DIVERSIFICATION OF HARVEST DECISIONS FOR EVEN-AGED STAND MANAGEMENT

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ABSTRACT

Diversification of management alternatives is an important,yet neglected issue in forestry decision making under conditions of uncertainty and when the forest owner is risk-averse. This paper presents a model for determining the optimal diversification strategy for clearcut decisions when future timber prices are stochastic. The model allows to divide an even-aged stand into two parts that can be harvested at different ages. The optimal division of the stand and the optimal harvest age for each part are determined by maximizing the expected utility of the net present value of the stand. Numerical results are presented for two mature Scots pine stands with different site qualities. The analysis assumes that timber prices in different years are independent and identically distributed and the value of bare land is constant and known with certainty. The results show that for each stand the optimal diversification strategy is to divide the stand into two parts of equal size and harvest one part a year later than the other. In comparison with the uniform decision (i.e. to harvest the entire stand at the same time), the diversified harvest strategy can significantly reduce the variance of the net present value at the cost of a slight decrease in the expected net present value. Whether it is optimal to diversify the harvest decision for a stand depends on the size of the stand, the fixed harvest cost, discount rate, and site quality. A sensitivity analysis shows that the degree of risk-aversion does not have any significant impact on the optimal harvest decision.

Keywords: Forest management, risk-aversion, rotation age, uncertainty.

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INTRODUCTION

In recent years interest in stochastic optimization of timber harvest decisions has increased greatly. The optimal time to clearcut an even-aged stand was traditionally formulated as the optimal rotation problem under the assumption that future timber yields and stumpage prices are known with certainty. However, it is widely recognized that both timber yields and stumpage prices in the future are

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stochastic. Because of uncertainty in future stand and market states, it is rarely optimal to harvest a stand at the Faustmann rotation age which is optimal given the certainty assumption. At the time when a stand reaches the Faustmann rotation age, timber prices could be much lower than the original prediction. Then, it would be beneficial for the forest owner to postpone the harvesting of the stand. For the same reason, it might be optimal to harvest a stand before it has reached the Faustmann rotation age. To incorporate future uncertainty into timber harvest decisions, one should determine an adaptive decision policy (a set of decision rules that specify the harvest level conditional on the realized state of nature). For even-aged stand management involving thinning decisions, adaptive decision policies have been developed using stochastic dynamic programming method (Lembersky & Johnson, 1975; Carlsson, 1995). For cases where only clear-cut decision is considered, age-dependent reservation prices (values) have been determined to guide the choice of optimal harvest time (Brazee & Mendelsohn, 1987; Lohmander, 1987; Gong, 1991; Forboseh *et al.*, 1996). In general, the adaptive decision policy can substantially increase the expected net present value (NPV) over the Faustmann rotation.

Another important issue in forest management decision-making under conditions of uncertainty is risk preferences. The adaptive decision models mentioned above implicitly or explicitly assume that the forest owner is risk-neutral. Recognizing that there are forest owners who are risk-averse, decision models have been developed to investigate the effects of risk-aversion on optimal even-aged stand management decisions. Caulfield (1988) incorporated risk-aversion into the rotation decision under fire risk using stochastic dominance analysis. Results from a test case show that the optimal rotation age may be shorter when the forest owner is risk-averse than in the risk-neutral case. Taylor & Fortson (1991) examined the optimal planting density and rotation age under uncertainty in survival rate, stumpage price and timber yields. They found that risk-averse forest owners should choose shorter rotation ages than do risk-neutral owners. Under conditions of uncertainty in annual stand growth, Valsta (1992) showed that the optimal rotation age would be longer if the forest owner is risk-averse and no thinning is allowed; If the rotation age is optimized simultaneously with thinning decisions, then the optimal rotation does not depend on the degree of risk-taking. In a recent study, Gong (1998a) developed a mothod for incorporating risk preferences into the evaluation of adaptive harvest policies for even-aged stand management. He found that the adaptive harvest policy is preferable to the optimal rotation age for risk-averse forest owners.

These studies have in common the implicit assumption that a uniform management program is chosen for all stands with the same characteristics. Optimal decisions are commonly determined based on analysis of the costs and revenues of managing one unit area of a stand, assuming that the optimal decision is independent of the size of the stand. It is widely acknowledged that a risk-averse investor may prefer to diversify an investment among different alternatives. With respect to forestry investments there are several dimensions of diversification, such as tree species, planting density, and rotation age (Gong, 1994). In principle, alternative management programs can be regarded as different investments. When uncertainty is recognized and the forest owner is risk-averse, it may be optimal to choose a different management program for each of the similar stands, or for each part of a single stand. In this case, the optimal decision is a combination of several management programs.

