OPTIMAL HARVESTING WITH AUTO-CORRELATED STUMPAGE PRICES

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ABSTRACT

We discuss the theoretical results from the use of Adaptive Management when stumpage prices are formed by a first-order autoregressive process. Random walks and random draws are analyzed as endpoints of a continuum of first-order autoregressive stumpage price process. Building on previous studies, we find that reservation prices exist and are optimal, and that the expected NPV of land and stumpage under Adaptive Management varies directly with the size of the spread of the stumpage price distribution. A key new result is that the size of the gains in the expected NPV of both land and stumpage from the use of Adaptive Management vary directly with the level of reversion to the mean. Using time series analysis, we examine a hardwood stumpage price and a pine stumpage price series in Virginia, USA. We find the series are autocorrelated.

Keywords: Adaptive Management, random draw, random walk, reservation prices, stumpage prices.

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INTRODUCTION

The inclusion of stumpage price risk in the modeling of harvesting decisions has been an active area of research since the late 1980's. A rationale for including stumpage price risk is the magnitude of observed stumpage price fluctuations over time, which are quite large for some commercial species in some regions.[1] Landowners decide to harvest or not to harvest at time t after they learn the stumpage price at time *t*, but before they know future stumpage prices. This process is known as Adaptive Management, and is modeled as a closed-loop, dynamic program or stopping rule problem.

[1] Stumpage price variations can be separated into deterministic, long-term trends, and short-run, random fluctuations.

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An important question is the size of the potential gains in expected net present value (NPV) from using Adaptive Management in response to stumpage price fluctuations. Previous studies indicate that the key to answering the question of potential gains is the underlying structure of the stumpage price fluctuations. The two most common distributional assumptions for stumpage price processes are a random walk and a random draw. In random walk models, the current stumpage price is assumed to fluctuate randomly around last period's or last instant's stumpage price. The potential gains from using Adaptive Management are zero, because the current stumpage is the best predictor of future stumpage prices; that is, the stumpage market is weakly efficient. In random draw models, the current stumpage price is assumed to fluctuate randomly around the mean stumpage price of a stationary distribution. The stumpage market is not efficient, and potential gains for a single landowner using an Adaptive Management strategy are potentially large and depend directly on the size of the fluctuations.[2]

Empirical evidence for stumpage price fluctuations is mixed. For some stumpage price series, the hypothesis of a random walk cannot be rejected (Washburn & Binkley, 1990; Thomson, 1992), while for other stumpage price series, the hypothesis of a random draw cannot be rejected (Lohmander, 1987; Brazee & Mendelsohn, 1988; Haight, 1991; Haight & Smith, 1991; Hultzkrantz, 1995). There are other stumpage price series in which prices are autocorrelated, and both the hypothesis of a random walk and the hypothesis of a random draw may be rejected (Lohmander, 1987; Haight & Holmes, 1991).

In our view random draws or random walks are chosen as null hypotheses because both are easier to work with than autocorrelated stumpage prices. We suspect that some of the series identified as random walks or random draws are probably partially autocorrelated. We base this suspi-

[2] Papers incorporating random walk models include Clarke & Reed (1989), Morck *et al*. (1989), Reed & Clarke (1990), Washburn & Binkley (1990), Thomson (1992), Hultkrantz (1993), Washburn & Binkley (1993) and Reed & Haight (1996). Papers employing random draw models include Lohmander (1987, 1988, 1992), Brazee & Mendelsohn (1988), Haight (1991), Haight & Smith (1991), Teeter & Caulfield (1991), Hultzkrantz (1995) and Forboseh *et al*. (1996).

cion on the presumption given to null hypotheses; that is, the information available on some truly autocorrelated stumpage price series is insufficient to reject a null hypothesis of a random draw or a random walk. Identification and analysis of autocorrelated stumpage prices are underrepresented in the literature. Norstrom (1976), Lohmander (1987), Haight & Holmes (1991), Yin & Newman (1995), Plantinga (1998) and Thorsen in this issue are notable exceptions.

