RED PINE MANAGEMENT FOR TIMBER, COMMERCIAL SEEDS AND AMENITIES: COMPARING NESTED MODELS

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ABSTRACT

The red pine (pinus koraiensis) is a commonly distributed tree species in northeastern China and Korea. It has a superior wood quality, which is valuable for construction and manufacturing purposes. In recent years, the seeds of this species have been extensively used as a source of delicious food and for medical purposes, and have acquired a considerable commercial value. Without special treatment, seed production is rather limited. It is greatly enhanced by removing the top of the tree at a mature age. Li & Löfgren (2000) derive optimal harvesting rules for such multiple use management of red pine trees. In this paper we amend their results, by introducing amenity values. We explore the optimality conditions, and we conduct a comparative static analysis. In particular, we introduce a technique to compare solutions between economic models that are different, but nested.

Keywords: Amenity values, Faustmann's formula, red pine management.

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INTRODUCTION

The red pine (*pinus koraiensis*) is a commonly distributed tree species in northeastern China and Korea. It has a superior wood quality, which is valuable for construction and manufacturing purposes. In recent years, the seeds of this species have been extensively used as a source of delicious food and for medical purposes, and have acquired a considerable commercial value. However, without special treatment, seed production is rather limited. It is greatly enhanced by removing the top of the tree at a mature age[1]. The critical trade-off is between the production of seeds

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[1] For details see Zhang & Zhang (1990).

and timber, since the removal of the top affects the potential timber production, due to the foregone growth of the removed treetop, and a hampered growth of the remaining stock.

Li & Löfgren (2000) derive optimal harvesting rules for such multiple use management of red pine trees. They give optimality conditions for cutting off the top of the tree, the intensity of removal, and, of course, the optimal date for final felling. The two harvesting dates will indirectly determine the optimal length of commercial seed production. They also derive comparative static results on how the rotation periods and the intensity of removal change, as a function of prices and the interest rate. In this paper we amend the theoretical contribution in Li and Löfgren (2000) by introducing amenity values. In the present context, the introduction of amenity values seems to us even more relevant than in the classical Faustmann rotation problem, which was first extended in this direction by Hartman (1976). The reason is that most people would agree that the removal of the tree top would strongly deteriorate both the beauty and recreation value of the stand. Since the top is removed at a rather mature age, one would expect that explicit consideration of the subsequent reduced amenity value would tend to shorten the optimal rotation age for the final felling.

The key difference between the Li & Löfgren (2000) paper and the present is that here amenities are considered to be produced over the whole period up to final felling, while seed production, by assumption, typically takes place after the top has been removed. In comparison with Hartman's original amenity model, our model generates two rotation periods, one for the top and one for the stem, and both rotation periods will be a function of the shape of the amenity value function.

The similarity with the original Li and Löfgren model, enables us to use some of their results for the comparative statics below. We start by introducing the model and the underlying key assumptions. We continue by exploring its optimality conditions, and we then conduct a comparative static analysis. In particular, we introduce a technique to compare solutions between economic models that are different, but nested. The paper concludes with a discussion of the relevance of our results for the management of other tree species.

THE MODEL

Following previous analyses of the classical rotation problem in forestry (see Faustmann (1849), Samuelson (1976), and Johansson & Löfgren (1985)), we make a few key simplifying assumptions. First, the capital market is perfect in that one can lend and borrow any amount of money at the ruling interest rate. The interest rate, the prices of timber and seeds, the values of amenities, and the production technology are all constant and known with certainty for all future periods. Second, we assume that forestlands can be bought, sold and rented in perfect markets. We make these assumptions to accord with the optimal rotation paradigm and to preserve simplicity. For practical management purposes, the model would have to be modified in accordance with actual conditions such as price trends and their future uncertainties, the imperfection of markets, and the effect of technological development.

We also abstract from restrictions on the cutting capacity or on the composition of the entire forest; this allows us to work at the stand level. Moreover, since all conditions are assumed stationary, the forest will be rotated according to constant rotation periods.

For convenience, we start the analysis at time zero with newly seeded land, cut the top at date t by a fraction α, and then carry out the final felling and regenerate the forest at date T, where $T > t$. During the whole interval $(0,T)$ amenities are produced.