The purpose of this paper is to demonstrate a method for determining the optimal diversification of clear-cut decisions for mature even-aged stands. Timber prices are the only stochastic variable considered in this study. Land value is assumed to be constant and known with certainty. A diversification strategy is formulated by dividing a stand into several parts which can be harvested at different ages. The optimal division of the stand and the optimal harvest ages are determined by maximizing the forest owner's expected utility of the NPV. The diversification problem is solved for two Scots pine stands with different site qualities in northern Sweden. Optimal diversification strategies are compared with uniform (undiversified) harvest decisions. A sensitive analysis is conducted to show the effects of land value, discount rate, fixed harvest cost, and the degree of risk-aversion on the optimal decisions.

THE MODEL

Consider a mature even-aged stand for which the only decision is when to clearcut the stand. Suppose that multiple products (timber assortsments) are produced at harvesting, and that the price of each product is a stochastic variable with a known distribution. The revenue from harvesting is stochastic and the optimal age at which the stand should be harvested depends on the preferences of the forest owner[1]. With risk-neutral preferences, the optimal harvest age can be determined by maximizing the expected NPV of the stand. If the forest owner is risk-averse, it may be optimal to harvest the entire stand at the same time, although the optimal harvest age could differ from the optimal age determined with risk-neutral preferences; It is also possible that the optimal decision is a diversified harvest strategy, i.e. to divide the stand into several parts to be harvested at different ages. In either case, the optimal decision can be determined by maximizing the forest owner's expected utility of the NPV. For this purpose, we need to determine the distribution of the NPV associated with each feasible decision alternative.

Assume that the expected price of the ith product (timber assortment) $E[p_i]$ is constant over time and known with certainty. The obtained price for the ith product when the stand is harvested at time t is:

$$p_i(t) = E[p_i] + e_i(t),$$

where $e_i(t)$, $t = 1,2,...$, represent the random variation of the ith product price at different time points. These are assumed to be independent and identically distributed normal variables[2] $N(0,s_i^2)$. At each time point the prices of different products may be correlated, however.

Let $y_i(t)$ be the yield of the ith product from harvesting 1m^3 of the standing timber at age t. The stumpage price at age t is modelled as:

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[1] When timber prices are stochastic, one can either decide in advance when to harvest the stand, or determine whether to harvest the stand or not at each age conditional on the observed timber prices at that age. This study assumes that the harvest age should be determined in advance.

[2] The assumption makes it easier to calculate the variance of the NPV associated with a diversification strategy. It also implies that the optimal decision is independent of the initial timber prices.

$$p(t) = \sum_{i=1}^{n} y_i(t) p_i(t) - CV,$$

where n is the number of products produced when the stand is harvested at age t, and CV is the constant marginal harvest cost. It follows from the assumption about the timber price processes that the stumpage prices at different ages are independent and normally distributed. The mean and variance of the stumpage price at age t are, respectively:

$$E[p(t)] = \sum_{i=1}^{n} y_i(t) E[p_i] - CV, \tag{1}$$

and

$$\sigma^2_{p(t)} = \sum_{i=1}^{n} \sum_{j=1}^{n} y_i(t) y_j(t) \rho_{ij} \sigma_i \sigma_j, \tag{2}$$

where ρ_{ij} is the correlation coefficient of the prices of products i and j.

Let s be the area of a stand, t^0 be the initial age of the stand, and $v(t)$ the volume of standing timber per ha at age t. Assume that the value of bare land L (SEK/ha) is constant and known with certainty[3]. The NPV of s_t ($0 \le s_t \le s$) ha of the stand harvested at age t ($t \ge t^0$) is:

$$W(s_t, t) = \begin{cases} \{s_t[p(t)v(t) + L] - CF\} e^{-r(t-t_0)} & \text{if } s_t > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where CF is the fixed harvest cost, and r is the discount rate. Given that the area harvested at age t is $s_t > 0$, the NPV of the revenues from harvesting is normally distributed because the stumpage price at any age t is a normal variable. Taking expectations on both sides of Equation (3) yields:

$$E[W(s_t, t)] = \begin{cases} (s_t[E[p(t)]v(t) + L] - CF) e^{-r(t-t^0)} & \text{if } s_t > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

[3] If the land is used for timber production in perpetuity, its value is the expected NPV of future rotations. The value of bare land is independent of the age at which the initial stand is harvested because the latter does not affect the forest owner's choices of silvicultural program and rotation age for future rotations. If the land is to be converted to another use, then L represents the expected value of that use.

