\varepsilon^2$.

For additive risk, the constraints (5b) and (5c) are $\hat{k}_1 = \bar{k}_1 - \frac{1}{2} A \sigma_\varepsilon^2$ and $\hat{k}_2 = \bar{k}_2 - \frac{1}{2} A g'^2 \sigma_\varepsilon^2$, respectively.[4]

B. *The Forest Owner Chooses Forest Stocks*

When periodic forest stocks are decision variables, biological risk is reflected in harvest revenue. For forest growth risk, future harvesting z (and thereby future consumption c_2) is stochastic and given by $\tilde{z} = k_1 + \tilde{\theta} g(k_1) - k_2$ when risk is multiplicative. The forest owner chooses stocks k_1 and k_2 so as to

$$\max_{\{k_1,k_2\}} V = u(\hat{c}) + v(k_1) + R^{-1} v(k_2) \tag{6a}$$

subject to

$$x = Q - k_1 \tag{6b}$$

$$\hat{z} = \bar{z} - \tfrac{1}{2} A g^2 \sigma_\theta^2 , \tag{6c}$$

where $\hat{z}$ the certainty equivalent of future harvesting,

[4] The expected value of $\tilde{k}_2$ can be obtained by using the second-order Taylor approximation. See, e.g., Dudewicz & Mishra (1988, pp. 263–265) on how to approximate the moments when the functions depend nonlinearly on stochastic variables. This leads to applications of Jensen's inequality, see Dudewicz & Mishra (1988, p. 298).

$\bar{z} = k_1 + g(k_1) - k_2$ and $\hat{c} = \bar{c} - \frac{1}{2}AR^{-2}p_2^2 g^2 \sigma_\theta^2$ is the certainty equivalent of the present value of consumption. For additive risk the constraint (6c) is replaced by $\hat{z} = \bar{z} - \frac{1}{2}A\sigma_\theta^2$ so that $\hat{c} = \bar{c} - \frac{1}{2}AR^{-2}p_2^2\sigma_\theta^2$.

Finally, for multiplicative forest stock risk, current harvesting becomes stochastic as $\tilde{x} = \tilde{\varepsilon}Q - k_1$, but future harvesting remains deterministic because $z = k_1 + g(k_1) - k_2$. The forest owner now chooses periodic stocks so as to

$$\max_{\{k_1, k_2\}} V = u\left(\hat{c}^*\right) + v\left(k_1\right) + R^{-1}v\left(k_2\right) \tag{7a}$$

subject to

$$\hat{x} = \bar{x} - \tfrac{1}{2}AQ^2\sigma_\varepsilon^2 \tag{7b}$$

$$z = k_1 + g\left(k_1\right) - k_2, \tag{7c}$$

where $\hat{c}^* = \hat{c}_1 + R^{-1}c_2$, $\bar{x} = Q - k_1$ and $\hat{x}$ is the certainty equivalent of random current harvesting. For additive risk the constraint (7b) is replaced by $\hat{x} = \bar{x} - \frac{1}{2}A\sigma_\varepsilon^2$. Thus we have eight different cases depending on whether the forest owner regards harvesting or stocks as the decision variable and on the source and type of risk.

ANALYTICS OF TIMBER SUPPLY UNDER BIOLOGICAL RISK AND MULTIPLE-USE OF FORESTS

This section is devoted to developing and comparing the resulting harvesting rules and their comparative statics in various cases. We follow the above-presented classification and start with the analysis for forest growth risk.