Let q denote the constant unit price of the tree top, p the constant unit price of the main stem, both relative to the seeds price, and $b(t,\alpha)$ the instantaneous production of seeds per unit of the remaining stock. The function $b(t,\alpha)$ is assumed to have the following properties: $b(t,0) = b(t,1)=0$, $b_\alpha(t,0) > 0$, and $b_{\alpha\alpha}(t,\alpha)<0$, i. e. the curve is bell shaped in α for all t. Seed production, as a function of the age of the tree, is assumed to have the following properties[2], $b(0,\alpha) = \lim_{t\to\infty} b(t,\alpha)=0$, and $b_t(0,\alpha) > 0$. The function is,

[2] As one referee points out, however, seed orchards regularly cull older trees.

in addition, assumed to be concave in t, i. e. $b_{tt}(t,\alpha) \leq 0$. These assumptions are only rough assumptions as there are no biological data available that would enable us to play with more delicate nuances[3].

The growth function of the tree is represented by $f(s,d)$, where $d = 0$ for $0 \leq s < t$, and $d = \alpha$, for $t \leq s$. The function is assumed to have the following properties $f \in C^2$, $f_t(t,d) > 0$ and $f_{tt}(t,d) < 0$, for all $t \geq 0$. We will also assume that growth deteriorates with α, and that this takes place at a non-decreasing rate, i.e. $f_\alpha(t,\alpha) < 0$, *and* $f_{\alpha\alpha}(t,\alpha) \geq 0$. It makes some biological sense to assume that the marginal growth with respect to time deteriorates in accordance with the share of the top that is cut, i. e. $f_{\alpha t}(t,\alpha) = f_{t\alpha}(t,\alpha) < 0$. We will also assume that the harm done to the tree by cutting an additional share of the top disappears as the tree grows older, i. e. $\lim_{t \to \infty} f_\alpha(t,\alpha) = 0$[4].

The amenity value function is written $g(s,d)$, where $d = 0$ for $0 \leq s < t$, and $d = \alpha$, for $t \leq s$ .The function is assumed to have the following properties: $g \in C^2$, $g_t(t,d) > 0$ and $g_{tt}(t,d) < 0$, for all t. We will also assume that the amenity value is convex, and does not increase with α, i.e. $g_\alpha(t,\alpha) \leq 0$, and $g_{\alpha\alpha}(t,\alpha) \geq 0$. In other words, we assume that the larger the fraction that is removed from the top, the less is the remaining amenity value, and that the marginal harm done by the removal of the top decreases (does not increase) with the fraction cut. We will, in addition, assume that the marginal harm done by cutting an additional fraction of the top disappears with the age of the tree, i. e. $\lim_{t \to \infty} g_\alpha(t,\alpha) = 0$. Finally, since we have no clear intuition about the sign of the cross derivative, we will assume that $g_{t\alpha}(t,\alpha) = 0$.

Planting and harvesting costs are both assumed to be zero, or, equivalently, that the prices are the net of the planting and harvesting costs.

The net present value of the top fraction cut at time t is $q\alpha f(t,0)e^{-rt}$. We assume that the subsequent growth of the tree is $(1-\alpha)f(t,\alpha)$, and the present value of the final felling at T becomes $p(1-\alpha)f(T,\alpha)e^{-rT}$. In other words, growth de-

[3] The property $b(t,0) = 0$ is unnecessary strong. Nothing fundamental changes if we allow seed production even if the top has not been removed.

[4] This assumption implies $\lim_{T \to \infty} f_{\alpha T} = 0$.

pends only on time and the fraction cut, not the timing of the topping. This is a simplification, since it is conceivable that the timber volume at a certain age after the top has been removed depends on when the top was removed.

The present value of amenities up to the removal of the top fraction sum to

$$\int_o^t g(s,0)e^{-rs}ds$$

and the present value of those created over the period $(t,T]$ can be written

$$\int_t^T g(s,\alpha)e^{-rs}ds\,.$$

Seed production takes place over the interval $(t,T]$, which yields

$$\int_t^T b(s,\alpha)(1-\alpha)f(s,\alpha)e^{-rs}ds=\int_t^T \omega(s,\alpha)f(s,\alpha)e^{-rs}ds$$

in present value. The land is released for a new generation at T, and the process starts all over again. The present value of all future generations can now be written:

$$J=\Big[q\alpha f(t,0)e^{-rt}+\int_0^t g(s,0)e^{-rs}ds+$$
$$\int_t^T\big[\omega(s,\alpha)f(s,\alpha)+g(s,\alpha)\big]e^{-rs}ds$$
$$+p(1-\alpha)f(T,\alpha)e^{-rT}\Big]\Big/1-e^{-rT}\,. \qquad (1)$$

Here the units have been chosen so that the price of seeds and the unit value of amenities are equal to one. The prices of timber should be thought of as relative to the price of seeds (or amenities).