The purpose of this paper is to extend understanding of autocorrelated stumpage prices by presenting an intuitive, non-technical approach to the simplest autocorrelation structure, a first-order autoregressive process. Due to space limitations and a desire for a simple presentation, we discuss and sketch proofs rather than offering formal proofs for propositions.[3] Key propositions are the existence of optimal reservation prices, potential gains from Adaptive Management from varying the spread of the stumpage price distribution, and potential gains from Adaptive Management from varying the degree of "autoregressiveness". To supplement existing analyses of autocorrelated stumpage series, we present two autocorrelated stumpage price series from the State of Virginia, USA. The structure of these series identifies a need for further research on stumpage prices with more complicated autocorrelation structures.

MODEL

The standard formulation of a first-order autogressive price process is (Hamilton, 1994):

$$P_t = A + BP_{t-1} + \varepsilon_t, \tag{1}$$

where A and B are parameters, P_t and P_{t-1} are stumpage prices at time t and t–1, and ε_t is an i.i.d., random disturbance with mean zero. Current stumpage price at time t depends on a constant, the previous price stumpage price, and the stochastic disturbance. Define $a = A/E(P)$, where $E(P)$ is the mean price. Equation (1) becomes:

$$P_t = aE(P) + BP_{t-1} + \varepsilon_t. \tag{2}$$

[3] Formal proofs are available on request from the authors.

Equation (2) demonstrates the influence of both the mean price and the previous price on the current stumpage price, and illustrates both the random walk and the random draw models. If $0 < a$, and $B < 1$, then stumpage prices are formed by a "proper" first-order regressive process. If $a = 0$ and $B = 1$, stumpage prices are formed by a random walk. For a random walk, the mean price has no influence on current stumpage price, and the previous price is the best predictor of the current stumpage price. If $a = 1$ and $B = 0$, stumpage prices are formed by a random draw. For a random walk, the previous price has no influence on the current stumpage price, and the mean price is the best predictor of the current price. If a and B are both positive and sum to 1, then prices form a stationary series, and there is reversion to the mean. This reversion is shown by substituting for previous prices from the current stumpage price, P_t, back to the initial stumpage price, P_0:[4]

$$P_t | P_0 = \sum_{i=1}^{t} \left[aE(P) + \varepsilon_i \right] B^{t-i} + B^t P_0 \tag{3}$$

First-order autoregressive stumpage prices are convenient to analyze, because if the distribution of the random disturbance is known, all information about unknown stumpage price P_t is represented by $E(P)$, P_{t-1}, parameters a and B, and the distribution of the random disturbance. The probability density function and cumulate density function for P_t is conditional only on P_{t-1}, i.e., the p.d.f. and c.d.f. are $f(P_t|P_{t-1})$ and $F(P_t|P_{t-1})$, respectively.

Landowners are assumed to maximize the expected net present value (NPV) from timber production. For simplicity, we will follow most previous studies and assume the landowner starts with bare land. To determine the expected NPV of bare land, it is necessary to predict future prices. Taking the expectations for Equation (3) and modifying subscripts, given P_t, we can predict future stumpage prices P_{t+s}, $s > 0$:

$$E\left(P_{t+s} | P_t\right) = \sum_{i=t+1}^{t+s} \left[aE(P) + E(\varepsilon_i) \right] B^{t+s-i} + B^s P_t. \tag{4}$$

[4] For more information on first-order autoregressive prices, see Hamilton (1994).

Since $E(\varepsilon_t)$ equals zero, the expected stumpage price at time $t{+}s$ given stumpage price t equals a weighted sum of price, P_t, and the mean price, $E(P)$.