Harvesting as the Decision Variable

A. Risky Forest Growth

When multiplicative biological risk is associated with forest growth, the first-order conditions for maximization of non-expected utility with respect to x and z can be obtained from (4a)

$$V_x = u'(c)p_1 - v'\left(k_1\right) - R^{-1}v'\left(\hat{k}_2\right)\left[1 + g'\left(1 - Ag\sigma_\theta^2\right)\right] = 0 \tag{8a}$$

$$V_z = u'(c)p_2 - v'(\hat{k}_2) = 0\,. \tag{8b}$$

These can be solved for the *harvesting rule*

$$Rp_1 - p_2(1+g'^*) = \frac{Rv'(k_1)}{u'(c)}, \tag{9}$$

where $g' = g'(Q-x)$ and $g'^* = g'(1-Ag\sigma_\theta^2)$ is the risk-adjusted growth rate under uncertain growth.

Current harvesting reflects the trade-offs between the harvest revenue, amenity valuation and risk aversion. Harvesting is carried out to the point where the difference between the marginal return and the opportunity cost of current harvesting (the LHS term) is equal to the marginal rate of substitution between consumption and amenity services (the RHS term). The size of risk and risk aversion affect the choice via the opportunity cost of harvesting. An increase in forest growth risk or in absolute risk aversion lessens the opportunity cost, and thus makes current harvesting more attractive.

Comparative statics for risky growth is reported in Table 1, columns 2 and 4 for quasi-linear target function.[5] An increase in the original timber stock increases current harvesting one-to-one thus having no effect on future harvesting. This results from the quasi-linear utility function. As the marginal utility is constant in harvest revenue and decreasing in amenity services, the forest owner takes the whole increase in the initial stock as harvest revenue. A rise in the multiplicative growth risk increases current and decreases future harvesting, respectively, if the forest owner is risk-averse. This reflects *precautionary behavior*. Higher growth uncertainty is like a lower expected return, which increases the relative attractiveness of current harvesting.

[5] From now on we will report the results only for quasi-linear preferences where $u'(c)=1$. This reflects our view that the forest owners are interested in the trade-off between harvest revenue and amenity services. This emphasis is present also in the rotation models with recreation, see Hartman (1976). If the utility function is concave in both consumption and amenities, the comparative statics can be decomposed into income and substitution effects. Total effects are generally ambiguous and the substitution effects are qualitatively similar to the overall effects for quasi-linear utility. The results are available from the authors upon request.

For additive growth risk the harvesting rule (9) reduces to

$$Rp_1 - p_2\left(1+g'\right) = \frac{Rv'\left(k_1\right)}{u'\left(c\right)}. \qquad (9')$$

While the marginal propensity to harvest is unity as for multiplicative risk, additive growth risk leaves harvesting unchanged for the reason that harvesting has no effect on risk.

B. Risky Forest Stock

If the initial forest stock is multiplicatively uncertain, the first-order conditions from (5a) with respect to x and z are

$$V_x = u'(c)p_1 - v'\left(\hat{k}_1\right) - R^{-1}v'\left(\hat{k}_2\right)\left[1+g' - Ag''g'Q^2\sigma_\varepsilon^2\right] = 0 \qquad (10a)$$

$$V_z = u'(c)p_2 - v'\left(\hat{k}_2\right) = 0. \qquad (10b)$$

These yield the *harvesting rule*

$$Rp_1 - p_2\left(1+g'^{**}\right) = \frac{Rv'\left(\hat{k}_1\right)}{u'\left(c\right)}, \qquad (11)$$

where $\hat{k}_1 = \bar{k}_1 - \frac{1}{2}AQ^2\sigma_\varepsilon^2$ and $g'^{**} = g'(1 - Ag''Q^2\sigma_\varepsilon^2) > 0$ is the risk-adjusted growth rate under uncertain stock. As under growth risk, the harvesting rule reflects the trade-off between the difference in the marginal return and the opportunity cost of harvesting, the marginal rate of substitution between amenity services and consumption, as well as risk-aversion. Stock risk both raises the opportunity cost of harvesting and the marginal valuation of current forest stock. Consequently, current harvesting is reduced.