The gradient condition for the optimal removal of the top can be simplified to read:

$$f_t(t,0)+\frac{g(t,0)-g(t,\alpha)}{\alpha q}-rf(t,0)-\frac{\omega(t,\alpha)f(t,\alpha)}{\alpha q}=0. \qquad (2)$$

The first two terms on the right-hand side are together the marginal revenue from letting the tree grow another period dt. The first term is the marginal revenue in terms of additional biomass, and the second term is the gain in net amenity value over the interval dt. Remember that $g(t,0) - g(t,\alpha) > 0$ by assumption. The sum of the third and fourth terms is the marginal cost of keeping the tree intact over the period dt. More precisely, the third term is the interest on the standing tree capital, and the fourth term is the marginal loss of revenue from seed production.

The gradient condition for the optimal fraction to be removed from the top can be simplified to read

$$\begin{aligned} &qf(t,0)e^{-rt}+\int_t^T\left[Q_\alpha(s,\alpha)+g_\alpha(s,\alpha)\right]e^{-rs}ds \\ &\qquad -p\left[f(T,\alpha)-(1-\alpha)f_\alpha(T,\alpha)\right]e^{-rT}=0. \qquad (3)\end{aligned}$$

The first term is the present value of the revenues, in terms of timber, from an addition unit fraction cut at time t. The second term is the present value of the marginal net impact on seed production and amenities from an additional unit fraction cut at time t. The final term is the present value of the marginal loss in timber production at the final felling, from cutting an additional unit fraction at time t. Here

$$Q_\alpha(s,\alpha)=\frac{\partial}{\partial\alpha}\left[\omega(s,\alpha)f(s,\alpha)\right]$$

can have any sign, and $g_\alpha(s,\alpha) < 0$, so the second term is not necessarily positive.

Finally, the gradient condition for the final felling can be rewritten in the following manner:

$$\begin{aligned} &Q(T,\alpha)+p(1-\alpha)f_T(T,\alpha)+g(T,\alpha)- \\ &\qquad r\left[p(1-\alpha)f(T,\alpha)+J(t,\alpha,T)\right]=0. \qquad (4)\end{aligned}$$

Equation (4) tells us that, at the optimal time for final felling, the sum of the value of seed production, the marginal growth of the timber and the amenities equals the interest on the value of the standing timber and the value of land.

COMPARATIVE STATICS

We denote the joint solution to the above problem:

$$\begin{aligned} t^a &= t^a(q,p,r) \\ \alpha^a &= \alpha^a(q,p,r) \\ T^a &= T^a(q,p,r) \end{aligned} \tag{5}$$

From the way we have defined the units, the price vector is given by (q,1,1,p). It is straightforward to show that the solution in (5) is homogeneous of degree zero in prices, i. e. the solution remains unchanged if all prices are scaled by a factor $\mu > 0$.

For $g(t,\alpha) \equiv 0$, the analysis collapses into the one in Li & Löfgren (2000). However, even if amenity values are included in the analysis, it is possible to use the calculations in Li & Löfgren (2000) to derive the comparative static results. By putting $Q(t,\alpha) + g(t,\alpha) = A(t,\alpha)$, the gradient conditions for the optimum can be rewritten in such a manner that they have the same qualitative shape as under standard red pine management, except for an important difference in the gradient condition for the optimal choice of t. The difference means that we are no longer able to sign the cross derivative $J_{t\alpha}$, but we will, to avoid being too taxonomic by treating too many cases, assume that it is positive as in our previous paper.