With predictions for future stumpage prices possible, and assuming the existence of an optimal set of reservation prices, a landowner's objective is to maximize the value of bare land conditional on the current stumpage price:

$$V\left(0|P_t\right) = -C + \sum_{s=t+1}^{t+N} \frac{1}{(1+r)^{s-t}} \prod_{j=1}^{k-1} F\left(A\left(j|P_{t+j-1}\right)\right) \times \int_{A(k|P_{t+k-1})}^{U} f\left(P_{t+k}|P_{t+k-1}\right)\left[P_{t+k}Y(k) + V\left(0|P_{t+k}\right)\right] dP_{t+k}, \quad (5)$$

where $A(k|P_{t+k-1})$ is the reservation price for age k stumpage given the price at time $t{+}k{-}1$, C is regeneration costs, N is an arbitrarily large, maximum harvest age, r is the discount rate, U is the upper endpoint of the stumpage price distribution, $V(0|P_t)$ is the expected NPV of bare land given the stumpage price in period t, and $Y(k)$ is stumpage volume at age k.

The expected NPV of bare land equals the discounted value of timber harvests and future values of bare land at harvest minus regeneration costs. Note that stumpage prices are conditional on the previous stumpage price, and expected land values are conditional on the stumpage price at the time of harvest. For analytical convenience, we assume that N is the maximum harvest age. Since N may be arbitrarily large, the assumption of a maximum harvest age is made without loss.

Similar to previous studies, an easy way to solve for the optimal reservation price for each combination of age and previous stumpage price is to form functional equations for stumpage of every possible age and previous stumpage price:

$$V\left(k|P_t\right) = \int_{A(k|P_{t-1})}^{U} f\left(P_t|P_{t-1}\right)\left[P_tY(k) + V\left(0|P_t\right)\right] dP_t + \frac{1}{1+r} F\left[A\left(k|P_{t-1}\right)\right] V\left(k+1|P_t\right), \quad (6)$$

where $V(k|P_t)$ is the expected NPV of land with age k stumpage given a stumpage price in period t. In each functional equation the value of land with stumpage age k at time t equals the value of timber harvests and bare land at time t when harvest occurs at time t plus the discounted value of timber harvests and bare land when harvest occurs in the future. In previous random draw studies, the stumpage price distribution is unconditional, and there are N–1 functional equations. Here with the stumpage price distribution conditional on the previous stumpage price, there are $N–1 \times M$ functional equations, where M is the number of discreet stumpage prices.

$N–1 \times M$ first-order necessary conditions follow from differentiating the functional equations with respect to reservation price:

$$P_t Y(k) + V\left(0|P_t\right) = \frac{1}{1+r} V\left(k+1|P_t\right). \tag{7}$$

Harvest is optimal when the harvest revenue of age k stumpage plus the expected NPV of bare land conditional on the current stumpage price equals the discounted expected NPV of land with stumpage age k+1 conditional on current stumpage price. Except for the conditionality on current and previous stumpage prices, these conditions are identical to conditions for random draw results.

For the reservation prices of Equation (7) to be optimal, the expected NPV of bare land must be maximized for every stumpage price. That is, the following M conditions must hold for every price, P_t:

$$V^*\left(0|P_t\right) = -C + \sum_{s=t+1}^{t+N} \frac{1}{(1+r)^s} \prod_{j=1}^{k-1} F\left(A^*\left(j|P_{t+j-1}\right)\right) \times \int_{A(k|P_{t+k-1})}^{U} f\left(P_{t+k}|P_{t+k-1}\right)\left[P_{t+k}Y(k) + V^*\left(0|P_{t+k}\right)\right] dP_{t+k}, \tag{8}$$

where *'s represent optimal levels. This contrasts with previous random draw studies in which only one condition for an unconditionally optimal expected NPV of bare land must hold.