The variance of the NPV is:

$$\sigma^2\left[W(s_t, t)\right] = \left[s_t v(t)\right]^2 \sigma^2_{p(t)} e^{-2r(t-t^0)}. \tag{5}$$

To diversify the harvest decision, we divide the stand into two parts[4]. Let (α, t_a, t_b) denote a diversified harvest strategy, where α is the percentage of the stand to be harvested at age t_a and the rest of the stand is harvested at age t_b. The area of the stand harvested at age t_a is $s_a = \alpha s$ and the area harvested at age t_b is $s_b = (1 - \alpha)s$. The NPV of the stand associated with the diversified harvest strategy is the sum of the NPVs of the two parts:

$$\Pi(\alpha, t_a, t_b) = \begin{cases} W(s, t_a) & \text{if } t_a = t_b \\ W(s_a, t_a) + W(s_b, t_b) & \text{otherwise.} \end{cases} \tag{6}$$

The NPV of the stand $\Pi(\alpha, t_a, t_b)$ is normally distributed. The mean and variance of the NPV are, respectively:

$$E\left[\Pi(\alpha, t_a, t_b)\right] = \begin{cases} E\left[W(s, t_a)\right] & \text{if } t_a = t_b \\ E\left[W(s_a, t_a)\right] + E\left[W(s_b, t_b)\right] & \text{otherwise,} \end{cases} \tag{7}$$

and

$$\sigma^2\left[\Pi(\alpha, t_a, t_b)\right] = \begin{cases} \sigma^2\left[W(s, t_a)\right] & \text{if } t_a = t_b \\ \sigma^2\left[W(s_a, t_a)\right] + \sigma^2\left[W(s_b, t_b)\right] & \text{otherwise.} \end{cases} \tag{8}$$

Let $u(x)$, $u'(x) > 0, u''(x) < 0$, be the utility function of the forest owner, where x denotes the realized NPV of the stand. Determination of the optimal harvest decision can be modelled as the following expected utility maximization problem:

$$\underset{\alpha, t_a, t_b}{\text{Max}} \int_{-\infty}^{+\infty} u(x) \frac{1}{\sqrt{2\pi}\,\sigma\left[\Pi(\alpha, t_a, t_b)\right]} \exp\left\{ \frac{-\left(x - E\left[\Pi(\alpha, t_a, t_b)\right]\right)^2}{2\sigma^2\left[\Pi(\alpha, t_a, t_b)\right]} \right\} dx, \tag{9}$$

where the mean of the NPV is defined by Equations (4) and (7), and the variance of the NPV by Equations (5) and (8).

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[4] Extension of the model to accommodate further division of the stand is straightforward.

Problem (9) is solved with the constraints $0 \le a \le 1$ and $t_a, t_b \ge t^0$. If the optimal harvest ages for the two parts of the stand are equal, then the entire stand will be harvested at the same time. Therefore, the optimal solution to problem (9) may be a diversified or a uniform harvest decision. When diversification is not allowed, the optimal harvest age is determined by solving problem (9) with a fixed $a = 1$ and an arbitrary value for t_b.

THE TEST CASE

Optimal harvest decisions are determined for two Scots pine stands (site indices 20 and 24, respectively) in northern Sweden. The initial states of the two stands are shown in Table 1. Two products, sawtimber and pulpwood, are recognized. Sawtimber and pulpwood yields are estimated using the following equations (Gong, 1998b).

$$sr(t) = -3.178 + 8.246 / d(t) - 0.00024441 d^2(t) + 1.02016 \ln d(t),$$
$$pr(t) = 5.3182 - 13.128 / d(t) + 0.00032417 d^2(t) - 1.3754 \ln d(t),$$

where $d(t)$ is basal area weighted average tree diameter at age t. The average tree diameter and the volume of standing timber in the future periods are estimated using the functions of Persson (1992). The distributions of future timber prices are determined based on the real prices (in 1990's value) of sawtimber and pulpwood in northern Sweden from 1968 to 1990. The expected prices of sawtimber and pulpwood are 506.8 and 222.4 SEK / m^3, respectively. The standard deviation is 61.3 SEK / m^3 for sawtimber price and 26.3 SEK / m^3 for pulpwood price. The coefficient of correlation of sawtimber and pulpwood prices is 0.74. The marginal harvest cost is 92.3 SEK / m^3.