Following Li & Löfgren (2000), this means that we single out three cases of sufficient conditions

A: $$A\left(T^a,\alpha^a\right) > rJ\left(t^a,\alpha^a,T^a\right)$$

B: $$A\left(T^a,\alpha^a\right) < rJ\left(t^a,\alpha^a,T^a\right)$$

C: $$A\left(T^a,\alpha^a\right) = rJ\left(t^a,\alpha^a,T^a\right)$$

These are nonstandard, since the functions are evaluated at the optimum. In other words, one cannot know in advance which comparative static results will apply. However, as soon as we know the solution we can say something about the comparative statics.

In case A, we are able to sign the effects from an increase of the price of timber. We have:

$$\frac{\partial t^a}{\partial p} < 0, \quad \frac{\partial \alpha^a}{\partial p} < 0, \quad and \quad \frac{\partial T^a}{\partial p} < 0. \tag{6}$$

Under condition B, we can sign the effect from the price of the top. One obtains:

$$\frac{\partial t^a}{\partial q} > 0, \quad \frac{\partial \alpha^a}{\partial q} > 0, \quad and \quad \frac{\partial T^a}{\partial q} < 0. \tag{7}$$

In the knife-edge case C, the results in Equation (6) and (7) are simultaneously valid.

By combining the results in (7), we have in addition in Case B that

$$\frac{\partial (T^a - t^a)}{\partial q} < 0,$$

i.e. an increase in the top price decreases the production period for seeds. It should also be mentioned that the conditions are sufficient, but not necessary. In other words, the results will hold under more general conditions as well.

COMPARING NESTED SOLUTIONS

It is, of course, also interesting to ask how the amenity augmented solution compares with the standard solution to red pine management. For example, does the introduction of an amenity value mean that it typically takes longer before the top is cut, and will the time for final felling also be extended beyond the harvesting age in the standard case? There is no unambiguous answer to this question, and they are difficult to handle, as we are dealing with discrete changes, and not infinitesimals, as is the case with the standard comparative statics. However, since the *standard case is nested* in *the amenity model*, by invoking global conditions for the second derivatives of the objective function, some combinations of qualitative differences can be excluded. To see this, let us look at the signs of the second order derivatives under Cases A, B above. In Case A, the sign configuration of the Hessian can, given our assumptions, be shown to be[5]

$$\begin{bmatrix} J_{tt} & J_{t\alpha} & J_{tT} \\ J_{\alpha t} & J_{\alpha\alpha} & J_{\alpha T} \\ J_{Tt} & J_{T\alpha} & J_{TT} \end{bmatrix} = \begin{bmatrix} - & + & 0 \\ + & - & + \\ 0 & + & - \end{bmatrix}$$

If we are willing to assume that the sign configuration in the above Hessian is valid globally under both models, we can conclude that, conditional on the fraction that is removed and the time for final felling, the optimal time for the removal of the top can be written as

$$t^a = t^a \underset{+}{(\alpha)}$$

where the sign under the argument denotes the sign of the partial derivative with respect to α. The only thing that differs between the first order conditions in the current case including amenities and the standard case with no amenities, is the second term in Equation (2), which is positive. The Hessian in the standard case has the same sign configuration as above, and from the assumed global strict concavity of the objective function in the controls, we can conclude that the following inequality will be valid

$$t^a(\alpha; p, q, r) > t^*(\alpha; p, q, r)$$

for all α >0, where the asterisk denotes that the function is derived from the first gradient solution in the standard case. Note, also that the functions are compared conditional on the same values of the exogenous variables. To simplify the notation, we will not spell out the parameter vector below.

By going over the remaining two gradient conditions in the same manner in both cases, we obtain the following system of inequalities

[5] See appendix. Note that J_{tT} = 0 in optimum since J_t = 0. In other words, it is restrictive to assume that it holds globally.

$$\begin{aligned} &t^a(\underset{+}{\alpha}) > t^*(\underset{+}{\alpha}) \\ &\alpha^a(\underset{+}{t},\underset{+}{T}) < \alpha^*(\underset{+}{t},\underset{+}{T}) \\ &T^a(\underset{+}{\alpha}) > T^*(\underset{+}{\alpha}) \end{aligned} \tag{8}$$

which hold for all t,α,T. Note that the functions are not evaluated at the optimal values of the controls, but the fact that they hold for all values of the controls can be used to exclude certain solution patterns. For example, assume that we want to check whether the following solution pattern is feasible