Comparative statics are given in Table 1, columns 2 and 4. Compared with the forest growth risk, there are two differences under multiplicative risk. First, a rise in the original timber stock increases current harvesting but by less than one-to-one. The forest owner does not take the whole increase in the initial stock in the form of harvest revenue, but also produces more current amenity services. This prevents the risk associated with future amenity services from becoming too high.[6] Second, higher stock risk decreases current harvesting, while the effect on future harvesting is ambiguous. The certainty equivalence of random forest stocks will lessen so that marginal value of amenity services increases. The forest owner reacts by harvesting less both today and tomorrow as a reflection of *diversification of risk*. The lower current harvesting, however, tends to increase future harvesting so that one cannot a priori determine the total effect of initial stock risk on future harvesting.

For additive forest stock risk, the harvesting rule [11] reduces to

$$Rp_1 - p_2\left(1+g'^{**}\right) = \frac{Rv'\left(\hat{k}_1\right)}{u'(c)}, \tag{11'}$$

where $\hat{k}_1 = \bar{k}_1 - \frac{1}{2}A\sigma_\varepsilon^2$ and $g'^{**} = g'(1 - Ag''\sigma_\varepsilon^2)$. Again timber supply behaves slightly differently under additive stock risk. The main difference is that timber supply is now linear in initial timber stock as under forest growth risk, because current harvesting (x) and initial stock (Q) have symmetric effects on the risk-adjusted growth g'^{**}.

Risk-Adjusted Timber Supply: Forest Stocks as the Decision Variable

We now consider the situation where the forest owner decides upon the size of current and future forest stocks, which makes timber supply, harvest revenue and consumption stochastic. To find out the timber supply responses, first one has to solve for periodic timber stocks and their response to changes in exogenous parameters, and, second, to use these values to develop the comparative statics of risk-adjusted timber supply.[7]

[6] Here we assume that amenity services are positively related to the volume of timber. It is easy to image cases where timber decay might increase amenity services. For instance, biodiversity of forests is positively related to the share of old and decaying timber. This model does not, however, capture directly the maturity of forests.

[7] By the term "risk-adjusted timber supply," we refer to the certainty equivalent value of timber supply, and will denote x and z by hat. We use this concept, because timber supply is now stochastic.

TABLE 1. HARVESTING AND STOCK AS THE DECISION VARIABLES.*

RISK SOURCE/TYPE	MULTIPLICATIVE RISK		ADDITIVE RISK	
Decision variables	Harvesting	Stock	Harvesting	Stock
Forest growth	$x_Q = 1$	$\hat{x}_Q = 1$	$x_Q = 1$	$\hat{x}_Q = 1$
	$x_{\sigma_\theta^2} > 0$	$\hat{x}_{\sigma_\theta^2} > 0$	$x_{\sigma_\theta^2} = 0$	$\hat{x}_{\sigma_\theta^2} = 0$
Forest growth	$z_Q = 0$	$\hat{z}_Q = 0$	$z_Q = 0$	$\hat{z}_Q = 0$
	$z_{\sigma_\theta^2} < 0$	$\hat{z}_{\sigma_\theta^2} < 0$	$z_{\sigma_\theta^2} = 0$	$\hat{z}_{\sigma_\theta^2} < 0$
Forest stock	$0 < x_Q < 1$	$0 < \hat{x}_Q < 1$	$x_Q = 1$	$\hat{x}_Q = 1$
	$x_{\sigma_\varepsilon^2} < 0$	$\hat{x}_{\sigma_\varepsilon^2} < 0$	$x_{\sigma_\varepsilon^2} < 0$	$\hat{x}_{\sigma_\varepsilon^2} < 0$
Forest stock	$z_Q = ?$	$\hat{z}_Q = 0$	$z_Q = 0$	$\hat{z}_Q = 0$
	$z_{\sigma_\varepsilon^2} = ?$	$\hat{z}_{\sigma_\varepsilon^2} = 0$	$z_{\sigma_\varepsilon^2} > 0$	$\hat{z}_{\sigma_\varepsilon^2} = 0$

* See Appendix 2 for the formulas of risk effects and the third section for the initial timber stock effects. A complete set of comparative statics is available from the authors upon request.