DISCUSSION OF THEORETICAL RESULTS

For both random walk and random draw models, the existence of optimal reservation prices have been previously demonstrated. The only differences in the proofs of the existence of first-order autoregressive prices and randomly drawn prices is the conditionality of the current stumpage price on the previous stumpage price, and the conditionality of the value of land with stumpage age 0 through N on the current stumpage prices. Rather than present a proof of the existence of optimal reservation prices, when stumpage prices are formed by a proper first-order autoregressive process, we refer interested readers to previous proofs of the existence of optimal reservation prices for random draw models. These proofs hold for a first-order autoregressive process, when $f(P_t|P_{t-1})$ is substituted for the probability density function for stumpage prices, $F(P_t|P_{t-1})$ is substituted for the cumulative distribution function for stumpage prices, and $V(k|P_t)$ is substituted for the expected NPV of bare land. A relatively accessible proof of the optimality of reservation prices is found in Forboseh *et al.*, p.66, Equations (19) through (23). Once the existence of an optimal reservation price strategy has been established, then optimal reservation prices can be calculated as presented in Equations (7) and (8).

Note that in some cases with the use of conditional distributions, it is possible to optimally separate stumpage prices into more than two regions. In a random draw model without autocorrelation, optimally there are only two regions of stumpage prices; that is, harvest should never occur below a reservation price, and should always occur above a reservation price. In a random walk model, Thomson (1992) describes three regions; that is, harvest should occur at both low prices and high prices. The rationale of harvesting at the high prices is the common reason, sell high. The rationale of harvesting at low prices is driven by an exogenous value of land; the landowner should harvest and receive an exogenous land value. Thorsen's paper in this issue also presents a model in which harvest should occur at low and at high prices. The intuition underlying these results is to harvest when farm income is low to avoid a higher tax rate on harvest revenues, and to harvest when farm income is high and likely to remain so to maximize harvest revenues. It is possible for the model presented here to show that with positive discounting that there are only two regions of stumpage prices. That is, harvest should never occur below a optimal reservation price, and should always occur above a reservation price.

Similar to random draw models, it can be demonstrated when stumpage prices are generated by a first-order autogressive process, that the expected NPV of bare land and the expected NPV of land occupied by stumpage increase with the spread of the random disturbance term. Although algebraically more complex, similar to random draw models a comparative statics approach is used. The additional complexity arises from the need to include M expected values of bare land conditional on the stumpage price at harvest, rather than one unconditional expected value of bare land.

It is possible to more fully answer the question of the size of the potential gains in expected NPV from using Adaptive Management in response to stumpage price fluctuations. Formally, this question is addressed through the use of comparative statics similar to the spread results discussed in the previous paragraph. After much algebra, it can be shown that, for a given stationary distribution of stochastic disturbances, with $a + B = 1$, the gains in expected NPV using Adaptive Management vary directly with the size of a and inversely with the size of B. Heuristically, this result follows directly from Equation (2). When a is small and B is relatively large, then the stumpage price series is almost a random walk, and the gains in expected NPV from Adaptive Management are small.[5] When a is large and B is relatively small, then the stumpage price series is almost a random draw, and the gains in expected NPV from Adaptive Management are potentially large. Random walk and random draw stumpage price processes are effectively the endpoints of a continuum of autoregressive stumpage price processes. This continuum also helps frame a range for market efficiency from weakly efficient for a random walk to highly inefficient for a random draw process.

[5] Note that optimal reservation prices can be derived even when the stumpage price series is a random walk. This is usually not done because there are no gains from mean-reversion under a random walk.

EMPIRICAL EVIDENCE

We have discussed Adaptive Management when stumpage prices are partially autocorrelated. Two ways to detect autocorrelation in real price data are to verify that prices follow an autoregressive process through time, or show that the process describing prices is not consistent with a random walk.[6] We now investigate the time dependence of prices using quarterly data reported for pine and hardwood stumpage prices in two regions in Virginia. The pine series is from the Piedmont region, where loblolly plantations are more prevalent, while the hardwood series is from the western part of the state.