$$\tilde{t}^a < \tilde{t}^*, \quad \tilde{\alpha}^a > \tilde{\alpha}^*, \quad \tilde{T}^a < \tilde{T}^*, \tag{9}$$

i.e. the removal of the top takes place earlier, the fraction cut is greater, and the final felling takes place earlier, when amenities are considered, than in the standard case. To see that this leads to a contradiction, pick an α such that $\tilde{\alpha}^a > \alpha > \tilde{\alpha}^*$. From the third equation in (8) we know that $T^a(\alpha) > T^*(\alpha)$. Now, plugging in a larger α on the left-hand and a smaller on the right hand side, means that, as the functions are increasing in α, the inequality would continue to hold. This contradicts our suggested solution where the opposite is true. We can conclude that, regardless of the relationship between the time for the removal of the top, the fraction of the top cut under amenities can never be greater under amenities than under the standard case, at the same time as the rotation period for final felling is shorter.

Intuition would tell us that the inclusion of amenities would tend to lengthen the period under which the top is kept intact, that the fraction removed is smaller, and that the time for final felling is extended in comparison with the standard case. In other words, we would expect a relationship between the solution vectors of the following form

$$\tilde{t}^a > \tilde{t}^*, \quad \tilde{\alpha}^a < \tilde{\alpha}^*, \quad \tilde{T}^a > \tilde{T}^*. \tag{10}$$

It is easy to show that (10) is not inconsistent with (8). This is, however, a rather weak conclusion, since it does not mean that the solutions have this property, only that this property cannot be excluded. If we exclude equalities[6], there are eight possible solution combinations. They are:

(1) $\tilde{t}^a > \tilde{t}^*, \quad \tilde{\alpha}^a > \tilde{\alpha}^*, \quad \tilde{T}^a > \tilde{T}^*$

(2) $\tilde{t}^a > \tilde{t}^*, \quad \tilde{\alpha}^a > \tilde{\alpha}^*, \quad \tilde{T}^a < \tilde{T}^*$

(3) $\tilde{t}^a > \tilde{t}^*, \quad \tilde{\alpha}^a < \tilde{\alpha}^*, \quad \tilde{T}^a < \tilde{T}^*$

(4) $\tilde{t}^a < \tilde{t}^*, \quad \tilde{\alpha}^a < \tilde{\alpha}^*, \quad \tilde{T}^a < \tilde{T}^*$

(5) $\tilde{t}^a < \tilde{t}^*, \quad \tilde{\alpha}^a > \tilde{\alpha}^*, \quad \tilde{T}^a > \tilde{T}^*$

(6) $\tilde{t}^a < \tilde{t}^*, \quad \tilde{\alpha}^a < \tilde{\alpha}^*, \quad \tilde{T}^a > \tilde{T}^*$

(7) $\tilde{t}^a < \tilde{t}^*, \quad \tilde{\alpha}^a > \tilde{\alpha}^*, \quad \tilde{T}^a < \tilde{T}^*$

(8) $t^a > \tilde{t}^*, \quad \tilde{\alpha}^a < \tilde{\alpha}^*, \quad \tilde{T}^a > \tilde{T}^*$

Under Case A, we can exclude combination (2), (5), and (7), while all other combinations are consistent with the inequalities in Equation (8). More formally, the theory under Case A is refuted if we observe any of these combinations.

If we assume that Case B is globally valid the sign of $J_{\alpha T} = J_{T\alpha}$ is negative, and the new sign configuration of the Hessian reads

$$\begin{pmatrix} - & + & 0 \\ + & - & - \\ 0 & - & - \end{pmatrix}$$

and the system of inequalities corresponding to Equation (8) changes to

$$\begin{aligned} &t^a(\underset{+}{\alpha}) > t^*(\underset{+}{\alpha}) \\ &\alpha^a(\underset{+}{t}, \underset{-}{T}) < \alpha^*(\underset{+}{t}, \underset{-}{T}) \\ &T^a(\underset{-}{\alpha}) > T^*(\underset{-}{\alpha}) \end{aligned} \tag{8a}$$

[6] There are no technical problems to include both strong and weak inequalities.