A. Risky Forest Growth

When growth risk is multiplicative, choosing periodic forest stocks in the target function (6a) yields

$$V_{k_1} = -u'(\hat{c})\times \left[p_1 - R^{-1}(1+g')p_2\left(1 - AR^{-1}p_2 g\sigma_\theta^2\right)\right] + v'(k_1) = 0 \quad (12a)$$

$$V_{k_2} = -u'(\hat{c})p_2 + v'(k_2) = 0. \quad (12b)$$

These can be solved for the *amenity production rule*

$$Rp_1 - p_2\left(1+g'^{***}\right) = \frac{Rv'(k_1)}{v'(k_2)}p_2, \quad (13)$$

where $g'^{***} = g'(1 - AR^{-1}p_2 g\sigma_\theta^2)$.

The amenity production rule looks qualitatively similar to that for harvesting (Equation [9]). The volume of forest is conserved so as to reflect trade-offs between the difference of the marginal return and opportunity cost, the marginal rate of substitution between current and future amenity services, and risk-aversion. Risk-aversion and uncertainty decrease the opportunity cost so that the certainty equivalent value of current harvesting increases. For additive risk the amenity production rule is reduced to

$$Rp_1 - p_2(1+g') = \frac{Rv'(k_1)}{v'(k_2)} p_2. \tag{13'}$$

Now uncertainty ceases to matter, because harvesting behavior has no effect on risk. Comparative statics results are given in Table 1, columns 3 and 5.[8] The properties of risk-adjusted timber supply are qualitatively quite similar to those when harvesting is the decision variable. Current timber supply is a linear function of the stock Q and precautionary behavior is reflected in higher current and lower future (risk-adjusted) timber supply. For additive risk, current timber supply is linear in Q, but uncertainty decreases only risk-adjusted future timber supply leaving current harvesting unchanged. This is because stock decision has no effect on risk.

B. Risky Forest Stock

Finally, for risky forest stock, the forest owner maximizes target function [7a] with respect to k_1 and k_2.

$$V_{k_1} = -u'(\hat{c}^*)\left[p_1 - R^{-1}(1+g'(k_1))\right] + v'(k_1) = 0 \tag{14a}$$

$$V_{k_2} = -u'(\hat{c}^*)p_2 + v'(k_2) = 0, \tag{14b}$$

where $\hat{c}^* = \hat{c}_1 + R^{-1}c_2$. These yield the *amenity production rule*

[8] Under multiplicative growth risk $k_{1Q} = k_{2Q} = 0$ and $k_{1\sigma_\theta^2} = R^{-1}g'gAp_2k_{1p_1} < 0$ and $k_{2\sigma_\theta^2} = 0$, while under multiplicative and additive stock and additive growth risk, one has $k_{1Q} = k_{2Q} = 0$ and $k_{1\sigma_\varepsilon^2} = k_{2\sigma_\varepsilon^2} = 0$. To find out the risk-adjusted timber supply effects, one has to differentiate equations [6b]-[6c] (and [7b] and [7c]) with respect to exogenous parameters. For instance, differentiating with respect to C gives $\hat{x}_Q = 1 - k_{1Q}$ and $\hat{z}_Q = (1+g')[1 - Ag\sigma_\theta^2])k_{1Q} - k_{2Q}$, and analogously for other cases. Applying the above comparative statics of stands gives the qualitative properties of the risk-adjusted timber supply for initial forest stock and growth risk in Table 1.

$$Rp_1 - p_2(1+g') = \frac{Rv'(k_1)}{v'(k_2)} p_2, \tag{15}$$

which is independent of risk both for multiplicative and additive risk. Changes in the initial stock and uncertainty have no effect, because by deciding upon periodic forest stocks, forest owners produce required future amenity services for certain. However, the certainty equivalent value of current timber supply $\hat{x} = Q - k_1 - \frac{1}{2}AQ^2$ depends negatively on risk, while future supply $z = k_1 + g(k_1) - k_2$ does not (see Table 1, columns 3 and 5).[9]

To recapitulate we have shown that:

Proposition 1. Under biological uncertainty a rise in the multiplicative (additive) forest growth risk increases (has no effect on) current harvesting, while a rise in forest stock risk always decreases it.