Previous work has established, albeit with mixed results, that price series of aggregate finished product markets (e.g., lumber) are not stable and may contain unit roots. Given the presence of unit roots, this work has determined that prices in different regional markets may be cointegrated. The discovery of cointegration suggests that the law of one price/market is a valid assumption for forest markets, and, more importantly, that price series should not be disaggregated into individual regions if they are not stable (Jung & Doroodian, 1994; Uri & Boyd, 1990; Murray & Wear, 1998).[7] The consensus from this literature is that only aggregated price series should be used to examine the impacts of policies or shifts in production across sectors.

Understanding the underlying error-generating process for disaggregated, regional, price series should not be abandoned, however. It is more likely that landowners make decisions using information about prices in their region instead of aggregated price series or linear combinations of (cointegrated) regional series. This is especially true for

[6] Of course, even if prices are not autoregressive through time, if the landowner forms adaptive expectations of future prices, then the distribution of prices used by the landowner to compute reservation prices will have autoregressive characteristics.

[7] Others have examined regional prices series. This work has focused on identifying nonstationarity through testing for unit roots. Haight & Holmes (1991) argue that unit roots may be a virtue of quarterly averaging of monthly series. However, Yin & Newman (1996) reject the presence of unit roots for some quarterly price series in the southern U.S. stumpage market. They go further and estimate autoregressive models with several lags to determine if markets are informationally efficient. In this section, we build upon this work to identify appropriate orders for stable price processes, and we determine the appropriate mix of autoregressive and moving average error components.

landowners who do not receive assistance from consultants. For these landowners, understanding and establishing whether independence of prices fails to hold in a disaggregated price series is not only legitimate, but consistent with the analysis here and most previous analyses of stumpage price risk.

To investigate empirical error processes, we use real stumpage price series obtained for two regions of Virginia, the predominantly pine piedmont region in the eastern part of the state, and the predominantly hardwood mountain region in the western part of the state. The series were constructed with the help of the Virginia Tech Extension program, and consist of TimberMart South (TMS) quarterly, seasonally unadjusted, prices for the periods 1975–1995. The series do not suffer from the statewide averaging present in published TMS data after 1993. The procedure to analyze each series involved assessing the stability and testing for the presence of unit roots, making the necessary transformations to ensure stability if unit roots were not discovered, and identifying appropriate moving average or autoregressive orders within the errors of the transformed series.

Pine Series

Initial plots of the data indicated both nonstationarity in the mean and variance. The series was log-transformed, and the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) were obtained. Table 1

Table 1. Sample Autocorrelation Functions (ACF) and Partial Autocorrelation Functions (PACF).

The table presents sample ACF and PACF for pine and hardwood sawtimber prices (pine, hw) and for the first differenced series (Dpine,Dhw). All prices are in log form. ACF and PACF are presented for the first four consecutive lags (standard errors are presented in parenthesis).

Series	ACF	PACF
pine	0.91,0.86,0.80,0.74,0.69 (0.11)	0.91,0.14,0.00,–0.03,–0.01 (0.11)
Dpine	–0.18,–0.20,–0.09,0.01,0.13 (0.12)	–0.18,–0.30,–0.23,–0.17,–.01,–0.10 (0.12)
hw	0.90,0.85,0.83,0.81 (0.11)	0.90,0.16,0.23,0.07 (0.11)
Dhw	–0.18,–0.27,–0.08,0.35, –0.19,–0.17 (0.11)	–0.18,–0.32,–0.24,0.22,–0.16,0.13 (0.11)

TABLE 2. AUGMENTED DICKEY-FULLER UNIT ROOT TEST STATISTIC VALUES.
The table presents augmented Dickey-Fuller unit root test statistic values for pine sawtimber prices in Virginia (pine), first differences (Dpine), hardwood sawtimber prices in Virginia (hw), and first differences (Dhw)). Pine tests include trend term, and the critical value is adjusted. All prices are log-transformed.