The analysis uses a normalized utility function of the following form:

$$u(x) = \frac{1 - \exp\{-bx/100000\}}{1 - \exp(-70b)}, \tag{10}$$

where $b > 0$ is a coefficient that determines the degree of risk-aversion: A larger value of b means a higher degree of risk aversion (see Figure 1).

TABLE 1. THE INITIAL STATE OF THE EXAMPLE STANDS.

Site Index	Age (Years)	Trees/Ha	Basal Area (m^2/ha)	Diameter (cm)	Volume (m^3/ha)
20	80	800	23.5	19.3	187.2
24	70	650	32.1	25.1	296.1

As the base case, the fixed harvest cost is 1765 SEK, stand area is 10 ha, land value equals 1500 SEK/ha for site index 20 and 3000 SEK/ha for site index 24, the discount rate is 3%, and the utility function coefficient equals 0.6. The effects of changing each of these parameters on the optimal harvest decision are determined by sensitivity analysis.

RESULTS

The optimal diversification decision and the optimal uniform decision are determined for each of the example stands with different land values (Table 2). For both stands, the diversified decision is superior to the uniform decision. Instead of harvesting the entire stand at the same time point, it is optimal to divide the stand into two parts of

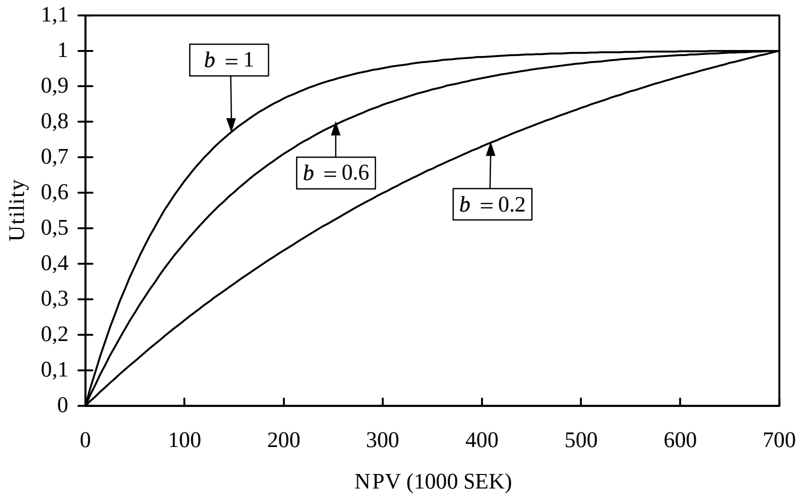

FIGURE 1. UTILITY FUNCTIONS WITH DIFFERENT DEGREES OF RISK-AVERSION.

equal size and harvest one of them a year later than the other. When the land value increases, the opportunity cost of leaving the stand to grow increases, and therefore the optimal harvest age decreases.

The effects of land value on the mean and standard deviation of the NPV associated with the diversified harvest decision are shown in Table 3. As the land value increases, the expected NPV increases for both stands. The land value affects the optimal harvest age and thus indirectly affects the standard deviation of the NPV. With a higher land value, the standard deviation of the NPV may be larger. In comparison with the uniform decision, both the mean and the standard deviation of the NPV associated with the diversified decision are smaller. However, the decrease in the standard deviation is much more significant than the reduction of the expected NPV. The reduction of the expected NPV in percentage of the expected NPV associated with the optimal uniform decision is less than 0.6%. The corresponding reduction of the standard deviation is more than

TABLE 2. EFFECTS OF LAND VALUE ON THE OPTIMAL DIVERSIFIED AND UNIFORM DECISIONS.
Stand area is 10 ha, fixed harvest cost equals 1765 SEK, and the discount rate is 3%.