Proposition 2. The marginal propensity to harvest is usually unity, but between zero and one for multiplicative stock risk.

Marginal Propensity to Harvest and the Size Distribution of Forest Stocks

According to Proposition 2 the marginal propensity to harvest may be non-constant only for multiplicative forest stock risk. In this case, when harvesting is the decision variable, the marginal propensities to harvest can be written as

$$x_Q = 1 - \frac{R^{-1}v'(\hat{k}_2)e + v''(\hat{k}_1)AQ\sigma_\varepsilon^2}{v''(\hat{k}_1) - R^{-1}v'(\hat{k}_2)a_x} \tag{16a}$$

$$z_Q = b + a(1 - x_Q), \tag{16b}$$

[9] For the effects of timber price risk and interest rate risk on timber supply and its properties, see Koskela & Ollikainen (1998b).The incidence and welfare effects of forest taxes under endogenous timber price risk are analyzed in Koskela & Ollikainen (1998a).

where $a = 1 + g'(1 - Ag''Q^2\sigma_\varepsilon^2) > 0$, $a_x = -g''(1 - Ag''Q^2\sigma_\varepsilon^2) > 0$, $e = 2Ag''g'Q\sigma_\varepsilon^2 < 0$, and $b = Q\sigma_\varepsilon^2(g'' - Ag'^2) < 0$.

Under the assumptions made thus far, comparative statics of (16a) and (16b) with respect to Q gives ambiguous results. Considering some special cases, however, is illustrative. If forest owners are risk-neutral, and forest growth is linear ($A = 0$ and $g'' = 0$), one can see that $x_Q = 1$ and $z_Q = 0$, so that $x_{QQ} = z_{QQ} = 0$. Assuming constant growth, but risk-aversion ($A > 0$ and $g'' = 0$) yields a non-linear response: $x_{QQ} = -A\sigma_\varepsilon^2 < 0$, and $z_{QQ} = AQ\sigma_\varepsilon^2(1 + g' - g'^2) > 0$. Finally, with $A = 0$ and $g'' < 0$, $x_{QQ} = 0$ but $z_{QQ} = \sigma_\varepsilon^2 g'' < 0$.

When forest stocks are the decision variable, one has for risk-adjusted timber supply

$$\hat{x}_Q = 1 - AQ\sigma_\varepsilon^2 < 0 \tag{17a}$$

$$\hat{z}_Q = 0. \tag{17b}$$

so that $\hat{x}_{QQ} = -A\sigma_\varepsilon^2 < 0$. These findings can be summarized as follows.

Proposition 3. Under multiplicative stock risk with harvesting being the decision variable, the marginal propensity to harvest decreases when forest owners are risk averse but forest growth is constant, when forest owners are risk-neutral and forest growth concave. If forest stocks are the decision variable, then the marginal risk-adjusted propensity to harvest decreases.

So there are situations where the marginal propensity is non-constant and has a tendency to decrease. One can then ask what happens if forest owners differ in terms of initial forest stocks? In particular, does the size distribution matter for aggregate timber supply?