Series	Test Statistic	Critical Value (10%)
pine	–3.882	–3.13
hw	–0.854	–2.57
Dpine	–3.79	–3.13
Dhw	–3.79	–2.57

presents the ACF and PACF for several lags, and for both the undifferenced pine prices (*pine*) and first-differenced pine prices (*Dpine*). For the undifferenced series, the sample ACF decays slowly, eventually becoming insignificant within (e.g., within 1.5 standard errors of zero) only at lag twelve. The PACF exhibits a spiked pattern characteristic of an AR(1) model. However, the ACF clearly indicates that differencing might be needed to achieve stationarity.

To determine whether differencing is a valid procedure, augmented Dickey-Fuller tests were conducted. The results are presented in Table 2. These tests reject the presence of a unit root (and thus a random walk) in the original series when a deterministic trend variable is included (this is consistent with the initial data plot for each series). Thus, differencing of the original series is appropriate to remove nonstationarity. For this differenced series (*Dpine*), unit root tests all reject for the presence of a unit root at the 10% level (for both models with and without a trend). Thus, we can conclude that the differenced series is stationary. A plot of the differenced series also indicates some seasonal variation of four-quarter length, which was verified with a Ballot-Bays table (e.g., Wei, 1990, pp. 159).[8]

[8] In fact, the seasonal pattern is not strong but appears to be quarterly-driven. We did not expect a seasonal pattern in Virginia stumpage prices, given that logging is usually possible year round. However, pine is typically found on ridgetop locations in the mountains or sandy unstable soil in the coastal plane. In these cases, logging in wet weather can be problematic. Also, logging can be seasonal given that unobserved mill capacity decisions are seasonal. Thus, later we examine both models that do and do not incorporate a seasonal specification. These are compared using model selection tests appropriate for time series models.

The pattern of the ACF/PACF and the series plot for the differenced pine prices suggest two possible candidate models, an ARIMA(1,1,1) or an ARIMA(1,1,2). The PACF clearly decays to zero as lags increase, with no significant correlates (i.e., >1.5 times the standard error) after the fifth lag. This clearly indicates a moving average process. However, the ACF does not totally rule out autoregressiveness. For example, while there are significant correlates at lags 1-2, 4-5, and 8 (which are consistent with a higher order MA process), there also appears to be a sinusoidal decay present in lags 4-7. Given the plots of the data, this AR component may in fact be seasonal.

Estimation produced the following three candidate models for the differenced log-transformed pine series (P_t^*) (standard errors are in parentheses):[9]

$$P_t^* = \underset{(.21\text{E-}06)}{0.90\text{E}{-}06} + \underset{(0.026)}{0.51}\, P_{t-1}^* + \underset{(0.030)}{0.48}\, P_{t-2}^* + \varepsilon_t - \underset{(0.008)}{0.971}\, \varepsilon_{t-1} \quad (9)$$

(AIC = 37.11; adjR^2 = .98)

$$P_t^* = \underset{(.13\text{E-}06)}{0.10\text{E}{-}04} + \underset{(0.001)}{0.91}\, P_{t-1}^* + \varepsilon_t - \underset{(0.04)}{1.3}\, \varepsilon_{t-1} - \underset{(0.02)}{0.94}\, \varepsilon_{t-2} \quad (10)$$

(AIC = 36.72; adjR^2 = .99)

$$P_t^* = \underset{(0.76\text{E-}07)}{0.17\text{E}{-}04} + \underset{(0.006)}{0.86}\, P_{t-4}^* + \varepsilon_t - \underset{(0.007)}{1.1}\, \varepsilon_{t-1} \quad (11)$$

(AIC = 34.29; adjR^2 = .97)

The first model is an ARIMA(1,1,1), the second is an ARIMA(1,1,2), and the third is an ARIMA(1,1,1) with seasonal AR roots. We also checked the feasibility of higher order AR processes, but as expected these resulted in lower AIC indices and adjusted R^2s. The residuals from each of the regressions above appear to have only white noise variation over time, with residuals centered around a mean of zero. The Akaike information criterion (AIC) and adjusted

[9] Because it was unclear whether a deterministic trend (such as the above models with a constant) or a stochastic trend (with time as a regressor) was appropriate, a stochastic trend model was also estimated. Unfortunately, the stochastic trend model did not work well, with the trend variable capturing most of the variation in the prices over time (a common result, i.e., see Greene, 1997).