LAND VALUE	DIVERSIFIED DECISION				UNIFORM DECISION	
	a^*	t_a^*	t_b^*	Utility	Harvest Age	Utility
(SEK/ha)		(Year)	(Year)		(Year)	
Site Index 20						
0	0.5	83	84	0.824	84	0.821
500	0.5	83	84	0.829	83	0.826
1000	0.5	82	83	0.834	83	0.831
1500	0.5	82	83	0.839	82	0.836
2000	0.5	81	82	0.844	81	0.841
2500	0.5	81	82	0.848	81	0.846
3000	0.5	80	81	0.853	80	0.851
Site Index 24						
0	0.5	71	72	0.968	71	0.965
1000	0.5	70	71	0.971	71	0.968
2000	0.5	70	71	0.973	70	0.971
3000	0.5	70	71	0.976	70	0.974
4000	0.5	70	71	0.978	70	0.976
5000	0.5	70	71	0.980	70	0.978
6000	0.5	70	71	0.982	70	0.980

Table 3. Effects of Land Value on the Mean and Standard Deviation of the NPV.

Stand area is 10 ha, fixed harvest cost equals 1765 SEK, and the discount rate is 3%.

Land Value (SEK/ha)	$E[NPV]$ (1000 SEK)	Percent. Reduct.[i]	s[NPV] (SEK)	Percent. Reduct.[ii]	Tradeoff $\Delta s/\Delta E[NPV]$
Site Index 20					
0	281.09	0.56	35835	29.20	9.26
500	285.60	0.57	35835	29.38	9.04
1000	290.16	0.57	35916	29.22	8.95
1500	294.80	0.57	35916	29.36	8.84
2000	299.54	0.57	35974	29.34	8.66
2500	304.32	0.59	35974	29.34	8.32
3000	309.24	0.56	36002	29.30	8.53
Site Index 24					
0	523.24	0.34	63044	29.36	14.49
1000	532.97	0.33	63133	29.26	14.46
2000	542.82	0.34	63133	29.32	14.18
3000	552.68	0.36	63133	29.32	13.13
4000	562.53	0.40	63133	29.32	12.22
5000	572.38	0.38	63133	29.32	11.43
6000	582.23	0.42	63133	29.32	10.74

[i] Percentage reduction of the expected NPV in comparison with the optimal uniform decision.

[ii] Percentage reduction of the standard deviation in comparioson with the optimal uniform decision.

29%. The forest owner can reduce the standard deviation of the NPV by more 8 SEK (10 SEK) by sacrificing 1 SEK expected NPV for the low (high) site index stand (Table 3, the last column).

The effects of discount rate on the optimal harvest age and on the mean and standard deviation of the NPV are shown in Table 4. The optimal division of the stand is to divide it into two parts of equal size (i.e. $\alpha^* = 0.5$) for both stands with different discount rates. The discount rate has significant impacts on the optimal harvest age for both stands: The optimal harvest age decreases as the discount rate increases. The cost of adopting the diversified harvest decision, as measured by the reduction of expected NPV in percentage of the expected NPV of the uniform decision, increases with increasing discount rate. The gain in terms of the reduction of the standard deviation for each unit of sacrificed expected NPV decreases when the discount rate

TABLE 4. EFFECTS OF DISCOUNT RATE ON THE MEAN AND STANDARD DEVIATION OF THE NPV.

Stand area is 10 ha, fixed harvest cost equals 1765 SEK, land value is 1500 SEK/ha for site index 20 and 3000 SEK/ha for site index 24.

DISCOUNT RATE (%)	HARVEST AGE t_a^* (Year)	t_b^* (Year)	E[NPV] (1000 SEK)	PERCENT. REDUCT.[i]	s [NPV] (SEK)	PERCENT. REDUCT.[ii]	TRADEOFF $\Delta s/\Delta E[NPV]$
Site Index 20							
1	143	144	494.61	0.19	57216	29.30	24.97
2	103	104	331.46	0.35	39389	29.34	13.99
3	82	83	294.80	0.57	35916	29.36	8.84
4	80	81	293.00	1.01	35823	29.65	5.04
5	80	80	295.99	0	50918	0	—
Site Index 24							
1	115	116	786.32	0.14	89203	29.29	32.44
2	84	85	592.42	0.22	67539	29.28	20.97
3	70	71	552.68	0.36	63133	29.32	13.13
4	70	71	549.93	0.86	62820	29.67	5.59
5	70	71	547.21	1.35	62512	30.02	3.59
6	70	71	544.51	1.83	62208	30.36	2.67
8	70	70	554.67	0	89323	0	—

[i] Percentage reduction of the expected NPV in comparison with the optimal uniform decision.