It is straightforward to show that combinations (4), (5), and (7), are inconsistent with the inequalities in (8a). Finally, under the knife edge case, Case C, the cross-derivatives $J_{\alpha T} = J_{T\alpha}$ are zero in optimum, but globally this will only hold under extremely simplifying assumptions, so we have chosen to skip the corresponding analysis in this case. The combinations that are permissible under both Case A and B are:

(1) $\tilde{t}^a > \tilde{t}^*, \quad \tilde{\alpha}^a > \tilde{\alpha}^*, \quad \tilde{T}^a > \tilde{T}^*$

(3) $\tilde{t}^a > \tilde{t}^*, \quad \tilde{\alpha}^a < \tilde{\alpha}^*, \quad \tilde{T}^a < \tilde{T}^*$

(6) $\tilde{t}^a < \tilde{t}^*, \quad \tilde{\alpha}^a < \tilde{\alpha}^*, \quad \tilde{T}^a > \tilde{T}^*$

(8) $t^a > \tilde{t}^*, \quad \tilde{\alpha}^a < \tilde{\alpha}^*, \quad \tilde{T}^a > \tilde{T}^*$

We note two things, the rotation period under amenities will typically (in three out of four cases) exceed the one in the standard case. One reason for this is that we have assumed that amenities increase with the age of the standing trees.[7] Moreover, we note that the fraction cut under amenities can only be larger if the top is kept longer than in the standard case. The intuition is that since the amenity value deteriorates with the fraction cut, a larger fraction cut can, under certain conditions, be compensated by an increase in the length of time the top is kept intact. In combination, (3), when the rotation period under amenities is shorter than in the standard case, the fraction cut under amenities is smaller than in the standard case.

Finally, given our assumptions, the following combinations can never be optimal:

(5) $\tilde{t}^a < \tilde{t}^*, \quad \tilde{\alpha}^a > \tilde{\alpha}^*, \quad \tilde{T}^a > \tilde{T}^*$

(7) $\tilde{t}^a < \tilde{t}^*, \quad \tilde{\alpha}^a > \tilde{\alpha}^*, \quad \tilde{T}^a < \tilde{T}^*$

In other words, it is never optimal under "conditions A and B" to remove the top earlier and cut a larger fraction than in the standard case.

[7] For a technique on how to compare the Faustmann and Hartmann rotation periods see Johansson & Löfgren (1988).

CONCLUDING COMMENTS

Although the topic of this paper is rather narrow important insights can be gained on the economic principles behind the optimal program for when the top of the tree should be cut to maintain both amenity values and seed production. These findings may be relevant for the joint production of other natural resources. For example, latex production and then lumber from the rubber trees in Central America and South-East Asia, pinyon pine nuts and then aromatic fuel-wood from the pinyon pine tree in the American South-West. They could also be relevant for maple syrup and later high quality hardwood lumber from the maples in the Northeast USA and Canada[8].

Further, the technique used here to narrow down the number of possible outcomes, despite being fairly simple, not incredibly sharp, and, perhaps, not fundamentally novel, may nevertheless also be applicable for much more general purposes. It would in principle work for all models that are nested, and it will generate test implications that are not generated by standard comparative statics.

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APPENDIX

First, we attempt to determine the signs of the second-order partial derivatives, which enter the Hessian matrix H. If we evaluate the derivatives at the optimum, we know from the fact that we are dealing with an optimum that $Sign J_{tt} = Sign J_{\alpha\alpha} = Sign J_{TT} < 0$. A straightforward derivation of the three first-order conditions (2), (4) and (6) gives that

$$J_{t\alpha} = \frac{\left[q\left(f_t(t,0) - rf(t,0)\right) - A_\alpha(t,\alpha)\right]e^{-rt}}{\left(1-e^{-rT}\right)}$$

$$J_{tT} = 0$$

$$J_{\alpha T} = \frac{\left\{A_\alpha(T,\alpha) - p(1-\alpha)\left(rf_\alpha(T,\alpha) - f_{\alpha T}(T,\alpha)\right) - p\left(f_T(T,\alpha) - rf(T,\alpha)\right)\right\}e^{-rT}}{1-e^{-rT}}$$