It is shown in Appendix 2 that the aggregate timber supply effect as a response to a less equal size distribution depends on the sign of x_{QQ} as follows

$$\frac{\partial X}{\partial I} \leq (>) 0 \quad \text{as} \quad x_{QQ} \leq (>) 0. \tag{18}$$

When the size distribution of initial forest stocks becomes more disperse, aggregate timber supply decreases, remains unchanged or increases depending on whether individual timber supply functions are strictly concave, linear or strictly convex in forest stock, respectively.[10] Clearly under multiple stock risk, one can conjecture on the basis of proposition 3 that current aggregate timber supply might well decrease when the size distribution becomes less equal.

CONCLUDING REMARKS

This paper has studied the effects of biological risk on timber supply behavior and amenity production of forests under the circumstances when forest owners have preferences over harvest revenue and amenity services. We have used the Kreps-Porteus-Selden non-expected utility approach, which distinguishes preferences over deterministic consumption and amenity services from preferences towards risk. Depending on whether the forest owner uses harvesting or forest stock as his decision variable, biological risk enters the target function differently. When harvesting is the decision variable, timber revenues are certain and stochasticity is reflected in the forest stocks and thereby in amenity services, while if he decides upon forest stocks, stochasticity is reflected in harvest revenue and thereby in consumption.

We have shown that both the source and type of risk matter for comparative statics.[11] The difference is most striking for current harvesting. Higher multiplicative growth risk increases current harvesting, and multiplicative stock risk decreases it, while higher additive risk decreases current harvesting only under stock uncertainty. These findings hold regardless of harvesting or stocks being the decision variable. The source and type of risk matters in another way as well. The marginal propensity to harvest out of stock is between zero and unity for multiplicative stock uncertainty, while unity for other cases. In the former case the marginal propensity to harvest is not usually constant thus suggesting that the size distribution of initial forest

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[10] See Carroll & Kimball (1996) for an analysis about when the consumption function under risk is a strictly concave function of income.

[11] In a very different context Newbery & Stiglitz (1981, ch.10) show how a number of results about the effects of commodity-price stabilization schemes depend critically on which assumption is made concerning the type of risk.

stocks matters for aggregate timber supply. In fact there is a conjecture that an increase in the dispersion of size distribution of forest stocks would decrease aggregate timber supply. This is clearly an interesting area for further research.

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APPENDIX 1. COMPARATIVE STATICS FOR MULTIPLE UNCERTAINTY

This appendix gives the comparative statics results for the effects of risk change in the case of multiple growth and stock uncertainty.

A. Harvesting as the Decision Variable

Growth Uncertainty

$$x_{\sigma_\theta^2} = -\frac{g'gAp_2}{\left(1+g'^{*}\right)} x_{p_2} > 0$$

$$z_{\sigma_\theta^2} = -\left(1+g'^{*}\right) x_{\sigma_\theta^2} + N^g < 0$$

Stock Uncertainty

$$x_{\sigma_\varepsilon^2} = \frac{-\frac{1}{2}AQ^2\left(g''R^{-1}v'\left(\hat{k}_2\right)+v''\left(k_1\right)\right)}{\left(1+g'^{**}\right)} x_{p_2} < 0$$

$$z_{\sigma_\varepsilon^2} = -\left(1+g'^{**}\right) x_{\sigma_\varepsilon^2} + N^s = ?$$

where

$$N^g = -\Delta_g^{-1}\left\{\tfrac{1}{2}R^{-1}Ag^2v''\left(\hat{k}_2\right)\left[v''\left(k_1\right)+R^{-1}v'\left(\hat{k}_2\right)\left(g''^{*}-g'^2A\sigma_\theta^2\right)\right]\right\} < 0$$

$$N^s = \Delta_s^{-1}\left\{\tfrac{1}{2}Q^2\left(g''-Ag'^2\right)R^{-1}v''\left(\hat{k}_2\right)\left(R^{-1}v'\left(\hat{k}_2\right)g''+v''\left(\hat{k}_1\right)\right)\right\} < 0 .$$