[ii] Percentage reduction of the standard deviation in comparioson with the optimal uniform decision.

increases. When the discount rate is sufficiently high, the reduction in the standard deviation of the NPV by diversifying the harvest time is not larger enough to compensate the associated reduction in the expected NPV. Therefore, the uniform decision is preferred to the diversified decision when the discount rate is high.

A critical factor which affects the forest owner's preferences between the diversified and uniform harvest decisions is the fixed harvest cost. If the fixed harvest cost is very low, the difference between the diversified and uniform decisions in the expected NPV would be small. Then, it may be optimal to divide the stand into parts to be harvested at different ages because the variance of the NPV could be reduced significantly by so doing. As the fixed harvest cost increases, the expected NPV of the diversified harvest strategy decreases more rapidly than the expected NPV of the uniform decision, but the fixed harvest cost does

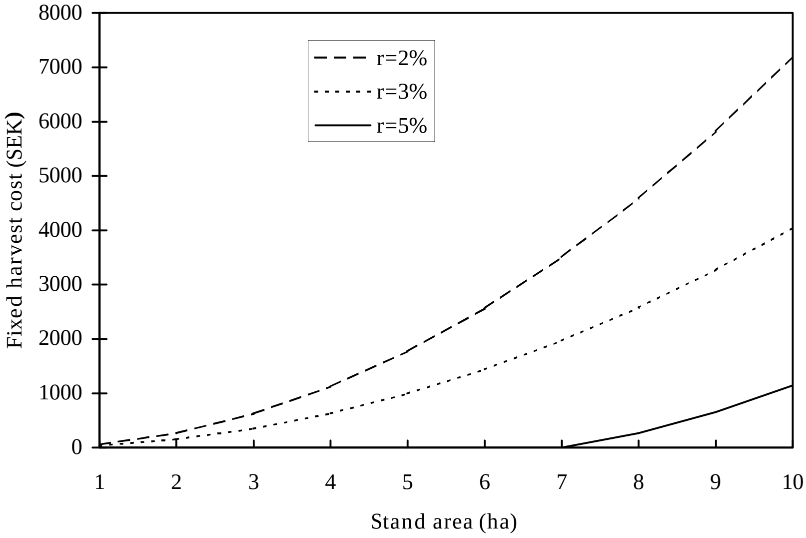

FIGURE 2. THE HIGHEST FIXED HARVEST COST BELOW WHICH IT IS OPTIMAL TO DIVERSIFY THE HARVEST AGE (SITE INDEX 20).

not directly affect the variance of the NPV. Therefore, if the fixed harvest cost is sufficiently high, it would be optimal to harvest the entire stand at the same time rather than harvesting the stand at two different ages and inducing the fixed harvest cost twice.

The highest fixed harvest costs below which the diversified harvest decision is preferred to the uniform decision with different stand sizes and discount rates are shown in Figures 2 and 3. For each discount rate, the highest fixed harvest cost the forest owner can afford to choose the diversified harvest strategy increases as the area of the stand increases for both site qualities. Given the size of the stand, the highest fixed harvest cost below which it is optimal to choose the diversified harvest strategy decreases when the discount rate increases. A comparison of Figures 2 and 3 shows that, for each discount rate and stand size, the highest fixed harvest cost below which it is optimal to diversify the harvest time is much higher for site index 24 than for site index 20. Therefore, whether or not it is optimal to diversify the harvest decision for an even-aged stand depends on the size and site quality of the stand, the fixed harvest cost, and the discount rate.