They are all evaluated at the optimum solution point $\Psi^* = (t^*, \alpha^*, T^*)$. The other elements in H can be easily obtained by invoking Young's theorem, yielding $J_{\alpha t} = J_{t\alpha}$, $J_{\alpha T} = J_{T\alpha}$ *and* $J_{Tt} = 0$. For $J_{\alpha T}$ the sign cannot be determined without further restrictions, since it depends on where the optimal solution lies. From the assumptions about the asymptotic properties of $g_\alpha(t,\alpha)$, $f_\alpha(T,\alpha)$ *and* $\omega_\alpha(T,\alpha)$, it follows that $A_\alpha(T,\alpha)$ goes to zero when $T \to \infty$. We will assume that its magnitude is of second order also at T^*. For the same reason $f_{\alpha T}(T^*,\alpha^*) \approx 0$. This means that the sign $J_{\alpha T}$ depends on the sign of

$$p(1-\alpha)\left\{f_T(T,\alpha) - rf(T,\alpha)\right\} = rJ(T,\alpha) - A(T,\alpha),$$

where the equality follows from the first order condition (4). If this expressions is negative we have $J_{\alpha T} > 0$ (Case A), and if the opposite holds $J_{\alpha T} < 0$ (Case B). It should be remembered that although seemingly sharp, these conclusions are only valid if T^* is large enough.

Next, we sign the vectors $v(p)$, $v(q)$ and $v(r)$ in (10) whose elements are given by

$$J_{tq}=\frac{\alpha\left(f_t(t,\alpha)-rf(t,\alpha)\right)e^{-rt}}{1-e^{-rT}}$$
$$J_{\alpha q}=\frac{f(t,\alpha)e^{-rt}}{1-e^{-rT}}$$
$$J_{Tq}=-\frac{r\alpha f(t,\alpha)e^{-r(t+T)}}{\left(1-e^{-rT}\right)^2}$$
$$J_{tp}=0$$
$$J_{\alpha p}=\frac{\left[(1-\alpha)f_\alpha(T,\alpha)-f(T,\alpha)\right]e^{-rT}}{1-e^{-rT}}$$
$$J_{Tp}=\frac{\left((1-\alpha)f_T(T,\alpha)-rf(T,\alpha)\right)e^{-rT}}{1-e^{-rT}}-\frac{(1-\alpha)rf(T,\alpha)e^{-2rT}}{\left(1-e^{-rT}\right)^2}$$
$$J_{tr}=\frac{-q\alpha f(t,\alpha)e^{-rt}}{1-e^{-rT}}$$
$$J_{\alpha r}=\frac{-tqf(t,\alpha)e^{-rt}-\int_t^T A_\alpha(T,\alpha)se^{-rs}ds+pT\left\{f(T)-f_\alpha(T,\alpha)\right\}e^{-rT}}{1-e^{-rT}}$$
$$J_{Tr}=-\frac{(1-\alpha)pf(T,\alpha)+J+rJ_r}{e^{rT}-1}$$

which are all evaluated at the optimum (t^*,α^*,T^*). Since the date t^* for removing the top is determined by

$$f_t(t,0)=rf(t,0)+\omega(t,\alpha)f(t,\alpha)/\alpha q\,,$$

we have $f(t,\alpha)-rf(t,\alpha)>0$, so that $J_{tq}>0$. From the expressions above, it is obvious that $J_{\alpha q}>0$, $J_{Tq}<0$, $J_{\alpha p}<0$ and $J_{tr}<0$. If $rJ(T,t,\alpha)>A(T,\alpha)$ {Case B} we have $J_{Tp}>0$, provided we can neglect the influence of the last term, which is small. Strictly speaking, however, the sign is ambiguous. Under Case A, we have $J_{Tp}>0$. Moreover, $J_{\alpha r}$ and J_{Tr} cannot, in general, be signed.

With the above expressions, we can easily sign the elements in the inverse matrix (9). Since J is jointly concave in the variables (t,α,T), it is true that $J_{tt}J_{\alpha\alpha}>J_{T\alpha}^2$ and $J_{TT}J_{\alpha\alpha}>J_{T\alpha}^2$, i.e. the main effect dominates the cross-effect. Recall that $|H|<0$ for our maximization problem, we can then express the signs of H^{-1} as

$$sign\left(H^{-1}\right)=\begin{bmatrix}- & - & -\\ - & - & -\\ - & - & -\end{bmatrix}$$

for Case A, and

$$sign\left(H^{-1}\right)=\begin{bmatrix}- & - & +\\ - & - & +\\ + & + & -\end{bmatrix}$$

for Case B.