R^2 (adjR^2) are highest for the nonseasonal ARIMA(1,1,2) model. The residuals of this model also have skewness and kurtosis parameters that are more favorable than those of the ARIMA(1,1,1) model. Thus, we conclude that the best model is the ARIMA(1,1,2).

Hardwood Series

An identical procedure for identification and estimation was conducted for the Virginia hardwood price series. The process explaining hardwood prices is most likely consistent with a random walk. Table 1 shows the sample ACF for both the differenced and undifferenced series in log form. Table 2 presents augmented Dickey-Fuller test statistics for log-transformed prices. The sample ACF of the undifferenced series, *hw*, persists for several lag lengths, indicating some type of nonstationarity. The spiked PACF of the undifferenced series seems to indicate either an AR or some type of ARMA process,[10] since there are spikes at early lags, some significant spikes at higher lags, and alternating signs across some lags.

Plots of the data, like the pine series, exhibited nonstationarity in both the mean and the variance indicating that log transformation and differencing are both required to remove nonstationarity. However, differencing may not be a valid procedure for hardwood prices; Dickey-Fuller tests indicate the presence of a unit root for the undifferenced series at the 10% level of significance. However, unit roots can be rejected for the differenced series at the 10% level in both models with and without a trend term.

Given that the differenced series appears to be stable, we estimate models applied to the differenced series despite the presence of the unit root for the undifferenced series. The PACF/ACF plots for the differenced series can be explained by an IMA (1,2) or an ARIMA (1,1,2) process. The PACF clearly damps and appears to be sinusoidal, which is consistent with a moving average process. The ACF has some significant and insignificant spikes at early lags, but then starts to damp at lags greater than length

[10] In cases where an ARMA model may be present, it is rare for the AR or MA process to be greater than an order equal to 2 (e.g., see Wei, 1990, Chapter 6). Although higher order ARMA estimates may appear significant, in fact they may be capturing autoregressive or moving average behavior of a lesser order.

four. The significant spikes in the ACF seem to point to a IMA(1,2) process, but the fact that the ACF looks cyclical after those spikes could indicate some autoregressive component. Thus, we estimate both an IMA(1,2) and an ARIMA(1,1,2) model, and we include a constant trend term. Estimation yields the following results,

$$H_t^* = \underset{(.41\text{E-}07)}{0.12\text{E}{-}03} + \varepsilon_t \underset{(0.029)}{-1.591\,\varepsilon_{t-1}} \underset{(0.018)}{-\,0.94\,\varepsilon_{t-2}} \tag{12}$$

(AIC = 32.12; adjR^2 = .91)

$$H_t^* = \underset{(.13\text{E-}06)}{0.11\text{E}{-}04} + \underset{(0.001)}{0.90\,H_{t-1}^*} - \varepsilon_t \underset{(0.032)}{-\,1.31\,\varepsilon_{t-1}} \underset{(0.018)}{-\,0.96\,\varepsilon_{t-2}} \tag{13}$$

(AIC = 36.80; adjR^2 = .99)

where H^* represents the log-transformed first-differenced hardwood price series. Clearly the AIC is in favor of the ARIMA model, which has a highly significant first-order correlation. Residuals for this model also had better estimated skewness and kurtosis sample parameters. A seasonal model was not attempted, primarily because of the unit root problem that existed with the data and the lack of strong evidence against an autoregressive process in the sample ACF/PACF estimates. Moreover, for the differenced series, the residuals for both models above appeared to represent white noise, and the ACF/PACF did not indicate a pattern of seasonality. Seasonal moving average models were not attempted due to the difficulty in interpretation, and because the ACF/PACF did not exhibit significant (>1.5 times the standard error) spikes at four quarter intervals.

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