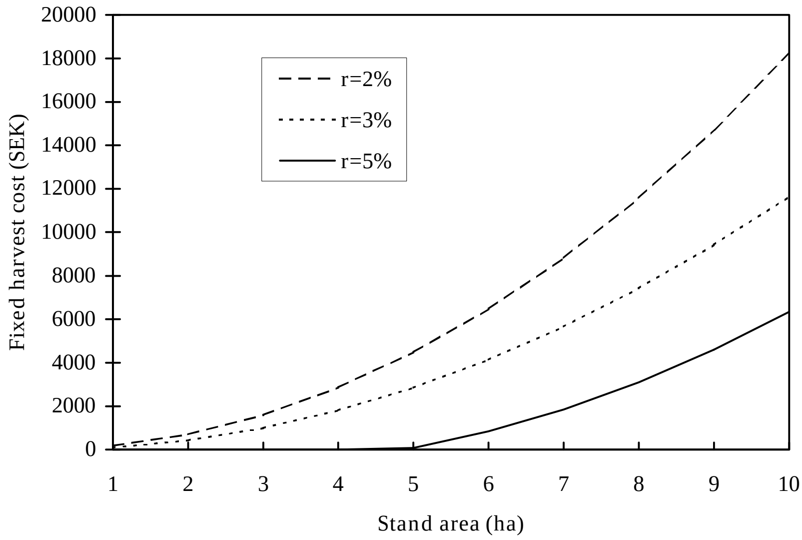

FIGURE 3. THE HIGHEST FIXED HARVEST COST BELOW WHICH IT IS OPTIMAL TO DIVERSIFY THE HARVEST AGE (SITE INDEX 24).

The effects of the degree of risk-aversion on the optimal decision are determined under the base case assumptions (the stand area is 10 ha, fixed harvest cost equals 1765 SEK, land value is 1500 SEK / ha for the site index 20 stand and 3000 SEK / ha for site index 24, and the discount rate is 3%). For the site index 20 stand, it is optimal to harvest the entire stand at the age of 82 years when $b = 0.2$. With higher values of b (higher degree of risk-aversion), the optimal decision is to harvest half of the stand at the age of 82 years and the other half one year later. For the better site quality stand, it is optimal to harvest half of the stand at the age of 70 years and the other half at 71 years with different values of b (0.2, 0.4, 0.6, 0.8, 1). These results show that the degree of risk-aversion has little impacts on the optimal decision. If diversification is not allowed, the optimal harvest age is 82 years for site index 20 and 70 years for the site index 24 stand with different values of b ranging from 0.2 to 1. These harvest ages are also optimal under the assumption of risk-neutral preferences. Given the numerical assumptions, risk-aversion does not affect the optimal harvest age unless the possibility of diversifying the harvest time is incorporated into the decision model.

CONCLUSIONS

Because future stand and market states are not known with certainty, forest management outcomes are stochastic and the optimal decision depends on the forest owner's risk preferences. A risk-averse forest owner may prefer a distribution of outcome with a lower mean and a lower variance to the one which has the maximum expected (mean) outcome. One way to reduce the variation of forest management outcomes is to diversify the management decision, i.e. to choose different management programs for different parts of a forest or stand. This paper shows how the optimal diversification strategy for clearcut decision in even-aged stand management with stochastic timber prices can be determined.

The main conclusion of this study is that risk-aversion could change the optimal harvest strategy, but has little impact on the optimal rotation age. Results from the test case show that, if the forest owner is risk-averse, then it may be optimal to divide an even-aged stand into parts to be harvested at different time points. By using the diversified harvest strategy, the forest owner can reduce the variance of the NPV significantly, although the expected NPV of the stand associated with the diversified harvest strategy is smaller than when the entire stand is harvested at a single time point. The sensitivity analysis shows that whether or not it is optimal to diversify the harvest decision for an even-aged stand depends on the size and site quality of the stand, the fixed harvest cost, and the discount rate. The possibility that it is optimal to diversify the harvest time is larger if the stand is large, the site quality is high, and the fixed harvest cost and discount rate are low. On the other hand, the degree of risk-aversion has little impacts on the optimal decision.

The study deals with a very simple situation, as its purpose is to present a vew way of thinking about the optimal harvest decision problem when the forest owner is risk averse. For practical applications, the decision model presented should be refined in several aspects. First, the model was formulated for determining the optimal harvest decision for a single stand with the possibility of dividing the stand into two parts which can be harvested at different ages. A forest owner typically owns many stands, a number of these can be harvested in the near future. If the forest owner is risk-averse, the optimal harvest decisions for different stands are not independent of each other, and thus should be determined simultaneously. For a large stand, it might be optimal to divide the stand into more than two parts. In practical applications, the model should be extended to include all the stands for which the harvest age need to be determined, and to allow for a division of each stand into a sufficiently large number of parts. Secondly, the model assumes that timber prices in different years are independent and identically distributed. If this assumption is not justified, the model should be modified to distinguish among the price distributions in different periods and to incorporate the effects of serial correlation of prices on the mean and variance of the net present value.

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