B. Stocks as the Decision Variable

Growth Uncertainty

$$\hat{x}_{\sigma_\theta^2} = -\frac{R^{-1}g'gAp_2}{\left(1+g'^{*}\right)} x_{p_2} > 0$$

$$\hat{z}_{\sigma_\theta^2} = -\left(1+g'^{*}\right) x_{\sigma_\theta^2} - \tfrac{1}{2}Ag^2 < 0$$

Stock Uncertainty

$$\hat{x}_{\sigma_\varepsilon^2} = -\tfrac{1}{2}AQ^2 < 0$$

$$\hat{z}_{\sigma_\varepsilon^2} = 0$$

APPENDIX 2. THE SIZE DISTRIBUTION OF STOCK AND AGGREGATE TIMBER SUPPLY

The aggregate timber supply can be defined as follows

$$X = \int_{Q_{\min}}^{Q_{\max}} x(Q...)h(Q,I)dQ \;, \tag{A1}$$

where $x(Q...)$ denotes individual timber supply, $h(Q,I)$ the frequency function for the distribution of initial forest stocks, I indicator of the dispersion of the size distribution of forest stocks, and $Q_{\min}$ and $Q_{\max}$ are the biggest and smallest forest stock, respectively.

If the size distribution of forest stock becomes more dispersed, the new cumulative distribution function is initially above and eventually below the original distribution function $H(Q,I)$, i.e., there is a higher frequency of both low and high values of Q. This mean-preserving increase in the dispersion of the size distribution of forest stocks can be formally defined by

$$\int_{Q_{\min}}^{s} H_I(Q,I)dQ > 0, \text{ as } s < Q_{\max} \tag{A2}$$

$$\int_{Q_{\min}}^{Q_{\max}} H_I(Q,I)dQ = 0, \tag{A3}$$

where H_I is the derivative of the cumulative density function H with respect to inequality parameter I (see, e.g., Diamond & Stiglitz, 1974). Equation (A2) describes the dispersion-increasing shift, while (A3) means that the aggregate forest stock does not change. Differentiating (A1) with respect to I yields

$$\frac{\partial X}{\partial I} = \int_{Q_{\min}}^{Q_{\max}} x(Q,...)h_I(Q,I)dQ \;. \tag{A4}$$

Next it is shown how one can move from (A4) to a more easily interpretable Equation (18) of the text. Integrating (A4) by parts with $u = x(Q,...)$ and $h_I(Q,I) = dv$ gives first

$$\begin{aligned}\int_{Q_{\min}}^{Q_{\max}} x(Q,...)h_I(Q,I)dQ &= x(Q,...)H_I\Big|_{Q_{\min}}^{Q_{\max}} - \int_{Q_{\min}}^{Q_{\max}} x_Q(Q,...)H_I(Q,I)dQ \\ &= -\int_{Q_{\min}}^{Q_{\max}} x_Q(Q,...)H_I(Q,I)dQ,\end{aligned} \tag{A5}$$

where $H_I(Q_{\max},I) = H_I(Q_{\min},I) = 0$. Using next $u = x_Q(Q,...)$ and $H_I(Q_{\max},I) = dv$ and again integrating by parts and utilizing (A2) and (A3), one gets

$$\begin{aligned}&-\int_{Q_{\min}}^{Q_{\max}} x_Q(Q,...)H_I(Q,I)dQ = \\ &-x_Q(Q,...)\left(\int_{Q_{\min}}^{s} H_I(s,I)ds\right)\Bigg|_{Q_{\min}}^{Q_{\max}} + \int_{Q_{\min}}^{Q_{\max}} x_{QQ}(Q,...)\left(\int_{Q_{\min}}^{s} H_I(s,I)ds\right)dQ \\ &= \int_{Q_{\min}}^{Q_{\max}} x_{QQ}(Q,...)\underbrace{\left\{\int_{Q_{\min}}^{s} H_I(s,I)ds)\right\}}_{+}dQ,\end{aligned} \tag{A6}$$

which is Equation (17) in the text.